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Electricity Markets: Designing Auctions Where Suppliers Have Uncertain Costs

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Electricity markets: Designing auctions where suppliers have uncertain costs.*

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Abstract

We analyze how market design influences the bidding behavior in multi-unit procurement auctions where suppliers have uncertain costs and are uncertain about the availability of production units, as in wholesale electricity markets. We find that the competitiveness of market outcomes improves with increased market transparency. We identify circumstances where the auctioneer prefers uniform to discriminatory pricing, and vice versa. We also identify circumstances where it should be market efficiency enhancing to restrict the number of steps in the bid-schedules.

Key words: cost uncertainty, asymmetric information, uniform-price auction, discriminatory pricing, market transparency, wholesale electricity market, treasury auction, bidding format, Bayesian Nash equilibria

JEL codes: C72, D43, D44, L13, L94

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1 Introduction

We analyze multi-unit auctions where producers submit offers before demand, production capacities and production costs are fully known. Our model accounts for asymmetric information in suppliers' production costs and considers unexpected outages and intermittent output, such as those from renewable energy sources. Our analysis is, for example, of relevance for European wholesale electricity markets, where the European Commission has introduced regulations that increase the market transparency, so that uncertainties and information asymmetries are reduced. According to EU No. 543/2013, the hourly production in every single plant should be published. EU No. 1227/2011 (REMIT) mandates all electricity market participants to disclose insider information, such as scheduled availability of plants. We are interested in how such regulations and the auction format influence the competitiveness of the resulting market outcomes when suppliers face various uncertainties about their competitor's costs and their own output. Our results are also applicable to multi-unit auctions that, for example, trade securities and emission permits.

In electricity markets, marginal cost can be estimated from public engineering data on plant characteristics and input fuel price indexes. Still, a generation unit owner has private information about the actual price paid for its input fuel and how the plant is maintained and operated, which creates cost uncertainty about a firm's competitors. We believe that the cost uncertainty and information asymmetry is greatest in hydro-dominated markets. The opportunity cost of using water stored in the reservoir behind a specific generation unit is typically estimated by solving a stochastic dynamic program based on estimates of the probability distribution of future water inflows and future offer prices of thermal generation units, which can leave significant scope for differences across market participants in their estimates of the generation unit-specific opportunity cost of water. This uncertainty is exacerbated by political risks such as the possibility of regulatory intervention and each producers' subjective beliefs about the probability of these events occurring during the planning period. The influence of these political risks on cost uncertainty are likely to be greatest during extreme system conditions when water is scarce and the probability of regulatory intervention is high.

We consider a multi-unit auction with two capacity-constrained producers facing an uncertain demand, where offers from both suppliers must be accepted in high demand states. These accepted offers are either paid a uniform or a discriminatory price. In the uniform-price procurement auctions, all accepted offers are paid the clearing price, which is set by the highest accepted offer price. In a discriminatory auction, all accepted offers are paid their own offer price. The uncertain demand is realized after offers have been submitted. Similar to von der Fehr and Harbord (1993), each firm offers its entire production capacity at one unit price in a one shot game. We generalize von der Fehr and Harbord (1993) by introducing uncertain interdependent costs. Analogous to Milgrom and Weber's (1982) auction for single objects, each firm makes its own estimate of production costs based on private imperfect information that it receives, and then makes an offer.[1] As is customary in game theory, we refer to this private information as a private *signal*. Similar to Milgrom and Weber (1982), we solve for a Bayesian NE and consider signals that are drawn from a bivariate distribution that is known to the suppliers.

There are no welfare losses in our setting, because demand is inelastic, each producer has constant marginal costs and offers its whole capacity at one price. Moreover, producers are symmetric ex-ante and offer prices are increasing with respect to a each supplier's cost signal. Thus our analysis focuses on bidding behaviour and how the auction design and information structure influence the payoff of the auctioneer.

As in von der Fehr and Harbord's (1993) study of the uniform-price auction, our results depend on whether producers are pivotal or not. A producer is pivotal if its competitors do not have enough production capacity to meet the realized demand. Producers are never pivotal in single object auctions with at least two participants, while the number of pivotal producers in wholesale electricity markets depends on the season and the time-of-day (Genc and Reynolds, 2011), but also on market shocks. Pivotal status indicators as measures of the ability to exercise unilateral market power have been evaluated by Bushnell et. al. (1999) and Twomey et al. (2005) and have been applied by the Federal Energy Regulator Commission (FERC) in its surveillance of electricity markets in U.S. Such binary indicators are supported by von der Fehr and Harbord's (1993) pure-strategy NE in uniform-price auctions, where the market price is either at the marginal cost of highest cost accepted supplier or the reservation price, depending on whether producers are pivotal or non-pivotal with certainty. Our equilibrium is more subtle, the pivotal status is typically uncertain before offers are submitted and the expected market price increases when producers are expected to be pivotal with a larger margin.

Most wholesale electricity markets use uniform pricing. One exception is the real time market in Britain, which uses discriminatory pricing.[2] We show that equilibrium offers in a discriminatory auction are determined by the expected sales of the highest and lowest bidder, respectively. In our setting, the variance in these sales after offers have been submitted – due to demand shocks, outages and intermittent renewable production – will not influence the bidding behaviour of producers in the discriminatory auction. Bidding in the uniform-price auction is also insensitive to this variance in sales, as long as these shocks are not sufficiently large to occasionally change the pivotal status of at least one producer. Even if the possibility of large shocks would influence bidding behaviour in uniform-price auctions, it is still the case that the probability that producers are pivotal does not influence payoffs for given expected sales and independent signals.

We show that uniform and discriminatory pricing are equivalent when signals are independent. An auctioneer tends to favour discriminatory pricing when sig-

[1] Milgrom and Weber (1982) analyse a single-object sales auction, so in their setting each agent estimates the value of the good that the auctioneer is selling.

[2] In addition, some special auctions in the electricity market, such as counter-trading in the balancing market and/or the procurement of power reserves, sometimes use discriminatory pricing (Holmberg and Lazarzcyk, 2015; Anderson et al., 2013).

nals become more correlated at higher values. The opposite is true when signals are less correlated at higher values. Advantages and disadvantages with uniform pricing tend to be amplified if producers are pivotal with a higher probability. We also argue that bidding formats that, as in practice, restrict the number of steps in a producer's supply schedule reduce the mark-ups of offer prices over marginal cost in uniform-price auctions and makes the uniform-price auction more attractive to consumers relative to the discriminatory auction, especially when producers have common uncertainties in their costs.

Independent of the auction format, we find that mark-ups decrease if producers' signals are more positively correlated, i.e. when they receive similar information before offers are submitted. This is related to Vives (2011) who finds that mark-ups decrease when producers receive less noisy cost information before competing in a uniform-price auction. It is also known from previous work that disclosure of information before bids are submitted improves competition in single object auctions (Milgrom and Weber, 1982). Taken together, these results suggest that publicly available information of relevance for production costs – such as weather conditions, fuel prices, prices of emission permits – is likely to improve the competitiveness of market outcomes. It is also easier for a producer to estimate the marginal cost of its competitors if the market operator discloses detailed historical bid data and/or detailed production data. Thus, our results support the argument that the transparency increasing measures of the European Commission should improve the performance of European electricity markets. In addition, information provision about outcomes from financial markets just ahead of the operation of related physical markets should lower the market uncertainty. Therefore, trading of long-term contracts which help producers predict future electricity prices, should lower this uncertainty and reduce the extent of informational asymmetries among suppliers about the opportunity cost of water.

Extending this logic further, our results suggest that regulatory risks are particularly harmful for competition in hydro-dominated wholesale electricity markets, especially when water is scarce, because of the potential informational asymmetries about the likelihood of regulatory interventions. Thus, we recommend clearly defined contingency plans for intervention by the regulator in case of extreme system conditions. This could potentially mitigate the extraordinarily high-priced periods that typically accompany low-water conditions in hydro-dominated markets such as California, Colombia, and New Zealand.

Because increased transparency lowers the payoff of producers in our model, we would not expect producers to agree to voluntarily disclose production cost-relevant information. This has similarities to Gal-Or (1986) who shows that producers that play a Bertrand equilibrium would try to conceal their costs from each other.

According to our results, increased transparency would only be helpful up to a point, because there is a lower bound on equilibrium mark-ups when producers are pivotal. Another caveat is that we only consider a single shot game. As argued by von der Fehr (2013), there is a risk that increased transparency in European electricity markets can facilitate tacit collusion in a repeated game.

Our study focuses on procurement auctions, but the results are analogous for multi-unit sales auctions. Purchase constraints in sales auctions correspond to production capacities in our setting.[3] As an example, U.S. treasury auctions have the 35% rule, which prevents a single bidder from buying more than 35% of the securities sold. Spectrum auctions by the Federal Communications Commission (FCC) have similar rules. California has purchase limits in its auction of Greenhouse Gas emission allowances. Purchase constraints are used to avoid the outcome where a single bidder purchases the vast majority of the good sold. On the other hand, such constraints increase the probability that a bidder will be pivotal and/or make bidders pivotal with a larger margin. The latter would make bidding less competitive and purchase prices would go down. The supply of treasury bills, which corresponds to the auctioneer's demand in our model, is typically uncertain when bids are submitted due to an uncertain amount of non-competitive bids (Wang and Zender, 2002) or because the auctioneer wants to wait for the latest market news before finally announcing its supply of treasury bills.

Most treasury auctions around the world use discriminatory pricing (Bartolini and Cottarelli, 1997). An important exception is the U.S. Treasury, which switched from the discriminatory format to the uniform-price format during the 1990s. Based on our results, we believe that a bidding format that restricts the number of steps in the bid-schedules would increase auction sales revenues to the U.S. Treasury. Given that bidders' marginal valuation of securities should be fairly constant, such bid constraints should improve competition without introducing any significant welfare losses. Our results also show that it is beneficial for auctioneers of securities to disclose market relevant information before the auction starts.

The remainder of the paper is organized as follows. Section 2 compares our paper with the previous literature. Section 3 formally introduces our model, which is analysed for auctions with discriminatory and uniform-pricing in Section 4. The paper is concluded in Section 5. All proofs are in the Appendix.

2 Comparison with related studies

In our setting and in Fabra et al. (2006), uniform and discriminatory pricing are equivalent when firms are non-pivotal with certainty. Independent of the auction format, the payoff is then zero for the producer with the highest offer price and the other producer is paid its own offer price. This corresponds to the first-price single-object auction, which is studied by Milgrom and Weber (1982). Our main methodological contribution is that we generalize their model to the pivotal case, where competitors of each producer do not have enough total production capacity to meet all of the demand. Our model also generalizes Parisio and Bosco (2003), which is restricted to producers with independent private costs in uniform-price auctions.

[3] To some extent, bidders' financial constraints would also correspond to production capacities. Financial constraints of bidders partly explain the bidding behaviour in security auctions (Che and Gale, 1998).

Divisible-good auctions often have restrictions on how many offer prices each producer can submit or, equivalently, how many steps a producer is allowed to have in its supply function. In practice, a producer is normally allowed to choose a supply-schedule with more than the one offer price that is considered here, in von der Fehr and Harbord (1993) and Fabra et al. (2006). Such bid constraints do normally not influence equilibrium bids for producers with constant marginal costs in discriminatory auctions, each such producer often finds it optimal to offer its whole production capacity at one price anyway (Genc, 2009; Anderson et al., 2013; Ausubel et al., 2014). For such cases and if there are no restrictions in the bidding format, Ausubel et al. (2014) show that auctioneers would often prefer discriminatory to uniform pricing.[4]

Restrictions on the number of offer prices per supplier typically have more impact on the equilibrium outcome in uniform-price auctions. In our study, where each producer offers its entire production capacity at one price, it does not matter how sensitive a producer's cost is to the competitor's signal. Results are the same irrespective of whether costs are private, common or anything in between those two extremes. This is very different in Vives (2011). The reason is that producers in his setting choose linear supply functions and can therefore condition their output on every price. To a larger extent than in our model, his bidding format therefore allows producers to indirectly condition their output on the competitor's information. If costs are common or positively interdependent, a producer therefore has an incentive to reduce output when the price is unexpectedly high (when the competitor has received a high cost signal) and increase the output when the price is unexpectedly low (when the competitor has received a low cost signal). This will make supply functions steeper or even downward sloping, which will significantly harm competition. If costs have a common uncertainty, then mark-ups in a uniform-price auction can be as high as for the monopoly case (Vives, 2011).

To summarize Vives (2011), he shows that more information (i.e. less noisy signals) before a uniform-price auction starts improves competition, but mark-ups increase if a producer learns information, or rather conditions its supply on the competitor's information, during the auction. To avoid this latter anti-competitive effect, it should be optimal to restrict the bid format to give producers less freedom to condition their output on the price and competitors' signals. Most wholesale electricity markets and other multi-units auctions already have such constraints. In case, marginal costs are constant, we conjecture that it is optimal to only allow one offer-price per bidder, as in our study.

In empirical studies of auctions, Armantier and Sbaï (2006;2009) and Hortaçsu and McAdams (2010) find that the treasury would prefer uniform pricing in France and Turkey, respectively, while Kang and Puller (2008) find that discriminatory pricing would be best for the treasury in South Korea. According to our discussion above, the ranking of auction formats could depend on details in the bidding

[4] The theoretical studies by Holmberg (2009) and Hästö and Holmberg (2006) find that an auctioneer would prefer discriminatory pricing if constraints in the bidding format are neglected and costs are common knowledge. Pycia and Woodward (2015) show that pay-as-bid and uniform-price auctions are revenue equivalent if costs are common knowledge and the auctioneer chooses both the reservation price and its supply of goods optimally.

format. Wolak (2007) and Kastl (2012) have developed structural econometric models that account for constraints in the bidding format, which could be useful in future assessment of the empirical rankings of auction formats.

As for example illustrated by Wilson (1979), Klemperer and Meyer (1989), Green and Newbery (1992) and Ausubel et al. (2014), there can be multiple NE in divisible-good auctions when some accepted offers are never price-setting. This is not an issue in our discriminatory auction or in our general model of the uniform-price auction where each producer's pivotal status is uncertain. However, in the special case where producers are pivotal with certainty in a uniform-price auction there is, in addition to the symmetric Bayesian equilibrium that we calculate, also an asymmetric high-price equilibrium (von der Fehr and Harbord, 1993). This equilibrium is very unattractive for consumers, because the highest offer, which sets the clearing price, is at the offer cap or reservation price. Thus for circumstances when the high-price equilibrium exists and is selected by producers, then the uniform-price auction is worse than the discriminatory auction for an auctioneer (Fabra et al., 2006).

In order to facilitate comparisons with previous studies, we derive results for the limit where the cost uncertainty decreases until the costs are almost surely common knowledge. In this limit, our model of the discriminatory auction corresponds to the classical Bertrand game. We get the competitive outcome with zero mark-ups for this limit when non-pivotal producers have weakly affiliated signals[5], both for uniform and discriminatory pricing. This result agrees with the competitive outcomes for non-pivotal producers in von der Fehr and Harbord (1993) and in Fabra et al. (2006). If signals are independent and producers pivotal, it follows from Harsanyi's (1973) purification theorem that in the limit when costs are almost surely common knowledge, our Bayesian Nash equilibria for uniform-price and discriminatory auctions correspond to the mixed-strategy NE analysed by Anderson et al. (2013), Anwar (2006), Fabra et al. (2006), Genc (2009), Son et al. (2004) and von der Fehr and Harbord (1993).[6] Analogous mixed strategy NE also occur in the Bertrand-Edgeworth game (Edgeworth, 1925; Allen and Hellwig, 1986; Beckmann, 1967; Levitan and Shubik, 1972; Maskin, 1986; Vives, 1986; Deneckere and Kovenock, 1996; Osborne and Pitchik, 1986).

3 Model

There are two risk-neutral producers in the market. Each producer $i \in \{1, 2\}$ receives a private signal s_i with imperfect cost information. The joint probability density $\chi(s_i, s_j)$ is continuously differentiable and symmetric, so that $\chi(s_i, s_j) \equiv$

[5] Affiliated signals are positively correlated signals, such that if the signal of one player increases, then it increases the probability that the competitor has a high signal relative to the probability that the competitor has a low signal.

[6] Blázquez de Paz (2014) generalizes these mixed-strategy NE to consider transmission constraints.

$\chi(s_j, s_i)$. Moreover, $\chi(s_i, s_j) > 0$ for $(s_i, s_j) \in (\underline{s}, \overline{s}) \times (\underline{s}, \overline{s})$.[7] We say that signals are weakly affiliated when[8]

$$\frac{\chi(u, v')}{\chi(u, v)} \leq \frac{\chi(u', v')}{\chi(u', v)}, \tag{1}$$

where $v' \geq v$ and $u' \geq u$. Thus, if the signal of one player increases, then it (weakly) increases the probability that its competitor has a high signal relative to the probability that its competitor has a low signal. It can be shown that signals are weakly affiliated if and only if $\ln \chi(u, v)$ is supermodular (Krishna, 2010). We say that signals are weakly unaffiliated when the opposite is true, i.e.

$$\frac{\chi(u, v')}{\chi(u, v)} \geq \frac{\chi(u', v')}{\chi(u', v)}, \tag{2}$$

where $v' \geq v$ and $u' \geq u$. Note that independent signals are both weakly affiliated and weakly unaffiliated. We let

$$F(s_i) = \int_{-\infty}^{s_i} \int_{-\infty}^{\infty} \chi(u, v)\, dv du$$

denote the marginal distribution, i.e. the unconditional probability that supplier i receives a signal below s_i. Moreover,

$$f(s_i) = F'(s_i).$$

As in von der Fehr and Harbord (1993), we consider the case when each firm's marginal cost is constant up to its production capacity constraint $\widetilde{q}_i$.[9] But in our setting, marginal costs and possibly also $\widetilde{q}_i$ are uncertain when offers are submitted. The production capacities of the two producers could be correlated, but they are symmetric information and we assume that they are independent of production costs and signals. In Europe, this assumption could be justified by the fact that any insider information on production capacities have to be disclosed to the market according to EU No. 1227/2011 (REMIT). Capacities are symmetric ex-ante, so that $\mathbb{E}[\widetilde{q}_i] = \mathbb{E}[\widetilde{q}_j]$. Realized production capacities are assumed to be observed by the auctioneer when the market is cleared.[10]

We refer to $c_i(s_i, s_j)$ as the marginal cost of producer i, but actually costs are not necessarily deterministic for given s_i and s_j. More generally, $c_i(s_i, s_j)$ is the expected marginal cost conditional on all information available among producers in the market. We use the convention that a firm's own signal is placed first in its list

[7] We do not require $\chi(s_i, s_j) > 0$ at the boundary, but $\frac{\chi_1(u,\overline{s})}{\chi(u,\overline{s})} = \frac{\chi_2(\overline{s},u)}{\chi(\overline{s},u)}$ is assumed to be bounded for $u \in [\underline{s}, \overline{s}]$.

[8] Milgrom and Weber (1982) call such signals affiliated. We write *weakly* affiliated to stress that the condition is also satisfied for independent signals.

[9] This corresponds to flat demand in the sales auction of Ausubel et al. (2014).

[10] Alternatively, similar to the market design of the Australian wholesale market, producers could first choose bid prices and later adjust production capacities at those prices just before the market is cleared. Anyway, we assume that the reported production capacities are publicly verifiable, so that bidders cannot choose them strategically.

of signals. Firms' marginal costs are symmetric ex-ante, i.e. $c_i(s_i, s_j) = c_j(s_i, s_j)$. But costs and information about costs are normally asymmetric ex-post, after private signals have been observed. We assume that

$$\frac{\partial c_i(s_i, s_j)}{\partial s_i} > 0, \tag{3}$$

so that a firm's marginal cost increases with respect to its own signal. A firm has more information on its own cost than about the competitor's cost, so that

$$\frac{\partial c_i(s_i, s_j)}{\partial s_i} \geq \frac{\partial c_j(s_j, s_i)}{\partial s_i}.$$

This also means that the firm with the highest cost receives the highest signal. A firm's marginal cost is allowed to decrease somewhat with respect to the competitor's signal, but we require that:

$$\frac{dc_i(s, s)}{ds} \geq 0. \tag{4}$$

Thus, if both producers would by (coincidence) receive the same signal s, then a producer's marginal cost is increasing with respect to that same signal. The special case with independent signals and $\frac{\partial c_i(s_i,s_j)}{\partial s_j} = 0$ corresponds to the private independent cost assumption, which for example is used in the analysis by Parisio and Bosco (2003). The common cost/value assumption that is often used in the auction literature corresponds to that $c_i(s_i, s_j) \equiv c_j(s_j, s_i)$.

We frequently refer to the limit where production costs are almost surely common knowledge. Formally, we define:

Definition 1 *Production costs are almost surely common knowledge when* $\frac{dc_i(s,s)}{ds} = 0$ *for* $s \in [\underline{s}, \overline{s})$.

Thus, a producer could, in principle, still have private information about its marginal cost off the diagonal of its cost function, but we show that such information will not influence the bidding behaviour.

As in von der Fehr and Harbord (1993), demand can be uncertain $D \in \left[\underline{D}, \overline{D}\right]$. It could be correlated with the production capacities, but demand is assumed to be independent of the production costs and signals. In addition, it is assumed that all outcomes are such that $0 \leq D \leq \widetilde{q}_i + \widetilde{q}_j$, so that there is always enough production capacity to meet the realized demand. As in von der Fehr and Harbord (1993), demand is inelastic up to a reservation price $\overline{p}$. Analogous to Milgrom and Weber (1982), we assume that the reservation price is set at the highest relevant marginal cost realization, i.e. $\overline{p} = c_i(\overline{s}, \overline{s})$ for $i \in \{1, 2\}$. This assumption can be motivated by the fact that an auctioneer would lower its procurement cost by lowering the reservation price whenever $\overline{p} > c_i(\overline{s}, \overline{s})$.

After firms have received their private signals, each firm submits an offer with one unit price for its whole capacity in a one-shot game. We let $p_i(s_i)$ be the chosen offer price of firm $i \in \{1, 2\}$ when it observes the signal s_i. The auctioneer accepts offers in order to minimize its procurement cost. In a uniform-price auction, the highest accepted offer price sets the uniform market price for all accepted offers. In a discriminatory auction, each accepted offer is paid its individual offer price.

Similar to classical auction theory, we solve for symmetric Bayesian Nash equilibria with the following properties: (i) the chosen offer price of firm $i \in \{1, 2\}$ is a twice differentiable function of its signal s_i and (ii) the chosen offer price is strictly monotonic in the firm's signal, i.e. $p_i'(s_i) > 0$ for $s_i \in (\underline{s}, \overline{s})$. Thus, the inverse $p_i^{-1}(p)$ always exists in equilibrium. Strict monotonicity also implies that ties occur with measure zero. Hence, the rationing rule will not influence the expected profit of producers in the equilibria for which we solve.

Ex-post, we denote the winning producer, which has a low offer price and gets a high output, by subscript H. The losing producer, which has a high offer price and gets a low output, is denoted by the subscript L. Winning and losing producers have the following expected outputs:

$$q_H = \mathbb{E}\left[\min\left(\widetilde{q}_H, D\right)\right] \tag{5}$$

and

$$q_L = \mathbb{E}\left[\max\left(0, D - \widetilde{q}_H\right)\right]. \tag{6}$$

The payoff of each producer is given by its revenue minus its realized production cost.

4 Analysis

We first analyse discriminatory pricing, where the offer price of a producer sets its own transaction price. Uniform-pricing is more complicated to analyse as it depends on producers' pivotal status whether the lowest or highest offer price sets the clearing price.

4.1 Discriminatory pricing

Each firm is paid as bid under discriminatory pricing. The demand uncertainty and production capacity uncertainties are independent of the cost uncertainties. Thus, the expected profit of firm i when receiving signal s_i is:

$$\begin{aligned}\pi_i(s_i) = {} & \left(p_i(s_i) - \mathbb{E}\left[c_i(s_i, s_j) \mid p_j \geq p_i\right]\right) \Pr\left(p_j \geq p_i \mid s_i\right) q_H \\ & + \left(p_i(s_i) - \mathbb{E}\left[c_i(s_i, s_j) \mid p_j \leq p_i\right]\right)\left(1 - \Pr\left(p_j \geq p_i \mid s_i\right)\right) q_L.\end{aligned} \tag{7}$$

In the Appendix, we show that:

Lemma 1 *In markets with discriminatory pricing:*

$$\begin{aligned}\frac{\partial \pi_i(s_i)}{\partial p_i} = {} & \Pr\left(p_j \geq p_i \mid s_i\right) q_H + \left(1 - \Pr\left(p_j \geq p_i \mid s_i\right)\right) q_L \\ & + \left(p_i - c_i\left(s_i, p_j^{-1}(p_i)\right)\right) \frac{\partial \Pr(p_j \geq p_i \mid s_i)}{\partial p_i}\left(q_H - q_L\right).\end{aligned} \tag{8}$$

The first two terms on the right-hand side of (8) correspond to the price effect. This is what the producer would gain in expectation from increasing its offer price by one unit if the acceptance probabilities were to remain unchanged. However, on the margin, a higher offer price lowers the probability of being the winning bidder by $\frac{\partial \Pr(p_j \geq p_i | s_i)}{\partial p_i}$. Switching from being the winning to the losing bidder reduces the accepted quantity by $q_H - q_L$. We refer to $\frac{\partial \Pr(p_j \geq p_i | s_i)}{\partial p_i} (q_H - q_L)$ as the quantity effect, the quantity that is lost on the margin from a marginal price increase. The mark-up for lost sales, $p_i - c_i \left(s_i, p_j^{-1}(p_i)\right)$, times the quantity effect gives the lost value of the quantity effect. This is the last term on the right-hand side of (8). The quantity effect and its associated loss is determined by cases where the competitor, producer j, is bidding really close to p_i, which corresponds to the competitor receiving the signal $p_j^{-1}(p_i)$. This explains why $c_i \left(s_i, p_j^{-1}(p_i)\right)$ is the relevant cost in the mark-up for lost sales.

We solve for a symmetric Bayesian NE and henceforth, we often drop firm-specific subscripts when discussing this equilibrium. Producers may receive different signals but, in equilibrium, they react in the same way to a private signal s as implied by the function $p(s)$. In the symmetric equilibrium, the price effect, i.e. $\Pr(p_j \geq p | s) q_H + (1 - \Pr(p_j \geq p | s)) q_L$, is given by:

$$\frac{\int_s^{\overline{s}} \chi(s, s_j)\, ds_j q_H + \int_{\underline{s}}^{s} \chi(s, s_j)\, ds_j q_{L,}}{\int_{\underline{s}}^{\overline{s}} \chi(s, s_j)\, ds_j} \tag{9}$$

where $\chi(s, s)$ is the joint probability density for signals. As shown in the Appendix, the quantity effect, i.e. $\frac{\partial \Pr(p_j \geq p | s)}{\partial p} (q_H - q_L)$, is given by

$$-(q_H - q_L) \frac{\chi(s, s)}{p'(s) \int_{\underline{s}}^{\overline{s}} \chi(s, s_j)\, ds_j}, \tag{10}$$

where $\frac{\chi(s,s)}{p'(s) \int_{\underline{s}}^{\overline{s}} \chi(s,s_j) ds_j}$ represents the probability density in terms of offer prices. Hence, the lost value due to the quantity effect, i.e. $(p - c(s, s)) \frac{\partial \Pr(p_j \geq p | s)}{\partial p} (q_H - q_L)$, is equal to:

$$(p - c(s, s))(q_H - q_L) \frac{\chi(s, s)}{p'(s) \int_{\underline{s}}^{\overline{s}} \chi(s, s_j)\, ds_j}. \tag{11}$$

We find it useful to introduce the following exogenous function, which is proportional to the quantity effect and inversely proportional to the price effect for a given $p(s)$.

Definition 2

$$H^*(s) := \frac{\chi(s, s)(q_H - q_L)}{\int_s^{\overline{s}} \chi(s, s_j)\, ds_j q_H + \int_{\underline{s}}^{s} \chi(s, s_j)\, ds_j q_L}. \tag{12}$$

As will be shown below, the function $H^*(s)$ captures the essential aspects of the information structure and auction format and the essential properties of demand and the production capacities.

Equilibrium offers are chosen optimally for each signal. The implication is that the price effect equals the lost value of the quantity effect for each signal. Thus, it follows from (9), (11) and (12) that equilibrium offers can be determined from the following ordinary differential equation (ODE):

$$p'(s) - (p - c(s,s)) H^*(s) = 0. \tag{13}$$

The solution to this ODE is presented in the following proposition, which also establishes when the solution is an equilibrium.

Proposition 1 *The symmetric Bayesian Nash equilibrium offer in a discriminatory auction is given by:*

$$p(s) = c(s,s) + \int_s^{\overline{s}} \frac{dc(v,v)}{dv} e^{-\int_s^v H^*(u)du} dv \tag{14}$$

if

$$\frac{d}{ds}\left(\frac{\int_x^{\overline{s}} \chi(s,s_j)\, ds_j q_H + q_L \int_{\underline{s}}^x \chi(s,s_j)\, ds_j}{\chi(s,x)}\right) \geq 0, \tag{15}$$

for all $s, x \in (\underline{s}, \overline{s})$. The equilibrium exists for more general probability distributions when $\frac{dc(v,v)}{dv} > 0$. In the limit when costs are almost surely common knowledge, the symmetric Bayesian Nash equilibrium can be simplified to:

$$p(s) = c(\underline{s},\underline{s}) + e^{-\int_s^{\overline{s}} H^*(u)du} (\overline{p} - c(\underline{s},\underline{s})), \tag{16}$$

for $s \in [\underline{s}, \overline{s})$.

The term $\int_s^{\overline{s}} \frac{dc(v,v)}{dv} e^{-\int_s^v H^*(u)du} dv$ in (14) corresponds to a mark-up. It follows from (13) that the mark-up is proportional to how sensitive the competitor's offer price is to its signal, i.e. $p'(s)$. Thus, it is understandable that the mark-up increases when the competitor's cost is more sensitive to its signal, i.e. $\frac{dc(s,s)}{ds}$ is large. Given that $H^*(s)$ is proportional to the quantity effect and inversely proportional to the price effect, it also makes sense that a high $H^*(s)$ results in more competitive offers with lower mark-ups. We also note from Definition 2 that $H^*(s)$ and $p(s)$ are determined by the expected sales of the high price bidder and the low price bidder, but $H^*(s)$ and $p(s)$ are independent of the variances of those sales. This would be different if signals were not independent of production capacities and demand. In the limit when firms' marginal costs are almost surely common knowledge, as in (16), the signals only serve the purpose of coordinating producers' actions as in a correlated equilibrium (Osborne and Rubinstein, 1994).

Another conclusion that we can draw from Proposition 1 is that bidding behaviour is only influenced by properties of $c_i(s_i, s_j)$ at points where $s_i = s_j$. Thus, for a given diagonal of the cost function, it does not matter for our analysis whether the costs are private, so that $\frac{\partial c_i(s_i,s_j)}{\partial s_j} = 0$, or common, so that $\frac{\partial c_i(s_i,s_j)}{\partial s_j} = \frac{\partial c_i(s_i,s_j)}{\partial s_i}$. As noted above, the reason is that when solving for the locally optimal offer price, a producer is only interested in cases where the competitor is bidding really close to p_i. In a symmetric equilibrium, this occurs when the competitor receives a similar signal. The properties of $c(\cdot)$ for signals where $s_i \neq s_j$ could influence the expected production cost of a firm, but not its bidding behaviour. This would be different if each producer submitted an offer with multiple offer prices or even a continuous supply function as in Vives (2011), so that a producer could indirectly condition its output on the competitor's information.

Before drawing further conclusions from Proposition 1, we introduce the following definition:

Definition 3 *We say that two pairs of probability density functions and marginal cost functions* $\left\{\chi^A(s_i, s_j), c^A(s_i, s_j)\right\}$ *and* $\left\{\chi^B(s_i, s_j), c^B(s_i, s_j)\right\}$ *are equivalent in expectation if:*
(i) the pairs have the same expected marginal cost conditional on a producer's private signal s

$$\mathbb{E}\left[c^A(s, s_j)\middle| s\right] = \mathbb{E}\left[c^B(s, s_j)\middle| s\right],$$

(ii) the same marginal cost for common signals s

$$c^A(s, s) = c^B(s, s),$$

(iii) and the same marginal density

$$\int_{\underline{s}}^{\bar{s}} \chi^A(s, s_j)\, ds_j = \int_{\underline{s}}^{\bar{s}} \chi^B(s, s_j)\, ds_j.$$

It can be shown from Definition 2, (5) and (6) that $H^*(u)$ increases with respect to the production capacity $\widetilde{q}$. The reason is that the quantity effect increases when the difference between producers' expected outputs, $q_H - q_L$, increases. Thus higher production capacities, and less restrictive purchase constraints in analogous sales auctions, will make bidding more competitive. It also follows from Definition 2 that $H^*(u)$ increases when the density at $\chi(s, s)$ increases relative to both $\int_{\underline{s}}^{s} \chi(s, s_j)\, ds_j$ and $\int_{s}^{\bar{s}} \chi(s, s_j)\, ds_j$. The reason is simply that the quantity effect from increasing one's offer price increases if, conditional on the reception of a signal s, it becomes more likely that the competitor receives a similar signal s and chooses a similar offer price. Thus, we can conclude from Proposition 1 that

Corollary 1 *Mark-ups in the discriminatory auction are lower when* $\widetilde{q}$ *increases and are lower for the pair* $\left\{\chi^A(s_i, s_j), c^A(s_i, s_j)\right\}$ *in comparison to the pair* $\left\{\chi^B(s_i, s_j), c^B(s_i, s_j)\right\}$*, if the two pairs are equivalent in expectation and if signals in* $\chi^A(s_i, s_j)$ *are more positively correlated signals in the sense that*

$$\frac{\chi^A(s, s)}{\int_{\underline{s}}^{s} \chi^A(s, s_j)\, ds_j} > \frac{\chi^B(s, s)}{\int_{\underline{s}}^{s} \chi^B(s, s_j)\, ds_j}$$

and

$$\frac{\chi^A(s, s)}{\int_{s}^{\bar{s}} \chi^A(s, s_j)\, ds_j} > \frac{\chi^B(s, s)}{\int_{s}^{\bar{s}} \chi^B(s, s_j)\, ds_j}.$$

In particular, if increased transparency makes signals more positively correlated without changing any cost realisation, then this will lower mark-ups.

This result implies that in hydro-dominated markets, mark-ups would also decrease for increased political and regulatory transparency that make signals more positively correlated without changing realisations of the opportunity cost.

Proposition 1 can be simplified in the special case when signals are independent.

Proposition 2 *If signals are independent, the symmetric Bayesian Nash equilibrium offer in a discriminatory auction is given by:*

$$p(s) = c(s,s) + \int_s^{\overline{s}} \frac{dc(v,v)}{dv} \left(\frac{(1-F(v))\, q_H + F(v)\, q_L}{(1-F(s))\, q_H + F(s)\, q_L} \right) dv. \tag{17}$$

If costs are almost surely common knowledge, then (17) can be simplified to:

$$p(s) = c(\underline{s},\underline{s}) + \left(\frac{q_L}{((1-F(s))\, q_H + F(s)\, q_L)} \right) (\overline{p} - c(\underline{s},\underline{s})) \ \text{ for } s \in [\underline{s},\overline{s}), \tag{18}$$

where $F(s)$ is a firm's (marginal) probability distribution for receiving the signal s.

In the limit when costs are almost surely independent of the signals, the independent signals effectively become randomization devices, which the producers use to randomize their offers. In this case, the functional form of the probability density for signals is of no importance, because a producer will decide its offer price based on the probability that the competitor received a lower signal, $F(s)$. Thus, bidding behaviour would not change if the probability distribution were transformed by the monotonic function $p(s)$ into a new signal $P = p(s)$, i.e. a signal that directly gives the price that a firm should choose. The price signal has the probability distribution $G(P) = F(p^{-1}(P))$. If we rewrite (18), we get that

$$G(P) = \frac{q_H}{q_H - q_L} - \frac{\overline{p} - c}{P - c} \frac{q_L}{q_H - q_L}. \tag{19}$$

This probability distribution of offer prices corresponds to the mixed-strategy NE that is calculated for discriminatory auctions by Fabra et al. (2006). This confirms Harsanyi's (1973) purification theorem that a mixed-strategy NE is equivalent to a pure-strategy Bayesian NE, where costs are almost surely common knowledge and signals are independent.

Finally, we note that divisible-good models of discriminatory auctions, where each producer chooses a single offer price, are identical to the Bertrand model. Thus, the results in this section are also relevant for the Bertrand-Edgeworth game.

4.1.1 Non-pivotal case

Only the lowest offer price is accepted when $\widetilde{q}_i > D$ for all $i \in \{1,2\}$ and all outcomes, so that producers are non-pivotal with certainty, i.e. $q_L = 0$ and $q_H = \mathbb{E}[D]$. This simplifies the expressions to the below result, which corresponds to Milgrom and Weber's (1982) result for first-price indivisible-good auctions. If producers are non-pivotal with certainty, then the winning offer sets its own price also in the uniform-price auction. Thus, there is no difference between a discriminatory and a uniform-price auction in this special case.

Proposition 3 *The symmetric Bayesian Nash equilibrium of producers that are non-pivotal with certainty in auctions with uniform or discriminatory pricing is given by:*

$$p(s) = c(s,s) + \int_s^{\overline{s}} \frac{dc(v,v)}{dv} e^{-\int_s^v H(u)du} dv, \tag{20}$$

where

$$H(s) := \frac{\chi(s,s)}{\int_s^{\overline{s}} \chi(s,s_j)\, ds_j}. \tag{21}$$

This is an equilibrium if the signals are weakly affiliated. The equilibrium exists for more general distributions when $\frac{dc(v,v)}{dv} > 0$. *In the limit when costs are almost surely common knowledge, the equilibrium offer in (20) is perfectly competitive, i.e.* $p(s) = c(\underline{s}, \underline{s})$ *for* $s \in [\underline{s}, \overline{s})$.

Private information gives an informational rent, so if costs are asymmetric information, then also non-pivotal bidders have a positive mark-up. But mark-ups are zero in the limit when costs are almost surely common knowledge. This concurs with von der Fehr and Harbord (1993) and Fabra et al. (2006), where mark-ups are zero in auctions with both uniform and discriminatory pricing, if producers are non-pivotal with certainty and marginal costs are constant and common knowledge. Thus, we generalize their result for non-pivotal producers to weakly affiliated signals. The same generalization applies to the Bertrand game.

4.2 Uniform-pricing

As mentioned earlier, Proposition 3 also applies to non-pivotal producers in a uniform-price auction. Below we consider producers that are pivotal with certainty. Later, we will consider the general case where the pivotal status of producers is uncertain.

Definition 4 *Producers are pivotal with certainty if it is always the case that* $\widetilde{q}_H < D \leq \widetilde{q}_H + \widetilde{q}_L$.

The highest offer sets the market price in a uniform-price auction when producers are pivotal with certainty. The demand and production capacity uncertainties are independent of the signals and cost uncertainties. Thus, when producers are pivotal with certainty, the expected profit of firm i when receiving signal s_i is:

$$\begin{aligned} \pi_i(s_i) &= \mathbb{E}\left[p_j - c_i(s_i,s_j) \mid p_j \geq p_i\right] \Pr\left(p_j \geq p_i \mid s_i\right) q_H \\ &+ \left(p_i(s_i) - \mathbb{E}\left[c_i(s_i,s_j) \mid p_j \leq p_i\right]\right)\left(1 - \Pr\left(p_j \geq p_i \mid s_i\right)\right) q_L. \end{aligned} \tag{22}$$

Lemma 2 *In a uniform-price auction with producers that are pivotal with certainty, we have:*

$$\begin{array}{c} \frac{\partial \pi_i(s_i)}{\partial p_i} = \left(1 - \Pr\left(p_j \geq p_i \middle| s_i\right)\right) q_L \\ + \frac{\partial \Pr(p_j \geq p_i | s_i)}{\partial p_i} \left(p_i - c_i\left(s_i, p_j^{-1}\left(p_i\right)\right)\right)\left(q_H - q_L\right). \end{array} \tag{23}$$

The first-order condition for the uniform-price auction is similar to the first-order condition of the discriminatory auction in Lemma 1, but there is one difference. In contrast to the discriminatory auction, the lowest bidder does not gain anything from increasing its offer price in a uniform-price auction when producers are pivotal with certainty. Thus, the price effect has one term less in the uniform-price auction, which reduces the price effect. There is a corresponding change in the H function which is proportional to the quantity effect and inversely proportional to the price effect.

Definition 5

$$\hat{H}(s) = \frac{(q_H - q_L)\,\chi(s,s)}{q_L \int_{\underline{s}}^{s} \chi(s,s_j)\, ds_j}.$$

Proposition 4 *The symmetric Bayesian Nash equilibrium offer in a uniform-price auction where producers are pivotal with certainty is given by*

$$p(s) = c(s,s) + \int_{s}^{\overline{s}} \frac{dc(v,v)}{dv} e^{-\int_s^v \hat{H}(u)du} dv \tag{24}$$

if signals are weakly unaffiliated. The equilibrium exists for more general probability distributions when $\frac{dc(v,v)}{dv} > 0$*. In the limit when costs are almost surely common knowledge, the symmetric Bayesian Nash equilibrium offer simplifies to:*

$$p(s) = c(\underline{s},\underline{s}) + e^{-\int_s^{\overline{s}} \hat{H}(u)du}\left(\overline{p} - c(\underline{s},\underline{s})\right), \tag{25}$$

for $s \in [\underline{s}, \overline{s})$*.*

Equation (24) has properties similar to the corresponding expressions for the discriminatory auction in Proposition 1. But the ratio of the quantity and price effects differs. It follows from Definitions 2 and 5 that $\hat{H}(s) > H^*(s)$ or, equivalently, that the price effect is relatively smaller in the uniform price auction as compared to a discriminatory auction. Thus, producers make offers with lower mark-ups in uniform-price auctions. On the other hand, in a uniform-price auction, the losing offer with the highest offer price sets the transaction price for both accepted offers, so in the end it is not self-evident that a uniform-price auction would lower the procurement cost of an auctioneer. We will analyse this further in Section 4.3.

Analogous to the discriminatory case, it can be shown from Definition 5, (5) and (6) that $\hat{H}(u)$ increases with respect to the production capacity $\widetilde{q}$. It also follows from Definition 5 that $\hat{H}(u)$ increases when the density at $\chi(s,s)$ increases relative to $\int_{\underline{s}}^{s} \chi(s,s_j)\, ds_j$. Thus, we can conclude from Proposition 4 and Definition 3 that

Corollary 2 *Mark-ups in a uniform-price auction where producers are pivotal with certainty are lower when $\widetilde{q}$ increases and are lower for the pair $\{\chi^A(s_i, s_j), c^A(s_i, s_j)\}$ in comparison to the pair $\{\chi^B(s_i, s_j), c^B(s_i, s_j)\}$, if the two pairs are equivalent in expectation and if the signals in $\chi^A(s_i, s_j)$ are more positively correlated in the sense that*

$$\frac{\chi^A(s,s)}{\int_{\underline{s}}^{s} \chi^A(s, s_j)\, ds_j} > \frac{\chi^B(s,s)}{\int_{\underline{s}}^{s} \chi^B(s, s_j)\, ds_j}.$$

In particular, if increased transparency makes signals more positively correlated without changing any cost realisation, then this will lower mark-ups.

It follows from Definition 5, (5), (6) and Proposition 4 that

Corollary 3 *In a uniform-price auction with certain demand and production capacities, it is optimal for firm i to choose its offer price as follows:*
i) $p_i(s) = c_i(s_i, s_i)$ in the limit where $\widetilde{q} \nearrow D$, so that $q_L \searrow 0$, i.e. when firms are just pivotal.
ii) $p_i(s) = \overline{p}$ in the limit where $2\widetilde{q} \searrow D$, i.e. when both firms always produce at full capacity., so that $q_L = q_H$.

The first property corresponds to Milgrom and Weber's (1982) results for second-price sales auctions, because the lowest bidder gets to produce (almost) the whole demand while the highest bidder sets the uniform market price. By comparing Proposition 3 and Corollary 3, we note that the comparative statics analysis of our symmetric equilibrium has a discontinuity at the critical point where producers' capacities switch from being nonpivotal with certainty to being pivotal with certainty. Somewhat counter-intuitively, offer prices decrease at this critical point, even if demand increases. The reason for this is that the offer that sets the market price also switches at this point, which drastically changes the bidding behaviour. Non-pivotal firms set their own price and use similar bidding strategies as in a first-price procurement auction, i.e. firms' mark-ups are strictly positive for uncertain costs. On the other hand, as implied by the first property of Corollary 3, producers make offers without mark-ups when firms are just pivotal. The following proves that the auctioneer's revenues may also shift downwards in a comparative statics analysis at the critical point where producers' capacities switch from being nonpivotal with certainty to being pivotal with certainty.

Proposition 5 *If producers' signals are weakly affiliated, then the expected payoff of the auctioneer is weakly larger for just pivotal producers than for producers that are just non-pivotal with certainty in markets with uniform pricing. The expected revenues are the same for the two cases when the signals are independent.*

Proposition 4 can be simplified in the special case when the signals are independent.

Proposition 6 *The symmetric Bayesian Nash equilibrium offer in a uniform-price auction where producers are pivotal with certainty is given by:*

$$p(s) = c(s,s) + \int_{s}^{\overline{s}} \frac{dc(v,v)}{dv} \left(\frac{F(s)}{F(v)} \right)^{\frac{(q_H - q_L)}{q_L}} dv \tag{26}$$

if signals are independent. If, in addition, costs are almost surely common knowledge, then (26) can be simplified to

$$p(s) = c(\underline{s}, \underline{s}) + (F(s))^{\frac{(q_H - q_L)}{q_L}} (\overline{p} - c(\underline{s}, \underline{s})) \text{ for } s \in [\underline{s}, \overline{s}), \tag{27}$$

where $F(s)$ is a firm's marginal distribution for receiving the signal s.

We can use an argument similar to the one that we used for the discriminatory auction to show that the limit result in (27) corresponds to the mixed-strategy NE that is derived for uniform-price auctions by von der Fehr and Harbord (1993). (27) can also be used to calculate the expected uniform clearing price.

Proposition 7 *If the signals are independent, the costs are almost surely common knowledge and producers are pivotal with certainty, then the expected market price in the uniform-price auction is given by:*

$$\overline{p} - \frac{(\overline{p} - c)(q_H - q_L)}{q_H + q_L},$$

where $c = c(\underline{s}, \underline{s})$.

In the special case with certain demand and certain production capacities that are pivotal, we have $q_H = \widetilde{q}$ and $q_L = D - \widetilde{q} > 0$, so that the expected market price is given by

$$\overline{p} - \frac{(\overline{p} - c)(2\widetilde{q} - D)}{D}. \tag{28}$$

Figure 1 plots the relation in (28), which gives a comparative statics analysis of the expected transaction price with respect to the (expected) demand level. As shown by Proposition 10 in the next section, the expected transaction price is the same for discriminatory pricing when signals are independent. In Figure 1, we also plot the high-price equilibrium in von der Fehr and Harbord (1993). In this equilibrium, the market price jumps directly from the competitive price with zero mark-ups up to the reservation price when demand increases at the critical point where producers switch from being non-pivotal to being pivotal with certainty in a uniform-price auction. This contrasts with our equilibrium, where the expected market price increases continuously as demand increases. The expected market price does not reach the reservation price until demand equals the total production capacity in the market. With more firms in the market, the expected price in our model would stay near the marginal cost until demand is near the total production capacity in the market, where the expected price will take off towards the reservation price. This would be reminiscent of what is often called "hockey-stick pricing" that is typical for wholesale electricity markets (Hurlbut et al., 2004; Holmberg and Newbery, 2010).

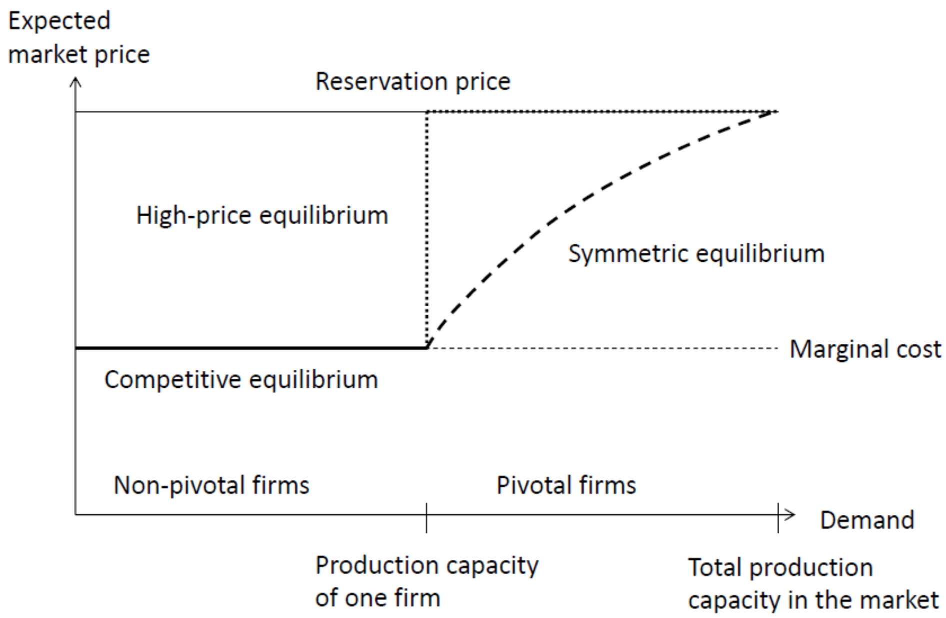

Figure 1: Comparative statics analysis for our symmetric equilibrium and von der Fehr and Harbord's (1993) high-price equilibrium in a uniform-price auction, when costs are almost surely common knowledge and signals are indepedent.

4.2.1 Uncertain pivotal status

In the general case, the pivotal status of producers is uncertain when offers are submitted. Unlike the discriminatory auction, this additional uncertainty makes the uniform-price auction more complicated to analyse. The problem is that the lowest bidder, which has the highest output, would set its own transaction price, as in a discriminatory auction, for outcomes when the highest bidder is non-pivotal, while the highest bidder would set the transaction price of the lowest bidder when the highest bidder is pivotal. Thus, unlike the discriminatory auction, the payoff of the winning producer depends on the probability that the highest bidder is non-pivotal. We denote this probability by Π^{NP}. In our setting, the pivotal status of producers is independent of signals.

Lemma 3 *In a uniform-price auction, where the pivotal status of producers is uncertain, we have:*

$$\begin{aligned} \frac{\partial \pi_i(s_i)}{\partial p_i} = & \Pr\left(p_j \geq p_i | \, s_i\right) q_H^{NP} \Pi^{NP} + \left(1 - \Pr\left(p_j \geq p_i | \, s_i\right)\right) q_L \\ & + \frac{\partial \Pr(p_j \geq p_i | s_i)}{\partial p_i} \left(p_i - c_i\left(s_i, p_j^{-1}\left(p_i\right)\right)\right)\left(q_H - q_L\right), \end{aligned} \tag{29}$$

where

$$q_H^{NP} = \mathbb{E}\left[\widetilde{q}_H | \, \widetilde{q}_H \geq D\right].$$

Thus, the quantity effect is similar as when producers are pivotal with certainty. But the price effect depends on the probability that the highest bidder is non-pivotal. Increasing an offer price contributes to the price effect when a producer's offer is price-setting, i.e. when the producer is pivotal and has the highest offer price or when the producer has the lowest offer price and the highest bidder is non-pivotal. There is a corresponding change in the H function.

Definition 6

$$\widetilde{H}(s) = \frac{\chi(s,s)(q_H - q_L)}{\int_s^{\overline{s}} \chi(s,s_j)\, ds_j q_H^{NP} \Pi^{NP} + \int_{\underline{s}}^{s} \chi(s,s_j)\, ds_j q_L}.$$

Proposition 8 *The symmetric Bayesian Nash equilibrium offer in a uniform-price auction where producers' pivotal status is uncertain is given by*

$$p(s) = c(s,s) + \int_s^{\overline{s}} \frac{dc(v,v)}{dv} e^{-\int_s^v \widetilde{H}(u)du} dv \tag{30}$$

if

$$\frac{d}{ds}\left(\frac{\int_x^{\overline{s}} \chi(s,s_j)\, ds_j q_H^{NP} \Pi^{NP} + q_L \int_{\underline{s}}^{x} \chi(s,s_j)\, ds_j}{\chi(s,x)}\right) \geq 0. \tag{31}$$

The equilibrium exists for more general probability distributions when $\frac{dc(v,v)}{dv} > 0$*. In the limit when costs are almost surely common knowledge, the symmetric Bayesian Nash equilibrium offer simplifies to:*

$$p(s) = c(\underline{s},\underline{s}) + e^{-\int_s^{\overline{s}} \widetilde{H}(u)du} \left(\overline{p} - c(\underline{s},\underline{s})\right), \tag{32}$$

for $s \in [\underline{s}, \overline{s})$*.*

We note that as Π^{NP} increases towards 1, the bidding behaviour in the uniform-price auction gets closer to offers in the discriminatory auction, which concurs with our discussion in Section 4.1.1. In the other extreme, when Π^{NP} decreases towards 0, bidding gets closer to the uniform-price auction with producers that are pivotal with certainty. For a given q_H^{NP}, producers will increase their offer prices when Π^{NP} increases. This may seem counterintuitive, but this is to compensate for the fact that there is a higher risk that the market price is set by the lowest offer price rather than the highest offer price. We can draw the following conclusion from Proposition 8 and Definition 6.

Corollary 4 *Mark-ups in an auction with uniform-pricing are lower when* $\widetilde{q}$ *increases and are lower for the pair* $\left\{\chi^A(s_i,s_j), c^A(s_i,s_j)\right\}$ *in comparison to the pair* $\left\{\chi^B(s_i,s_j), c^B(s_i,s_j)\right\}$*, if the two pairs are equivalent in expectation and if the signals in* $\chi^A(s_i,s_j)$ *are more positively correlated signals in the sense that*

$$\frac{\chi^A(s,s)}{\int_{\underline{s}}^{s} \chi^A(s,s_j)\, ds_j} > \frac{\chi^B(s,s)}{\int_{\underline{s}}^{s} \chi^B(s,s_j)\, ds_j}$$

and

$$\frac{\chi^A(s,s)}{\int_s^{\overline{s}} \chi^A(s,s_j)\, ds_j} > \frac{\chi^B(s,s)}{\int_s^{\overline{s}} \chi^B(s,s_j)\, ds_j}.$$

In particular, if increased transparency makes signals more positively correlated without changing any cost realisation, then this will lower mark-ups.

Proposition 8 can be simplified in the special case when signals are independent.

Proposition 9 *The symmetric Bayesian Nash equilibrium offer in a uniform-price auction where the pivotal status of producers is uncertain is given by:*

$$p(s) = c(s,s) + \int_{s}^{\overline{s}} \frac{dc(v,v)}{dv} \left(\frac{(1-F(v))\, q_H^{NP}\Pi^{NP} + F(v)\, q_L}{(1-F(s))\, q_H^{NP}\Pi^{NP} + F(s)\, q_L} \right)^{\frac{(q_H - q_L)}{q_H^{NP}\Pi^{NP} - q_L}} dv \tag{33}$$

if the signals are independent. If, in addition, costs are almost surely common knowledge, then (26) can be simplified to

$$p(s) = c(\underline{s},\underline{s}) + \left(\frac{q_L}{((1-F(s))\, q_H^{NP}\Pi^{NP} + F(s)\, q_L)} \right)^{\frac{(q_H - q_L)}{q_H^{NP}\Pi^{NP} - q_L}} (\overline{p} - c(\underline{s},\underline{s})), \tag{34}$$

where $F(s)$ is a firm's marginal distribution for receiving the signal s.

As shown by von der Fehr and Harbord (1993), the alternative equilibrium (the high-price equilibrium), where the highest offer price is at the reservation price, does not exist when the pivotal status of producers is uncertain. The reason is that the lowest bidder would find it optimal to choose an offer just below the reservation price, if its offer price will set the transaction price with a positive probability. But this means that the high-price bidder, in its turn, would find it optimal to deviate and slightly undercut the low-price bidder.

4.3 Ranking of auction formats

We already know from Section 4.1.1 that the two auction formats are equivalent in the non-pivotal case. Below we show that there are cases where the two auction formats are equivalent also when producers are pivotal with a positive probability, so that $q_L > 0$.

Lemma 4 *If signals are independent and costs are almost surely common knowledge, then the expected profit for a producer is given by*

$$\pi(s) = q_L (\overline{p} - c(\underline{s},\underline{s})), \tag{35}$$

for both the uniform-price and the discriminatory auction and irrespective of the probability that the highest bidder is pivotal.

There is a simple intuition for this equivalence result. If signals are independent and costs are almost surely common knowledge, then our Bayesian NE corresponds to a mixed-strategy NE. In a mixed-strategy NE a producer gets the same expected payoff irrespective of the chosen offer price, as long as the price is chosen with a positive probability in equilibrium. Thus irrespective of the auction format, the expected payoff can be calculated from the case when a producer chooses the reservation price and is almost surely undercut by the competitor, which gives the payoff in (35). The proposition below generalizes this to uncertain costs.

Proposition 10 *If signals are independent then the expected profit for a producer is the same for the uniform-price and the discriminatory auction and independent of the probability that the highest bidder is pivotal.*

In comparison to independent signals, it follows from Corollaries 1 and 4 that more positively correlated signals will reduce mark-ups in both auctions for a given marginal distribution $F(s)$. If correlation of signals depends on the magnitude of the signals, then there are cases where prices increase in one auction and decrease in the other, in comparison to the case with independent signals. In such cases, it is straightforward to rank the auction formats. We can for example deduce the following from Definition 2, Proposition 1, Definition 6 and Proposition 8.

Corollary 5 *Assume that the cost uncertainty is sufficiently large so that an equilibrium exists in both auctions. After observing a signal s, a producer will in expectation receive a higher payoff in a discriminatory auction in comparison to a uniform-price auction, if the correlation of the two signals decrease with respect to s at a sufficient rate, such that i) the expected prices are higher in the discriminatory auction than for independent signals with the same marginal distribution $F(s)$, i.e.:*

$$H^*(s) = \frac{\chi(s,s)(q_H - q_L)}{\int_s^{\overline{s}} \chi(s,s_j)\, ds_j q_H + \int_{\underline{s}}^{s} \chi(s,s_j)\, ds_j q_L} \leq \frac{f(s)(q_H - q_L)}{(1-F(s))\, q_H + F(s)\, q_L},$$

and ii) the expected prices are lower in the uniform-price auction than for independent signals, i.e.:

$$\widetilde{H}(s) = \frac{\chi(s,s)(q_H - q_L)}{\int_s^{\overline{s}} \chi(s,s_j)\, ds_j q_H^{NP} \Pi^{NP} + \int_{\underline{s}}^{s} \chi(s,s_j)\, ds_j q_L} \geq \frac{f(s)(q_H - q_L)}{(1-F(s))\, q_H^{NP} \Pi^{NP} + F(s)\, q_L}.$$

Thus an auctioneer tends to prefer uniform-pricing when the correlation of the two signals decrease with respect to s. Under these circumstances advantages with uniform-pricing tends to increase if producers are pivotal with a higher probability, i.e. Π^{NP} decreases, for fixed q_L and q_H. On the other hand, we can also show that:

Corollary 6 *Assume that the cost uncertainty is sufficiently large so that an equilibrium exists in both auctions. After observing a signal s, a producer will in expectation receive a lower payoff in a discriminatory auction in comparison to a uniform-price auction, if the correlation of the two signals increase with respect to s at a sufficient rate, such that i) the expected prices are lower in the discriminatory auction than for independent signals, i.e.:*

$$H^*(s) = \frac{\chi(s,s)(q_H - q_L)}{\int_s^{\overline{s}} \chi(s,s_j)\, ds_j q_H + \int_{\underline{s}}^{s} \chi(s,s_j)\, ds_j q_L} \geq \frac{f(s)(q_H - q_L)}{(1-F(s))\, q_H + F(s)\, q_L},$$

and ii) the expected prices are higher in the uniform-price auction than for independent signals, i.e.:

$$\widetilde{H}(s) = \frac{\chi(s,s)(q_H - q_L)}{\int_s^{\overline{s}} \chi(s,s_j)\, ds_j q_H^{NP} \Pi^{NP} + \int_{\underline{s}}^{s} \chi(s,s_j)\, ds_j q_L} \leq \frac{f(s)(q_H - q_L)}{(1-F(s))\, q_H^{NP} \Pi^{NP} + F(s)\, q_L}.$$

5 Concluding discussion

We consider a duopoly model of a divisible-good procurement auction with production uncertainty, such as a wholesale electricity market. Each producer receives a private signal with imperfect cost information from a bivariate probability distribution (known to each producer) and then chooses one offer price for its whole production capacity. Demand and production capacities could also be uncertain.

We solve for a symmetric Bayesian NE. We show that in expectation uniform and discriminatory pricing are equivalent when signals are independent. Consumers tend to favour discriminatory pricing when cost signals are more correlated for higher values of the signals. The opposite is true when signals are less correlated for higher values of the signals. Advantages and disadvantages with uniform pricing tend to be amplified if producers are pivotal with a higher probability.

Markups tend to fall for both auction formats if producers receive less noisy cost information. The latter concurs with a similar result in Vives (2011), who studies another bidding format, and with Milgrom and Weber's (1982) result for single object auctions. Taken together, these results support the measures taken by the European Commission to increase transparency in European wholesale electricity markets. On the other hand, in a repeated game, there is a risk that increased transparency will facilitate tacit collusion as argued by von der Fehr (2013).

We are concerned that cost uncertainty and asymmetric information could result in significant mark-ups in hydro dominated electricity markets with scarce water. This could help explain the extraordinarily high price-periods that typically accompany scarcity of water in such markets. One measure that could mitigate this is to clearly define contingency plans for intervention by the market operator and the regulator under extreme system conditions. In hydro-dominated markets, improved political transparency has similar pro-competitive effects as improved market transparency.

If producers are pivotal, then disclosure of information is only beneficial up to a point. A pivotal producer can deviate to the reservation price, which ensures it a minimum profit. Thus, there is a lower bound on how small equilibrium mark-ups can become.

We show that equilibrium offers in a discriminatory auction are determined by the expected sales of the producer with the highest and lowest offer price, respectively. The variance of these sales – due to demand shocks, production outages and volatile renewable production – will not influence the bidding behavior of producers. Bidding in the uniform-price auction is also insensitive to this variance, as long as it is not sufficiently large to occasionally change the pivotal status of at least one producer. Moreover, for given expected sales and independent signals, the probability that a producer is pivotal in a uniform-price auction does not influence expected payoffs.

Unlike Vives (2011), our results do not depend on the extent to which the cost uncertainty is private, interdependent or common. In his setting, producers choose linear supply functions and can therefore condition their output on every price. To a larger extent than in our model, his bidding format allows producers to condition their output on the competitor's information. This leads to equilibria that are very unfavourable for the auctioneer when costs have common uncertainties. Thus, our results and the results in Vives (2011) indicate that when the cost uncertainty is common or strongly interdependent, which should often be the case in wholesale electricity markets, then it should be optimal to limit the number of allowed steps in producers' supply functions in order to give producers less freedom to condition their output on competitors' signals. More information (i.e. less noisy signals) before a uniform-price auction starts improves competition, but mark-ups increase if a producer learns information, or rather conditions its supply on the competitor's information, during the auction. This also suggests that bidding formats, for which producers must choose piece-wise linear supply functions, as in the Nordic countries (Nord Pool) and France (Power Next), can be harmful for market competitiveness.

Results are analogous for multi-unit sales auctions, such as security auctions. In particular, given that bidders' marginal valuation of financial instruments should be fairly constant, we believe that it would be beneficial for an auctioneer of securities or emission permits to use a uniform-price auction with a bidding format that significantly restricts the number of steps in the bid-schedules. Purchase constraints in sales auctions make bidding less competitive, at least in a one shot game.

6 References

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Appendix

Before proving the lemmas and propositions that have been presented in the main text, we will derive some results that will be used throughout these proofs. By assumption, $p_j(s_j)$ is monotonic and invertible. Thus, we get

$$\begin{array}{rcl} \Pr\left(p_j \geq p_i \middle| s_i\right) & = & \frac{\int_{p_j^{-1}(p_i)}^{\overline{s}} \chi(s_i,s_j)ds_j}{\int_{\underline{s}}^{\overline{s}} \chi(s_i,s_j)ds_j} \\ \frac{\partial \Pr(p_j \geq p_i | s_i)}{\partial p_i} & = & \frac{-p_j^{-1\prime}(p_i)\chi\left(s_i,p_j^{-1}(p_i)\right)}{\int_{\underline{s}}^{\overline{s}} \chi(s_i,s_j)ds_j}, \end{array} \tag{36}$$

where the last result follows from Leibniz' rule. The results above and Leibniz' rule are used in the following derivations.

$$\begin{array}{rcl} \mathbb{E}\left[c_i(s_i,s_j)\middle| p_j \geq p_i\right] & = & \frac{\int_{p_j^{-1}(p_i)}^{\overline{s}} c_i(s_i,s_j)\chi(s_i,s_j)ds_j}{\int_{p_j^{-1}(p_i)}^{\overline{s}} \chi(s_i,s_j)ds_j} = \frac{\int_{p_j^{-1}(p_i)}^{\overline{s}} c_i(s_i,s_j)\chi(s_i,s_j)ds_j}{\Pr(p_j \geq p_i|s_i)\int_{\underline{s}}^{\overline{s}} \chi(s_i,s_j)ds_j} \\ \frac{\partial \mathbb{E}[c_i(s_i,s_j)|p_j \geq p_i]}{\partial p_i} & = & \frac{p_j^{-1\prime}(p_i)\chi\left(s_i,p_j^{-1}(p_i)\right)\int_{p_j^{-1}(p_i)}^{\overline{s}}\left(c_i(s_i,s_j)-c_i\left(s_i,p_j^{-1}(p_i)\right)\right)\chi(s_i,s_j)ds_j}{\left(\int_{p_j^{-1}(p_i)}^{\overline{s}} \chi(s_i,s_j)ds_j\right)^2} \\ & = & \frac{-\frac{\partial \Pr\left(p_j \geq p_i \middle| s_i\right)}{\partial p_i}\int_{p_j^{-1}(p_i)}^{\overline{s}}\left(c_i(s_i,s_j)-c_i\left(s_i,p_j^{-1}(p_i)\right)\right)\chi(s_i,s_j)ds_j}{\left(\Pr(p_j \geq p_i|s_i)\right)^2\int_{\underline{s}}^{\overline{s}} \chi(s_i,s_j)ds_j}. \end{array} \tag{37}$$

From (36) and (37), we have that:

$$\begin{array}{l} -\frac{\partial \mathbb{E}[c_i(s_i,s_j)|p_j \geq p_i]}{\partial p_i}\Pr\left(p_j \geq p_i \middle| s_i\right) - \mathbb{E}\left[c_i(s_i,s_j)\middle| p_j \geq p_i\right]\frac{\partial \Pr(p_j \geq p_i|s_i)}{\partial p_i} \\ = \left(\frac{\int_{p_j^{-1}(p_i)}^{\overline{s}}\left(c_i(s_i,s_j)-c_i\left(s_i,p_j^{-1}(p_i)\right)\right)\chi(s_i,s_j)ds_j}{\int_{p_j^{-1}(p_i)}^{\overline{s}} \chi(s_i,s_j)ds_j} - \frac{\int_{p_j^{-1}(p_i)}^{\overline{s}} c_i(s_i,s_j)\chi(s_i,s_j)ds_j}{\int_{p_j^{-1}(p_i)}^{\overline{s}} \chi(s_i,s_j)ds_j}\right)\frac{\partial \Pr(p_j \geq p_i|s_i)}{\partial p_i} \\ = -\frac{\int_{p_j^{-1}(p_i)}^{\overline{s}} c_i\left(s_i,p_j^{-1}(p_i)\right)\chi(s_i,s_j)ds_j}{\int_{p_j^{-1}(p_i)}^{\overline{s}} \chi(s_i,s_j)ds_j}\frac{\partial \Pr(p_j \geq p_i|s_i)}{\partial p_i} \\ = -c_i\left(s_i,p_j^{-1}(p_i)\right)\frac{\partial \Pr(p_j \geq p_i|s_i)}{\partial p_i}. \end{array} \tag{38}$$

Using the above equation, we can derive the following result:

$$\begin{array}{l} \left(1-\frac{\partial \mathbb{E}[c_i(s_i,s_j)|p_j \geq p_i]}{\partial p_i}\right)\Pr\left(p_j \geq p_i \middle| s_i\right) + \left(p_i - \mathbb{E}\left[c_i(s_i,s_j)\middle| p_j \geq p_i\right]\right)\frac{\partial \Pr(p_j \geq p_i|s_i)}{\partial p_i} \\ = \Pr\left(p_j \geq p_i \middle| s_i\right) + \left(p_i - c_i\left(s_i,p_j^{-1}(p_i)\right)\right)\frac{\partial \Pr(p_j \geq p_i|s_i)}{\partial p_i}. \end{array} \tag{39}$$

Similarly, from (36), we have that

$$
\begin{aligned}
1-\Pr\left(p_j \geq p_i \middle| s_i\right) &= \frac{\int_{\underline{s}}^{p_j^{-1}(p_i)} \chi(s_i,s_j)ds_j}{\int_{\underline{s}}^{\overline{s}} \chi(s_i,s_j)ds_j} \\
\mathbb{E}\left[c_i\left(s_i,s_j\right)\middle| p_j \leq p_i\right] &= \frac{\int_{\underline{s}}^{p_j^{-1}(p_i)} c_i(s_i,s_j)\chi(s_i,s_j)ds_j}{\int_{\underline{s}}^{p_j^{-1}(p_i)} \chi(s_i,s_j)ds_j} = \frac{\int_{\underline{s}}^{p_j^{-1}(p_i)} c_i(s_i,s_j)\chi(s_i,s_j)ds_j}{(1-\Pr(p_j\geq p_i|s_i))\int_{\underline{s}}^{\overline{s}} \chi(s_i,s_j)ds_j} \\
\frac{\partial \mathbb{E}[c_i(s_i,s_j)|p_j\leq p_i]}{\partial p_i} &= \frac{p_j^{-1\prime}(p_i)\chi\left(s_i,p_j^{-1}(p_i)\right)\int_{\underline{s}}^{p_j^{-1}(p_i)}\left(c_i\left(s_i,p_j^{-1}(p_i)\right)-c_i(s_i,s_j)\right)\chi(s_i,s_j)ds_j}{\left(\int_{\underline{s}}^{p_j^{-1}(p_i)} \chi(s_i,s_j)ds_j\right)^2} \\
&= \frac{-\frac{\partial \Pr\left(p_j\geq p_i \middle| s_i\right)}{\partial p_i}\int_{\underline{s}}^{p_j^{-1}(p_i)}\left(c_i\left(s_i,p_j^{-1}(p_i)\right)-c_i(s_i,s_j)\right)\chi(s_i,s_j)ds_j}{(1-\Pr(p_j\geq p_i|s_i))^2\int_{\underline{s}}^{\overline{s}} \chi(s_i,s_j)ds_j}.
\end{aligned}
\tag{40}
$$

It now follows from (40) that:

$$
\begin{aligned}
-\frac{\partial \mathbb{E}[c_i(s_i,s_j)|p_j\leq p_i]}{\partial p_i}\left(1-\Pr\left(p_j\geq p_i\middle| s_i\right)\right) + \mathbb{E}\left[c_i\left(s_i,s_j\right)\middle| p_j\leq p_i\right]\frac{\partial \Pr(p_j\geq p_i|s_i)}{\partial p_i} \\
= \frac{\partial \Pr(p_j\geq p_i|s_i)}{\partial p_i} c_i\left(s_i,p_j^{-1}\left(p_i\right)\right).
\end{aligned}
\tag{41}
$$

Discriminatory auction

Proof. (Lemma 1) It follows from (7) that

$$
\begin{aligned}
\frac{\partial \pi_i(s_i)}{\partial p_i} &= \left(1-\frac{\partial \mathbb{E}[c_i(s_i,s_j)|p_j\geq p_i]}{\partial p_i}\right)\Pr\left(p_j\geq p_i\middle| s_i\right)q_H \\
&+\left(p_i-\mathbb{E}\left[c_i\left(s_i,s_j\right)\middle| p_j\geq p_i\right]\right)\frac{\partial \Pr(p_j\geq p_i|s_i)}{\partial p_i}q_H \\
&+\left(1-\frac{\partial \mathbb{E}[c_i(s_i,s_j)|p_j\leq p_i]}{\partial p_i}\right)\left(1-\Pr\left(p_j\geq p_i\middle| s_i\right)\right)q_L \\
&-\left(p_i-\mathbb{E}\left[c_i\left(s_i,s_j\right)\middle| p_j\leq p_i\right]\right)\frac{\partial \Pr(p_j\geq p_i|s_i)}{\partial p_i}q_L.
\end{aligned}
\tag{42}
$$

Using (39) and the relation in (41) yields:

$$
\begin{aligned}
\frac{\partial \pi_i(s_i)}{\partial p_i} &= \Pr\left(p_j\geq p_i\middle| s_i\right)q_H + \left(p_i-c_i\left(s_i,p_j^{-1}\left(p_i\right)\right)\right)\frac{\partial \Pr(p_j\geq p_i|s_i)}{\partial p_i}q_H \\
&+c_i\left(s_i,p_j^{-1}\left(p_i\right)\right)\frac{\partial \Pr(p_j\geq p_i|s_i)}{\partial p_i}q_L \\
&+\left(1-\Pr\left(p_j\geq p_i\middle| s_i\right)\right)q_L - p_i\frac{\partial \Pr(p_j\geq p_i|s_i)}{\partial p_i}q_L,
\end{aligned}
$$

which gives (8). ■

Proof. (Proposition 1) We solve for symmetric strategies, so that $p_i(s) = p_j(s) = p(s)$ and $p_j^{-1}(p_i) = s$. Hence, we get the following first-order condition from (8).

$$
\begin{aligned}
\frac{\partial \pi_i(s_i)}{\partial p_i} &= \Pr\left(p_j\geq p\middle| s\right)q_H + \left(1-\Pr\left(p\geq p\middle| s\right)\right)q_L \\
&+\left(p-c_i\left(s,s\right)\right)\frac{\partial \Pr(p_j\geq p|s)}{\partial p}\left(q_H-q_L\right)=0.
\end{aligned}
$$

Using (36) and that $p_j^{-1\prime}(p_i) = \frac{1}{p'(s)}$, the condition can be written as follows:

$$
\int_s^{\overline{s}} \chi\left(s,s_j\right)ds_j q_H + \int_{\underline{s}}^{s} \chi\left(s,s_j\right)ds_j q_L - \frac{\left(p-c\left(s,s\right)\right)}{p'\left(s\right)}\chi\left(s,s\right)\left(q_H-q_L\right)=0.
$$

We can use the definition in (12) to write the first-order condition on the following form:

$$p'(s) - (p - c(s,s)) H^*(s) = 0. \tag{43}$$

Multiplication by the integrating factor $e^{\int_s^{\overline{s}} H^*(u)du}$ yields:

$$\begin{aligned} &p'(s)\, e^{\int_s^{\overline{s}} H^*(u)du} - pH^*(s)\, e^{\int_s^{\overline{s}} H^*(u)du} \\ &= -c(s,s)\, H^*(s)\, e^{\int_s^{\overline{s}} H^*(u)du}, \end{aligned}$$

so that

$$\frac{d}{ds}\left(p(s)\, e^{\int_s^{\overline{s}} H^*(u)du}\right) = -c(s,s)\, H^*(s)\, e^{\int_s^{\overline{s}} H^*(u)du}.$$

Next we integrate both sides from s to $\overline{s}$.

$$\begin{aligned} \overline{p} - p(s)\, e^{\int_s^{\overline{s}} H^*(u)du} &= -\int_s^{\overline{s}} c(v,v)\, H^*(v)\, e^{\int_v^{\overline{s}} H^*(u)du}\, dv \\ p(s) = \overline{p} e^{-\int_s^{\overline{s}} H^*(u)du} &+ \int_s^{\overline{s}} c(v,v)\, H^*(v)\, e^{-\int_s^{v} H^*(u)du} dv. \end{aligned}$$

We use integration by parts to rewrite the above expression as follows:

$$p(s) = \overline{p} e^{-\int_s^{\overline{s}} H^*(u)du} + \left[-c(v,v)\, e^{-\int_s^{v} H^*(u)du}\right]_s^{\overline{s}} + \int_s^{\overline{s}} \frac{dc(v,v)}{dv} e^{-\int_s^{v} H^*(u)du} dv,$$

which gives (14), because $c(\overline{s},\overline{s}) = \overline{p}$. It is clear from (14) that $p > c(s,s)$ for $s \in [\underline{s}, \overline{s})$. Hence, it follows from (43) that $p'(s) > 0$ for $s \in [\underline{s}, \overline{s})$.

It remains to show that $p(s)$ is an equilibrium. It follows from (8) and (36) that

$$\begin{aligned} \frac{\partial \pi_i(s)}{\partial p} &= \frac{\int_{p_j^{-1}(p)}^{\overline{s}} \chi(s,s_j)\, ds_j}{\int_{\underline{s}}^{\overline{s}} \chi(s,s_j)\, ds_j} q_H + \frac{\int_{\underline{s}}^{p_j^{-1}(p)} \chi(s,s_j)\, ds_j}{\int_{\underline{s}}^{\overline{s}} \chi(s,s_j)\, ds_j} q_L \\ &- \frac{p_j^{-1\prime}(p)\, \chi\left(s, p_j^{-1}(p)\right)}{\int_{\underline{s}}^{\overline{s}} \chi(s,s_j)\, ds_j} \left(p - c_i\left(s, p_j^{-1}(p)\right)\right)(q_H - q_L). \\ \frac{\partial \pi_i(s)}{\partial p} &= \frac{\chi\left(s, p_j^{-1}(p)\right)}{\int_{\underline{s}}^{\overline{s}} \chi(s,s_j)\, ds_j} \left(\frac{\int_{p_j^{-1}(p)}^{\overline{s}} \chi(s,s_j)\, ds_j}{\chi\left(s, p_j^{-1}(p)\right)} q_H + \frac{\int_{\underline{s}}^{p_j^{-1}(p_i)} \chi(s,s_j)\, ds_j}{\chi\left(s, p_j^{-1}(p)\right)} q_L \right. \\ &\left. - p_j^{-1\prime}(p)\left(p - c_i\left(s, p_j^{-1}(p)\right)\right)(q_H - q_L)\right). \end{aligned}$$

We know that $\frac{\partial \pi_i(s)}{\partial p} = 0$ for $s = p_j^{-1}(p)$. Thus whenever $\frac{d}{ds}\left(\frac{\int_x^{\overline{s}} \chi(s,s_j)ds_j q_H + q_L \int_{\underline{s}}^{x} \chi(s,s_j)ds_j}{\chi(s,x)}\right) \geq 0$, it follows from the above and (3) that $\frac{\partial \pi_i(s)}{\partial p} > 0$ when $s > p_j^{-1}(p) \iff p < p_j(s)$ and that $\frac{\partial \pi_i(s)}{\partial p} < 0$ when $s < p_j^{-1}(p) \iff p > p_j(s)$. Thus, $p(s)$ globally maximizes the profit of firm i for any signal s when the inequality in (15) is satisfied.

In the special case when costs are almost surely common knowledge, we have $\frac{dc(v,v)}{dv} \searrow 0$ for $v < \overline{s}$, so it follows from (14) that

$$p(s) \to c(\underline{s},\underline{s}) + e^{-\int_s^{\overline{s}} H^*(u)du} \int_s^{\overline{s}} \frac{dc(v,v)}{dv} dv,$$

which gives (16).

(Proposition 2) First we note that $\chi(s, s_j) = f(s) f(s_j)$ for independent signals, so the inequality

$$\begin{aligned} & \frac{d}{ds}\left(\frac{\int_x^{\overline{s}} \chi(s, s_j)\, ds_j q_H + q_L \int_{\underline{s}}^x \chi(s, s_j)\, ds_j}{\chi(s, x)}\right) \\ &= \frac{d}{ds}\left(\frac{\int_x^{\overline{s}} f(s) f(s_j)\, ds_j q_H + q_L \int_{\underline{s}}^x f(s) f(s_j)\, ds_j}{f(s) f(x)}\right) = \\ &= \frac{d}{ds}\left(\frac{\int_x^{\overline{s}} f(s_j)\, ds_j q_H + q_L \int_{\underline{s}}^x f(s_j)\, ds_j}{f(x)}\right) = 0 \geq 0 \end{aligned}$$

is satisfied. Thus, the global second-order condition in (15) is satisfied. Moreover, for independent signals, we have from Definition 2 that

$$\begin{aligned} H^*(s) &= \frac{f(s)(q_H - q_L)}{\int_s^{\overline{s}} f(s_j)\, ds_j q_H + \int_{\underline{s}}^s f(s_j)\, ds_j q_L} \\ &= -\frac{d}{ds} \ln\left(\int_s^{\overline{s}} f(s_j)\, ds_j q_H + \int_{\underline{s}}^s f(s_j)\, ds_j q_L\right). \end{aligned}$$

Thus, (14) can be written as in (17).

In case costs are almost surely common knowledge, so that $\frac{dc(v,v)}{dv} \searrow 0$ for $v < \overline{s}$, (17) can be simplified to (18) as follows:

$$\begin{aligned} p(s) &= c(\underline{s}, \underline{s}) + \left(\frac{q_L}{((1 - F(s)) q_H + F(s) q_L)}\right) \int_s^{\overline{s}} \frac{dc(v, v)}{dv} dv \\ &= c(\underline{s}, \underline{s}) + \left(\frac{q_L}{((1 - F(s)) q_H + F(s) q_L)}\right) (\overline{p} - c(\underline{s}, \underline{s})). \end{aligned}$$

■

Non-pivotal case

The following lemma is useful when deriving results for the non-pivotal case.

Lemma 5 *$e^{-\int_s^v H(u)du} > 0$ for $\underline{s} \leq s < v < \overline{s}$ and $e^{-\int_s^{\overline{s}} H(u)du} = 0$ for $\underline{s} \leq s < \overline{s}$.*

Proof. It follows from (21) that

$$H(u) = \frac{\chi(u, u)}{\int_u^{\overline{s}} \chi(u, s_j)\, ds_j} = -\frac{d}{du} \ln\left(\int_u^{\overline{s}} \chi(u, s_j)\, ds_j\right) + \frac{\int_u^{\overline{s}} \chi_1(u, s_j)\, ds_j}{\int_u^{\overline{s}} \chi(u, s_j)\, ds_j}. \quad (44)$$

The assumptions that we make for the joint probability density imply that $\frac{\int_u^{\overline{s}} \chi_1(u,s_j)ds_j}{\int_u^{\overline{s}} \chi(u,s_j)ds_j}$ is bounded. Thus, $e^{-\int_s^v H(u)du}$ is strictly positive, unless

$$
\begin{aligned}
e^{\left[\ln\left(\int_u^{\overline{s}} \chi(u,s_j)ds_j\right)\right]_s^v} &= e^{\ln\left(\int_v^{\overline{s}} \chi(v,s_j)ds_j\right)-\ln\left(\int_s^{\overline{s}} \chi(s,s_j)ds_j\right)} \\
&= \frac{\int_v^{\overline{s}} \chi\left(v, s_j\right) ds_j}{\int_s^{\overline{s}} \chi\left(s, s_j\right) ds_j}
\end{aligned}
$$

is equal to zero. This is the case if and only if $\int_v^{\overline{s}} \chi\left(v, s_j\right) ds_j = 0$. It follows from the assumptions that we make on the joint probability distribution that this is the case if and only if $v = \overline{s}$. ■

Proof. (Proposition 3) We have $q_L = 0$ in the non-pivotal case, so it is evident that $H^*(s)$ simplifies to (21). For weakly affiliated signals, we have $\frac{d}{ds}\left(\frac{\chi(s,s_j)}{\chi(s,x)}\right) \geq 0$ if $s_j \geq x$, which ensures that the global second-order condition in (15) is satisfied when $q_L = 0$. The result now follows from Proposition 1.

By definition, we have that $\frac{dc(v,v)}{dv} = 0$ for $s < \overline{s}$ when costs are almost surely common knowledge, so it follows from (20) that

$$
\begin{aligned}
p(s) &= c\left(\underline{s}, \underline{s}\right) + e^{-\int_s^{\overline{s}} H(u)du} \int_s^{\overline{s}} \frac{dc(v,v)}{dv} dv \\
&= c\left(\underline{s}, \underline{s}\right) + e^{-\int_s^{\overline{s}} H(u)du} \left(\overline{p} - c\left(\underline{s}, \underline{s}\right)\right).
\end{aligned}
$$

It now follows from Lemma 5 above that equilibrium offers are perfectly competitive for $s < \overline{s}$ when costs are almost surely common knowledge. ■

Uniform-price auction

The following derivations will be useful when analysing uniform-price auctions. It follows from (36) and Leibniz' rule that:

$$
\begin{aligned}
\mathbb{E}\left[p_j - c_i\left(s_i, s_j\right) \middle| p_j \geq p_i\right] &= \frac{\int_{p_j^{-1}(p_i)}^{\overline{s}} (p_j(s_j)-c_i(s_i,s_j))\chi(s_i,s_j)ds_j}{\int_{p_j^{-1}(p_i)}^{\overline{s}} \chi(s_i,s_j)ds_j} \\
&= \frac{\int_{p_j^{-1}(p_i)}^{\overline{s}} (p_j(s_j)-c_i(s_i,s_j))\chi(s_i,s_j)ds_j}{\Pr(p_j \geq p_i|s_i) \int_{\underline{s}}^{\overline{s}} \chi(s_i,s_j)ds_j}
\end{aligned}
$$

$$
\frac{\partial \mathbb{E}[p_j - c_i(s_i,s_j)|p_j \geq p_i]}{\partial p_i} = \frac{-\frac{\partial \Pr\left(p_j \geq p_i \middle| s_i\right)}{\partial p_i} \int_{p_j^{-1}(p_i)}^{\overline{s}} \left(p_j(s_i)-c_i(s_i,s_j)-\left(p_i-c_i\left(s_i,p_j^{-1}(p_i)\right)\right)\right)\chi(s_i,s_j)ds_j}{(\Pr(p_j \geq p_i|s_i))^2 \int_{\underline{s}}^{\overline{s}} \chi(s_i,s_j)ds_j}. \tag{45}
$$

Similar to (38), it can be shown that:

$$
\begin{aligned}
&\frac{\partial \mathbb{E}[p_j - c_i(s_i,s_j)|p_j \geq p_i]}{\partial p_i} \Pr\left(p_j \geq p_i \middle| s_i\right) + \mathbb{E}\left[p_j - c_i\left(s_i, s_j\right) \middle| p_j \geq p_i\right] \frac{\partial \Pr(p_j \geq p_i|s_i)}{\partial p_i} \\
&= \frac{\partial \Pr(p_j \geq p_i|s_i)}{\partial p_i} \left(p_i - c_i\left(s_i, p_j^{-1}(p_i)\right)\right).
\end{aligned} \tag{46}
$$

Proof. (Lemma 2) We have from (22) that

$$\begin{aligned}\frac{\partial\pi_i(s_i)}{\partial p_i} &= \frac{\partial\mathbb{E}[p_j-c_i(s_i,s_j)|p_j\geq p_i]}{\partial p_i}\Pr\left(p_j\geq p_i|\,s_i\right)q_H \\ &+\mathbb{E}\left[p_j-c_i\left(s_i,s_j\right)|\,p_j\geq p_i\right]\frac{\partial\Pr(p_j\geq p_i|s_i)}{\partial p_i}q_H \\ &+\left(1-\frac{\partial\mathbb{E}[c_i(s_i,s_j)|p_j\leq p_i]}{\partial p_i}\right)\left(1-\Pr\left(p_j\geq p_i|\,s_i\right)\right)q_L \\ &-\left(p_i-\mathbb{E}\left[c_i\left(s_i,s_j\right)|\,p_j\leq p_i\right]\right)\frac{\partial\Pr(p_j\geq p_i|s_i)}{\partial p_i}q_L.\end{aligned} \tag{47}$$

Next we use (41) and (46) to simplify this expression to (23). ■

Proof. (Proposition 4) Note that (23) is very similar to (8) and the statements can be proven in a very similar way to the proof of Proposition 1. In particular, it can be shown that the first-order condition is given by:

$$\int_{\underline{s}}^{s}\chi\left(s,s_j\right)ds_jq_L-\frac{\left(p-c\left(s,s\right)\right)}{p'\left(s\right)}\chi\left(s,s\right)\left(q_H-q_L\right)=0$$
$$p'\left(s\right)-p\hat{H}\left(s\right)=-c\left(s,s\right)\hat{H}\left(s\right).$$

The property of unaffiliated signals in (2) implies that $\frac{d}{ds}\left(\frac{\int_{\underline{s}}^{x}\chi(s,s_j)ds_j}{\chi(s,x)}\right)\geq 0$ for $x>s_j$, which is sufficient to ensure global optimality. ■

Proof. (Proposition 5) In the non-pivotal case, the lowest offer price sets the market price and the winning producer (with the lowest offer price) gets to produce the entire demand, which corresponds to a first-price procurement auction. In the just pivotal case, the highest offer price sets the market price and the winning producer gets to produce the entire demand, which corresponds to a second-price auction. Thus, the statement follows from Milgrom and Weber (1982). ■

Proof. (Proposition 6)

For independent signals we have $\chi\left(s,s_j\right)=f\left(s\right)f\left(s_j\right)$, so it follows from Definition 5 that

$$\begin{aligned}\hat{H}\left(u\right)&=\frac{\left(q_H-q_L\right)f\left(u\right)}{q_L\int_{\underline{s}}^{u}f\left(s_j\right)ds_j}\\ &=\frac{d}{du}\frac{\left(q_H-q_L\right)\ln\left(\int_{\underline{s}}^{u}f\left(s_j\right)ds_j\right)}{q_L}.\end{aligned}$$

Thus (24) can be written as in (26). Independent signals are weakly affiliated. This ensures that the sufficiency condition in Proposition 4 is satisfied.

In the special case when costs are almost surely common knowledge, we have by definition that $\frac{dc(v,v)}{dv}=0$ for $s<\overline{s}$, so it follows from (26) that

$$p\left(s\right)=c\left(s,s\right)+\left(F\left(s\right)\right)^{\frac{\left(q_H-q_L\right)}{q_L}}\int_{s}^{\overline{s}}\frac{dc\left(v,v\right)}{dv}dv,$$

where $F\left(s\right)$ is the marginal probability distribution. This gives (27), because by assumption $c\left(\overline{s},\overline{s}\right)=\overline{p}$. ■

Proof. (Proposition 7) We let $G(P)$ be the probability that a producer's offer price is below P. This is the same as the probability that s is below $p^{-1}(P)$. Hence, it follows from (27) that

$$G(P) = \left(\frac{P-c}{\overline{p}-c}\right)^{\frac{q_L}{q_H-q_L}}.$$

From the theory of order statistics we know that

$$G^2(P) = \left(\frac{P-c}{\overline{p}-c}\right)^{\frac{2q_L}{q_H-q_L}}$$

is the probability distribution of the highest offer price, which sets the price. Hence, the probability density of the market price is given by $2G(p)G'(p)$. Thus, the expected market price is given by:

$$\int_c^{\overline{p}} 2G(p)G'(p)\,pdp = \left[G^2(p)\,p\right]_c^{\overline{p}} - \int_c^{\overline{p}} G^2(p)\,dp$$
$$= \overline{p} - \left[\frac{(p-c)^{\frac{2q_L}{q_H-q_L}+1}}{\left(\frac{2q_L}{q_H-q_L}+1\right)(\overline{p}-c)^{\frac{2q_L}{q_H-q_L}}}\right]_c^{\overline{p}} = \overline{p} - \frac{(\overline{p}-c)(q_H-q_L)}{q_H+q_L}.$$

■

Proof. (Lemma 3) The demand and production capacity uncertainties are independent of the signals and cost uncertainties. Thus, when producers are pivotal with certainty, the expected profit of firm i when receiving signal s_i is:

$$\begin{aligned}\pi_i(s_i) = {} & \mathbb{E}\left[p_j - c_i(s_i,s_j)\,|\,p_j \geq p_i\right]\Pr(p_j \geq p_i|\,s_i)\,q_H^P\left(1-\Pi^{NP}\right) \\ & +\mathbb{E}\left[p_i(s_i) - c_i(s_i,s_j)\,|\,p_j \geq p_i\right]\Pr(p_j \geq p_i|\,s_i)\,q_H^{NP}\Pi^{NP} \\ & +\left(p_i(s_i) - \mathbb{E}\left[c_i(s_i,s_j)\,|\,p_j \leq p_i\right]\right)\left(1-\Pr(p_j \geq p_i|\,s_i)\right)q_L,\end{aligned} \tag{48}$$

where

$$q_H^P = \mathbb{E}\left[\widetilde{q}_H|\,\widetilde{q}_H < D\right].$$

It follows from differentiation of (48) and the relations in (39), (41) and (46) that:

$$\begin{aligned}\frac{\partial \pi_i(s_i)}{\partial p_i} = {} & \frac{\partial \Pr(p_j \geq p_i|s_i)}{\partial p_i}\left(p_i - c_i\left(s_i, p_j^{-1}(p_i)\right)\right)q_H^P\left(1-\Pi^{NP}\right) \\ & +\left(\Pr(p_j \geq p_i|\,s_i) + \left(p_i - c_i\left(s_i,p_j^{-1}(p_i)\right)\right)\frac{\partial \Pr(p_j \geq p_i|s_i)}{\partial p_i}\right)q_H^{NP}\Pi^{NP} \\ & +\frac{\partial \Pr(p_j \geq p_i|s_i)}{\partial p_i}\left(c_i\left(s_i,p_j^{-1}(p_i)\right) - p_i\right)q_L \\ & +\left(1-\Pr(p_j \geq p_i|\,s_i)\right)q_L,\end{aligned} \tag{49}$$

so

$$\begin{aligned}\frac{\partial \pi_i(s_i)}{\partial p_i} = {} & \frac{\partial \Pr(p_j \geq p_i|s_i)}{\partial p_i}\left(p_i - c_i\left(s_i,p_j^{-1}(p_i)\right)\right)\left(q_H^P\left(1-\Pi^{NP}\right) + q_H^{NP}\Pi^{NP} - q_L\right) \\ & +\Pr(p_j \geq p_i|\,s_i)\,q_H^{NP}\Pi^{NP} + \left(1-\Pr(p_j \geq p_i|\,s_i)\right)q_L,\end{aligned}$$

which can be simplified to (29), because $q^H = q_H^P\left(1-\Pi^{NP}\right) + q_H^{NP}\Pi^{NP}$. ■

Proof. (Proposition 8) The proof is similar to the proof of Proposition 1.

■

Proof. (Proposition 9) First we note that $\chi(s, s_j) = f(s) f(s_j)$ for independent signals, so the inequality

$$\begin{aligned} &\frac{d}{ds}\left(\frac{\int_x^{\overline{s}} f(s) f(s_j) ds_j q_H^{NP}\Pi^{NP} + q_L \int_{\underline{s}}^x f(s) f(s_j) ds_j}{f(s) f(x)}\right) \\ &= \frac{d}{ds}\left(\frac{\int_x^{\overline{s}} f(s_j) ds_j q_H^{NP}\Pi^{NP} + q_L \int_{\underline{s}}^x f(s_j) ds_j}{f(x)}\right) = 0 \geq 0 \end{aligned}$$

is satisfied. Thus, the global second-order condition in (31) is satisfied. Moreover, for independent signals, we have from Definition 6 that

$$\begin{aligned} \tilde{H}(s) &= \frac{f(s)(q_H - q_L)}{\int_s^{\overline{s}} f(s_j) ds_j q_H^{NP}\Pi^{NP} + \int_{\underline{s}}^s f(s_j) ds_j q_L} \\ &= -\frac{d}{ds}\ln\left((1 - F(s)) q_H^{NP}\Pi^{NP} + F(s) q_L\right) \frac{(q_H - q_L)}{q_H^{NP}\Pi^{NP} - q_L}. \end{aligned} \tag{50}$$

Thus, (30) can be written as in (33).

In case costs are almost surely common knowledge, so that $\frac{dc(v,v)}{dv} \searrow 0$ for $v < \overline{s}$, (33) can be simplified to (34) as follows:

$$\begin{aligned} p(s) &= c(\underline{s}, \underline{s}) + \left(\frac{q_L}{(1 - F(s)) q_H^{NP}\Pi^{NP} + F(s) q_L}\right)^{\frac{(q_H - q_L)}{q_H^{NP}\Pi^{NP} - q_L}} \int_s^{\overline{s}} \frac{dc(v, v)}{dv} dv \\ &= c(\underline{s}, \underline{s}) + \left(\frac{q_L}{((1 - F(s)) q_H^{NP}\Pi^{NP} + F(s) q_L)}\right)^{\frac{(q_H - q_L)}{q_H^{NP}\Pi^{NP} - q_L}} (\overline{p} - c(\underline{s}, \underline{s})), \end{aligned}$$

where $F(s)$ is the marginal probability distribution. ■

Ranking of auction formats

Proof. (Lemma 4)

For the discriminatory auction, it follows directly from (7) and (18) that

$$\begin{aligned} \pi(s) &= (p(s) - c(\underline{s}, \underline{s}))(1 - F(s)) q_H + (p(s) - c(\underline{s}, \underline{s})) F(s) q_L \\ &= q_L (\overline{p} - c(\underline{s}, \underline{s})), \end{aligned}$$

when costs are almost surely common knowledge and signals are independent. Going through the same calculation for the uniform-price auction is rather tedious, because the winning producer is sometimes paid the offer price of the losing producer, so the expected transaction price is less straightforward. Thus we use a different approach for the uniform-price auction. It follows from (49) that

$$\begin{gathered}\frac{\partial\pi_i}{\partial p_i}=\frac{\partial\Pr(p_j\geq p_i)}{\partial p_i}\left(p_i-c_i\left(\underline{s},\underline{s}\right)\right)q_H^P\left(1-\Pi^{NP}\right)\\+\left(\Pr\left(p_j\geq p_i\right)+\left(p_i-c_i\left(\underline{s},\underline{s}\right)\right)\frac{\partial\Pr(p_j\geq p_i)}{\partial p_i}\right)q_H^{NP}\Pi^{NP}\\+\frac{\partial\Pr(p_j\geq p_i)}{\partial p_i}\left(c_i\left(\underline{s},\underline{s}\right)-p_i\right)q_L\\+\left(1-\Pr\left(p_j\geq p_i\right)\right)q_L=0\end{gathered}\tag{51}$$

whenever signals are independent, $s_i<\overline{s}$ and costs are almost surely common knowledge. Hence, in equilibrium the expected payoff of a producer will not change if it changes the offer price in the range $[c,\overline{p}]$ for a given signal. This is expected as this special case corresponds to a mixed-strategy NE in accordance with Harsanyi's purification theorem. Thus to calculate the expected equilibrium payoff for producer i we can assume that it makes an offer at $\overline{p}$. The competitor plays the equilibrium strategy, so it will almost surely undercut $\overline{p}$, i.e. $\Pr\left(p_j\geq p_i\right)=0$. The expected profit of producer i can now be calculated from (48):

$$\pi_i\left(s\right)=\left(\overline{p}-c\left(\underline{s},\underline{s}\right)\right)q_L.$$

■

Proof. (Proposition 10) It follows from (48) and (30) that the expected revenue of a producer in a uniform price auction after observing the signal s is:

$$\begin{aligned}R\left(s\right)&=\frac{\int_s^{\overline{s}}p(s_j)\chi(s,s_j)ds_jq_H^P\left(1-\Pi^{NP}\right)+\int_s^{\overline{s}}p(s)\chi(s,s_j)ds_jq_H^{NP}\Pi^{NP}+\int_{\underline{s}}^{s}p(s)\chi(s,s_j)ds_jq_L}{\int_{\underline{s}}^{\overline{s}}\chi(s,s_j)ds_j}\\&=\frac{\int_s^{\overline{s}}\left(c(s_j,s_j)+\int_{s_j}^{\overline{s}}\frac{dc(v,v)}{dv}e^{-\int_{s_j}^{v}\widetilde{H}(u)du}dv\right)\chi(s,s_j)ds_jq_H^P\left(1-\Pi^{NP}\right)}{\int_{\underline{s}}^{\overline{s}}\chi(s,s_j)ds_j}\\&\quad+\frac{\int_s^{\overline{s}}\left(c(s,s)+\int_s^{\overline{s}}\frac{dc(v,v)}{dv}e^{-\int_s^{v}\widetilde{H}(u)du}dv\right)\chi(s,s_j)ds_jq_H^{NP}\Pi^{NP}}{\int_{\underline{s}}^{\overline{s}}\chi(s,s_j)ds_j}\\&\quad+\frac{\int_{\underline{s}}^{s}\left(c(s,s)+\int_s^{\overline{s}}\frac{dc(v,v)}{dv}e^{-\int_s^{v}\widetilde{H}(u)du}dv\right)\chi(s,s_j)ds_jq_L}{\int_{\underline{s}}^{\overline{s}}\chi(s,s_j)ds_j}.\end{aligned}\tag{52}$$

We can also use the expression above to calculate the expected revenue in the discriminatory auction by setting $\Pi^{NP}=1$. $R\left(s\right)$ can be rewritten as follows:

$$R\left(s\right)=\frac{\int_s^{\overline{s}}c\left(s,s\right)\chi\left(s,s_j\right)ds_jq_H}{\int_{\underline{s}}^{\overline{s}}\chi\left(s,s_j\right)ds_j}+\frac{\int_{\underline{s}}^{s}c\left(s,s\right)\chi\left(s,s_j\right)ds_jq_L}{\int_{\underline{s}}^{\overline{s}}\chi\left(s,s_j\right)ds_j}+\frac{\Theta\left(s\right)}{\int_{\underline{s}}^{\overline{s}}\chi\left(s,s_j\right)ds_j}.\tag{53}$$

$\Theta\left(s\right)$ is defined below. It captures how differences in the auction format and the probability that producers are pivotal influence the expected revenue.

$$\begin{gathered}\Theta\left(s\right)=\int_s^{\overline{s}}\int_{s_j}^{\overline{s}}\frac{dc(v,v)}{dv}e^{-\int_{s_j}^{v}\widetilde{H}(u)du}dv\chi\left(s,s_j\right)ds_jq_H^P\left(1-\Pi^{NP}\right)\\+\int_s^{\overline{s}}\int_s^{\overline{s}}\frac{dc(v,v)}{dv}e^{-\int_s^{v}\widetilde{H}(u)du}dv\ \chi\left(s,s_j\right)ds_jq_H^{NP}\Pi^{NP}\\+\int_{\underline{s}}^{s}\int_s^{\overline{s}}\frac{dc(v,v)}{dv}e^{-\int_s^{v}\widetilde{H}(u)du}dv\ \chi\left(s,s_j\right)ds_jq_L\\+\int_s^{\overline{s}}\left(c\left(s_j,s_j\right)-c\left(s,s\right)\right)\ \chi\left(s,s_j\right)ds_jq_H^P\left(1-\Pi^{NP}\right).\end{gathered}$$

Next, we change the order of integration for the double integral and adjust limits, so that the integrals describe the same domain of integration.

$$\begin{aligned}\Theta(s) =& \int_s^{\bar{s}} \frac{dc(v,v)}{dv} \int_s^v e^{-\int_{s_j}^v \widetilde{H}(u)du} \chi(s,s_j)\, ds_j dv q_H^P \left(1-\Pi^{NP}\right) \\ &+ \int_s^{\bar{s}} \frac{dc(v,v)}{dv} e^{-\int_s^v \widetilde{H}(u)du} \int_s^{\bar{s}} \chi(s,s_j)\, ds_j dv\ q_H^{NP}\Pi^{NP} \\ &+ \int_s^{\bar{s}} \frac{dc(v,v)}{dv} e^{-\int_s^v \widetilde{H}(u)du} \int_{\underline{s}}^s \chi(s,s_j)\, ds_j dv q_L \\ &+ \int_s^{\bar{s}} \left(c(s_j,s_j) - c(s,s)\right)\ \chi(s,s_j)\, ds_j q_H^P \left(1-\Pi^{NP}\right).\end{aligned}$$

Assume now that $\frac{dc(v,v)}{dv}$ is zero for v below $w \geq s$. In this case, we have:

$$\begin{aligned}\Theta(s) =& \int_w^{\bar{s}} \frac{dc(v,v)}{dv} \int_s^v e^{-\int_{s_j}^v \widetilde{H}(u)du} \chi(s,s_j)\, ds_j dv q_H^P \left(1-\Pi^{NP}\right) \\ &+ \int_w^{\bar{s}} \frac{dc(v,v)}{dv} e^{-\int_s^v \widetilde{H}(u)du} \int_s^{\bar{s}} \chi(s,s_j)\, ds_j dv\ q_H^{NP}\Pi^{NP} \\ &+ \int_w^{\bar{s}} \frac{dc(v,v)}{dv} e^{-\int_s^v \widetilde{H}(u)du} \int_{\underline{s}}^s \chi(s,s_j)\, ds_j dv q_L \\ &+ \int_w^{\bar{s}} \left(c(s_j,s_j) - c(w,w)\right)\ \chi(s,s_j)\, ds_j q_H^P \left(1-\Pi^{NP}\right),\end{aligned}$$

if $w \geq s$. We have $\frac{d\Theta(s)}{dw} = 0$ if $w < s$, otherwise

$$\begin{aligned}\frac{d\Theta(s)}{dw} =& -\frac{dc(w,w)}{dw} \int_s^w e^{-\int_{s_j}^w \widetilde{H}(u)du} \chi(s,s_j)\, ds_j q_H^P \left(1-\Pi^{NP}\right) \\ &- \frac{dc(w,w)}{dw} e^{-\int_s^w \widetilde{H}(u)du} \left(\int_{\underline{s}}^s \chi(s,s_j)\, ds_j q_L + \int_s^{\bar{s}} \chi(s,s_j)\, ds_j q_H^{NP}\Pi^{NP}\right) \\ &- \frac{dc(w,w)}{dw} \int_w^{\bar{s}} \chi(s,s_j)\, ds_j q_H^P \left(1-\Pi^{NP}\right).\end{aligned} \tag{54}$$

Next, we use that signals are independent, so $\chi(s,s_j) = f(s) f(s_j)$ and it follows from (50) that

$$\begin{aligned}\widetilde{H}(s) &= -\frac{d}{ds} \ln\left(\left(1-F(s)\right) q_H^{NP}\Pi^{NP} + F(s) q_L\right) \frac{(q_H - q_L)}{q_H^{NP}\Pi^{NP} - q_L} \\ \int_s^w \widetilde{H}(u)\, du &= \left[\frac{(q_H - q_L)\ln\left(\left(1-F(s)\right) q_H^{NP}\Pi^{NP} + F(s) q_L\right)}{q_L - q_H^{NP}\Pi^{NP}}\right]_s^w \\ &= \frac{(q_H - q_L)\ln\left(\frac{(1-F(w))q_H^{NP}\Pi^{NP} + F(w)q_L}{(1-F(s))q_H^{NP}\Pi^{NP} + F(s)q_L}\right)}{q_L - q_H^{NP}\Pi^{NP}} \\ e^{-\int_s^w \widetilde{H}(u)du} &= \left(\frac{\left(1-F(w)\right) q_H^{NP}\Pi^{NP} + F(w) q_L}{\left(1-F(s)\right) q_H^{NP}\Pi^{NP} + F(s) q_L}\right)^{\frac{-(q_H - q_L)}{q_L - q_H^{NP}\Pi^{NP}}}.\end{aligned} \tag{55}$$

We substitute (55) and that $\chi(s,s_j) = f(s) f(s_j)$ into (54)

$$\begin{aligned}\frac{d\Theta(s)}{dw} = & -\frac{dc(w,w)}{dw}\int_s^w \left(\frac{(1-F(w))q_H^{NP}\Pi^{NP}+F(w)q_L}{(1-F(s_j))q_H^{NP}\Pi^{NP}+F(s_j)q_L}\right)^{\frac{-(q_H-q_L)}{q_L-q_H^{NP}\Pi^{NP}}} f(s) f(s_j)\, ds_j q_H^P\left(1-\Pi^{NP}\right) \\ & -\frac{dc(w,w)}{dw}\left(\frac{(1-F(w))q_H^{NP}\Pi^{NP}+F(w)q_L}{(1-F(s))q_H^{NP}\Pi^{NP}+F(s)q_L}\right)^{\frac{-(q_H-q_L)}{q_L-q_H^{NP}\Pi^{NP}}} \int_s^{\overline{s}} f(s) f(s_j)\, ds_j q_H^{NP}\Pi^{NP} \\ & -\frac{dc(w,w)}{dw}\left(\frac{(1-F(w))q_H^{NP}\Pi^{NP}+F(w)q_L}{(1-F(s))q_H^{NP}\Pi^{NP}+F(s)q_L}\right)^{\frac{-(q_H-q_L)}{q_L-q_H^{NP}\Pi^{NP}}} \int_{\underline{s}}^{s} f(s) f(s_j)\, ds_j q_L \\ & -\frac{dc(w,w)}{dw} f(s)\int_w^{\overline{s}} f(s_j)\, ds_j q_H^P\left(1-\Pi^{NP}\right)\end{aligned} \tag{56}$$

Next we use the substitution $F = F(s_j)$, so that $dF = f(s_j)\, ds_j$.

$$\begin{aligned}\frac{d\Theta(s)}{dw} = & -\frac{dc(w,w)}{dw}\int_{F(s)}^{F(w)} \left(\frac{(1-F(w))q_H^{NP}\Pi^{NP}+F(w)q_L}{(1-F)q_H^{NP}\Pi^{NP}+Fq_L}\right)^{\frac{-(q_H-q_L)}{q_L-q_H^{NP}\Pi^{NP}}} f(s)\, dF q_H^P\left(1-\Pi^{NP}\right) \\ & -\frac{dc(w,w)}{dw}\left(\frac{(1-F(w))q_H^{NP}\Pi^{NP}+F(w)q_L}{(1-F(s))q_H^{NP}\Pi^{NP}+F(s)q_L}\right)^{\frac{-(q_H-q_L)}{q_L-q_H^{NP}\Pi^{NP}}} f(s)\left(1-F(s)\right) q_H^{NP}\Pi^{NP} \\ & -\frac{dc(w,w)}{dw}\left(\frac{(1-F(w))q_H^{NP}\Pi^{NP}+F(w)q_L}{(1-F(s))q_H^{NP}\Pi^{NP}+F(s)q_L}\right)^{\frac{-(q_H-q_L)}{q_L-q_H^{NP}\Pi^{NP}}} f(s) F(s)\, q_L \\ & -\frac{dc(w,w)}{dw} f(s)\left(1-F(w)\right) q_H^P\left(1-\Pi^{NP}\right).\end{aligned} \tag{57}$$

The first integral can be solved as follows:

$$\begin{aligned}&\int_{F(s)}^{F(w)} \left(\frac{\left(1-F(w)\right)q_H^{NP}\Pi^{NP}+F(w)\,q_L}{\left(1-F\right)q_H^{NP}\Pi^{NP}+Fq_L}\right)^{\frac{-(q_H-q_L)}{q_L-q_H^{NP}\Pi^{NP}}} dF \\ &= \left(\left(1-F(w)\right)q_H^{NP}\Pi^{NP}+F(w)\,q_L\right)^{\frac{-(q_H-q_L)}{q_L-q_H^{NP}\Pi^{NP}}} \int_{F(s)}^{F(w)} \left(\left(1-F\right)q_H^{NP}\Pi^{NP}+Fq_L\right)^{\frac{(q_H-q_L)}{q_L-q_H^{NP}\Pi^{NP}}} dF \\ &= \left(\left(1-F(w)\right)q_H^{NP}\Pi^{NP}+F(w)\,q_L\right)^{\frac{-(q_H-q_L)}{q_L-q_H^{NP}\Pi^{NP}}} \left[\frac{\left(\left(1-F\right)q_H^{NP}\Pi^{NP}+Fq_L\right)^{\frac{(q_H-q_L)}{q_L-q_H^{NP}\Pi^{NP}}+1}}{\left(q_H-q_H^{NP}\Pi^{NP}\right)}\right]_{F(s)}^{F(w)} \\ &= \left(\left(1-F(w)\right)q_H^{NP}\Pi^{NP}+F(w)\,q_L\right)^{\frac{-(q_H-q_L)}{q_L-q_H^{NP}\Pi^{NP}}} \left[\frac{\left(\left(1-F\right)q_H^{NP}\Pi^{NP}+Fq_L\right)^{\frac{(q_H-q_L)}{q_L-q_H^{NP}\Pi^{NP}}+1}}{q_H^P\left(1-\Pi^{NP}\right)}\right]_{F(s)}^{F(w)},\end{aligned}$$

because $q_H = q_H^{NP}\Pi^{NP} + q_H^P\left(1-\Pi^{NP}\right)$. Using this result, we can rewrite (57) as follows:

$$\begin{aligned}
\frac{d\Theta(s)}{dw} &= -\frac{dc(w,w)}{dw}\left((1-F(w))\,q_H^{NP}\Pi^{NP}+F(w)\,q_L\right)^{\frac{-(q_H-q_L)}{q_L-q_H^{NP}\Pi^{NP}}} f(s) \\
&\quad \left[\left((1-F)\,q_H^{NP}\Pi^{NP}+Fq_L\right)^{\frac{(q_H-q_L)}{q_L-q_H^{NP}\Pi^{NP}}+1}\right]_{F(s)}^{F(w)} \\
&\quad -\frac{dc(w,w)}{dw}\left(\frac{(1-F(w))q_H^{NP}\Pi^{NP}+F(w)q_L}{(1-F(s))q_H^{NP}\Pi^{NP}+F(s)q_L}\right)^{\frac{-(q_H-q_L)}{q_L-q_H^{NP}\Pi^{NP}}} f(s)\,(1-F(s))\,q_H^{NP}\Pi^{NP} \\
&\quad -\frac{dc(w,w)}{dw}\left(\frac{(1-F(w))q_H^{NP}\Pi^{NP}+F(w)q_L}{(1-F(s))q_H^{NP}\Pi^{NP}+F(s)q_L}\right)^{\frac{-(q_H-q_L)}{q_L-q_H^{NP}\Pi^{NP}}} f(s)\,F(s)\,q_L \\
&= -\frac{dc(w,w)}{dw}\left((1-F(w))\,q_H^{NP}\Pi^{NP}+F(w)\,q_L\right) f(s) \\
&\quad -\frac{dc(w,w)}{dw} f(s)\,(1-F(w))\,q_H^{P}\left(1-\Pi^{NP}\right) \\
&= -\frac{dc(w,w)}{dw}\left((1-F(w))\,q_H+F(w)\,q_L\right) f(s)
\end{aligned} \tag{58}$$

Hence, it follows that $\frac{d\Theta(s)}{dw}$ is independent of Π^{NP} and thus also independent of whether the auction has a uniform or discriminatory format. The same type of independency applies to $\frac{dR(s)}{dw}$. We have from Lemma 4 that the expected revenue $R(s)$ is the same independent of Π^{NP} and independent of the auction format, if $w \nearrow \overline{s}$ and signals are independent. From the reasoning above, it follows that the result in Lemma 4 can be generalized to any $w \in (\underline{s}, \overline{s})$. ■