Appendix

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THE MEANING OF HUMAN CAPITAL

Contents

This paper reports on some work carried out within the research project: "Education and the Labour Market", sponsored by the Chancellor's Office at the University of Stockholm. The problem treated here was originally taken up by one of my collaborators on this project, Åsa Sohlman, who intends to present a more extensive analysis of the human capital concept in a forthcoming report. I have profited much from her comments and suggestions.

Introduction

The idea of human capital is as old as the subject of political economy itself and the writings of classical economists are often interspersed with references to the economic analogy between men and machines. In the last two decades, following the seminal writings of Becker, much interest and research effort has been centered on the possibility of building empirical hypotheses around the notion of human capital, aiming e.g. at explaining relative earnings in terms of educational investments.

It is hardly surprising that in this sudden outpouring of writings and studies, many diverse and mutually inconsistent notions of human capital have been exemplified. "Human capital" has usually been treated as a primitive concept, whose exact meaning and relation to other concepts of economic theory is often left unspecified.

Human capital theory has by now matured to the stage where there is a plethora of empirical results to be evaluated, a rising number of competing alternative theoretical approaches and consequently a growing need for a critical appraisal of the central theoretical foundations of the theory.

What is the exact meaning of "human capital" and how should it be fitted into the general equilibrium theory on which human capital theory is supposed to be based?

This paper aims at giving at least a partial answer or one possible answer to these broad questions.

The discussion in the following is divided into two chapters.

The aim in the first chapter is simply to try to fit a very general concept of human capital into the framework of equilibrium analysis, without heeding the more specific needs and notions of so-called human capital theory.

If we really want to treat men as machines, i.e. as heterogeneous capital goods, taking part in production and being changed by production in various ways, how can this be formally represented? In contrast to machines, men are not tradable and investment in men means investment not in production but in change.

It will be shown that to make room for separate accounting of human capital and human investments the usual equilibrium framework must be modified both as to institutional assumptions and the specification of commodity space.

Once this is accomplished human resources can be socially accounted for in analogy with machines or buildings. This means, as it is spelled out in section 1.6, that we can have a human capital theory in the sense of a financial account for human capital and investments following the usual accounting rules for profit rate and rate of return on capital in equilibrium.

This may be enough to explicate the meaning of classical references to human capital in general but it certainly does not provide a basis for human capital theory in the modern sense of Becker. The second chapter is dedicated to an attempt to give a stepwise account of the further restrictions on the general equilibrium model, that are needed to provide a notion of human capital useful for the purpose of explaining relative earnings.

Searching for conceptual clarification is seldom exciting and often rather tedious. It is, however, a necessary starting-point for any critical reappraisal of a theory's viability and validity.

1 Human capital and general equilibrium

1.1 The context: A myopic general equilibrium

The Arrow-Debreu model of general equilibrium as presented in Debreu (1951), Arrow-Debreu (1954) and Debreu (1959), has a well-known intertemporal interpretation involving the assumption of dated commodities and of a complete set of spot and forward markets.

The general equilibrium framework we are going to use here is a respecified, reinterpreted and somewhat extended version of this intertemporal Arrow-Debreu model.

The production possibilities of the firms will be presented disaggregately and specified period by period under the assumption that outputs of a particular date depend only on inputs at the preceding date. This way of representing production by period in terms of a generalized activity analysis was first introduced by Malinvaud (1953 and 1972) and has later been used, discussed and developed in Bliss (1975).

The intertemporal equilibrium will also be reinterpreted as a temporary equilibrium, existing in each consecutive period. In the Arrow-Debreu model the economic agents make transaction decisions once and for all in the first period, after which they are faced with an over-all budget restriction and have access to a full complement of forward markets. In a temporary equilibrium model the agents are assumed to make decisions concerning transactions only in the current period. They only have access to spot markets including a bond market linking current transactions with the future. The relevant budget restriction also holds for current transactions only. From current prices and interest and expectations about future prices and interest the agents decide on an optimal allocation of their budgets. A temporary equilibrium will ensue when these decisions are such that supply equals demand on spot markets and the bond market.[x)]

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x) Early examples of the use of a temporary equilibrium concept are provided by Lindahl's classical paper from 1929 and Hick's discussion of a "Spot economy" in his 1939 book: Value and Capital. A survey of temporary equilibrium models and of the contemporaneous efforts to extend these models to deal with situations involving uncertainty and quantitative restrictions is given in Grandmont (1976).

If we assume that agents have perfect foresight and that their plans are fully consistent so that their price and interest expectations will all come true, we get a very special kind of temporary equilibrium or rather a sequence of such equilibria. This is very close to the original Arrow-Debreu model since in both cases the agents make all decisions with full knowledge of the true development

For our purposes here the use of a model of temporary equilibrium with perfect foresight affords the advantage that definitions and discussions of human capital concepts can be made simpler and more intuitive by being framed in terms of current period decisions. By suitable assumptions on the firm's financial arrangements the firms will also be seen to behave myopically in this model, i.e. they will make their decisions on the basis of current input-output possibilities only and will endeavor to maximize current dividends. That is the reason why we have chosen to call the model myopic general equilibrium - in the following abbreviated as MGE.

But if we wish to pursue the discussion in terms of an arbitrary period we must also remove the asymmetrical treatment in the Arrow-Debreu model of the first and last periods. This will be done - following Svensson (1976) - by allowing for initial debts in the first period and by letting the model extend into an infinity of future periods. There is, however, a price to be paid for this pedagogical convenience. Although equilibria with bankruptcy and with an infinity of commodities have been examined recently there can be no assurance here of either the existence or efficiency of this extended version of the model. A further extension in comparisons with most standard equilibrium models will be made here by allowing for the possibility of negative prices for some special goods. How this can be done and what it implies has already been shown - first by Arrow (1951) and Koopmans (1951).

As an obvious safeguard against perverse cases made possible by the infinite horizon we also have to assume convergence for the various discounted values involved. This will in particular exclude the case where expenditure is financed by indefinitely postponing the payment of an ever-increasing debt.[x)]

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cont.

of prices and interest over time. It differs solely in the respect that the over-all budget restriction in the Arrow-Debreu model is now broken down into a series of temporary restrictions linked by the bond market, forcing the agents to postpone their transaction decisions in a corresponding manner. It can also easily be shown that the perfect foresight temporary equilibrium is equivalent to the Arrow-Debreu model in the sense that any consumption and production allocation in one model can also be realized in the other. This has been shown by Guesnerie - Jaffrey (1974) for the exchange economy and by Svensson (1976) for the economy with production.

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x) The MGE-model here used is very closely related to Malinvaud's model (1953) and its specification has borrowed many traits from Svensson's paper (1976). It differs from Svensson's model mainly in the special institutional arrangements and the more general treatment of non-produced factors of production used here as a means of explicitly introducing human capital.

1.2 Production and consumption

In presenting the model we begin with its more general features; the special characteristics needed to deal with human capital will be introduced in the next section.

The economy has I consuming households and J production firms. In addition to this each household is assumed to own and operate one of I holding companies. The raison d'être for these holding companies will be given in the next section.

There are at each date at most U goods, including both commodities and services, both produced goods and non-producible factors. Time extends over $t = 0, 1, 2 \ldots$ up to infinity. We want to study the economy at an arbitrary date t. For the later specification of the full intertemporal equilibrium, however, it is most convenient if we choose to study the economy at $t = 1$.

At any chosen time $(t \geq 1)$ there are spot markets for all of the U goods and also a bond market, with prices, $p_t \in R^U$, and interest r_t.

A household i at time 1 can be characterized by a quadruple $(C_i, \precsim_i, \nu_{io}, c_{ibo})$ where $C_i \subset R_+^\infty$ (R^∞ is used here as an abbreviated notation for the countably infinite sequence $R^U \times R^U \ldots$) is the consumption possibility set of which the elements $c_i = (c_{i1}, c_{i2} \ldots) \in C_i$ are consumption sequences, with $c_{it} \in R_+^U$ $(t \geq 1)$ being consumption at date t.

$\nu_{io} \in R_+^{I+J}$ $(\nu_{ioi} = 1, \nu_{ioj} = 0, \ j = 1 \ldots (i-1)\ (i+1) \ldots I,$ $0 \leq \nu_{ioj} \leq 1, \ j = (I+1) \ldots (I+J))$ denotes initial shares for household i in holding companies and in production firms, respectively. c_{ibo} denotes initial bond holdings. $v_i = (v_{it})$, $\nu_{it} = (\nu_{itj})$ and $c_{ib} = (c_{ibt})$ are defined correspondingly.

The household in this model thus has no initial endowments. Its initial wealth is entirely of a financial nature, consisting of shares and bonds, while all physical capital is held and managed by the holding companies. Its disposible income in consecutive periods will therefore be made up of dividends from shares held and interest on bond holdings. For simplicity's sake we further assume for the moment that there are no durable consumption goods, so that the household's current purchases are fully consumed in each period.

Let us denote dividends in the various firms and companies at time $t, d_t = (d_{jt})$ $(j = 1 \ldots (I+J))$

and represent the value of all shares in the respective firms at the same time $v_t = (v_{jt})$ $(j = 1 \ldots (I+J))$.

The budget restriction of household i can then be written as:[x)]

$$(2.1) \quad p_t c_{it} + c_{ibt} + v_t(\nu_{it} - \nu_{i,t-1}) \leq (1 + r_{t-1}) c_{ib,t-1} + \nu_{i,t-1} d_t$$

The left-hand side of (2.1) consists of the different forms of household outlays - consumption purchases, bond purchases and net purchases of shares. The right-hand side contains two forms of household income - interest on bond holdings and dividends.

Each household i is assumed to maximize $\gtrsim_i$ on the set of all c_i, c_{ib} and ν_i that fullfill the budget restrictions with given c_{ibo} and ν_{io}.

For the production firm j we assume that its production possibilities in any period t can be represented by a set $(z_{jt}, q_{j,t+1}) \subset R^U \times R^U$ where z_{jt} is input at date t and $q_{j,t+1}$ is output at date t+1. We thus assume that output in any period depends on inputs in the preceding period but not on inputs at earlier dates.

Each period the production firm rents its inputs from the holding companies in the form of services or use of material goods and non-producible factors held by these companies. The firm pays, with interest, for the inputs advanced by the holding companies when these inputs have matured after one period into outputs. The firm is thus free from both financial worries and responsibility for the management of physical capital, since these functions are handled separately by the holding companies. These can therefore be looked upon as representing the pure capitalist functions while the production firms account only for current production decisions, i.e. handle the production technology. The outputs of the firms are sold to households for consumption or to holding companies for what we might call gross investment. The dividend at time t will be:

$$(2.2) \quad d_{jt} = p_t q_{jt} - (1+r_{t-1}) p_{t-1} z_{j,t-1} \quad (j = (I+1) \ldots (I+J))$$

The firm is supposed to maximize the discounted value of future dividends, where the discount factors are defined as:

$$(2.3) \quad \beta_1 = 1$$

$$\beta_t = \frac{1}{1 + r_{t-1}} \quad (t \geq 2)$$

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x) For a further development of this term, see footnote in the beginning of section 1.5 below.

From our assumptions above, however, it is apparent that, since each dividend depends only on current technology, prices and interest, this is equivalent to letting the firms maximize each consecutive dividend separately, i.e. having them act in a myopic fashion.

For the starting period we must assume that inputs in the previous period, z_{jo}, are given. With this restriction, the behavior rule for the firms can be stated simply. In each period t each firm maximizes its dividend as given by (2.2) above on the set of its production possibilities $(z_{jt-1}, q_{j,t})$.

1.3 The holding companies

In most standard models of this kind only two types of institutions are recognized: households and firms. Between them they then share the responsibility for managing the social capital. The household may e.g. be the owner not only of its own labor capacity but also of land resources, i.e. the households hold the nonproducible resources, while the firms hold the produced capital.

In this model we have chosen to make a further division. Mainly in order to afford human capital a treatment analogous to that of physical capital, we have separated the resource management function into a separate institution called a holding company. Each household is assumed to have its own holding company, which manages various amounts and sorts of the three different kinds of social capital: real estate, human resources and production capital, i.e. reproducible capital including everything from raw materials to durable machinery. What then remains for the household proper is mainly a consumption-saving function although we have found it convenient for later use of the model to let the household also retain the financial investment function, i.e. to be able to buy not only savings bonds but also shares in production firms. The function remaining for the production firms is the economic use of a given production technology.

The holding company owns and manages the human resources - the human capital - of the household. This arrangement may sound strange but means merely that for accounting reasons we have chosen to separate the resource managing functions of the household. This may be expressed by assuming that the current trend towards conducting household business through companies - mostly due in fact to tax reasons - has become the rule.

Each holding company finances its operations for each period separately by selling bonds to households on the bond-market. With this money it purchases the various kinds of non-human capital in the beginning of each period. It then immediately rents out its capital to production firms. In the special case of human capital the holding company also rents part of it - the leisure hours - back to the household at the current market wage. At the end of each period it gets back the capital - or what is left of it - and a rent payment with interest. It then sells out the capital and uses the proceeds together with rent- and interest income to pay back the bond loans, while the surplus is given back as dividends to the household as sole owner of the company.

Since there is no trade in holding company shares there will be no market price. The same is true of human capital, for which a discounted value can be defined, but no market price.

The explication of the human capital concept in general equilibrium conditions would undoubtedly be much simpler and more straightforward if we dared, for modelling purposes, assume some kind of trade in human resources. Before proceeding, it may be worthwhile to comment on the reasons why this does not seem possible.

The reasons are usually regarded as obvious. You cannot go out on the market and buy human beings - there is no slave market. If this argument is taken to imply that labor contracts are always such that they only cover the short-term disposal of labor for specific tasks, then this may not be wholly convincing.x)

Any attempt to introduce human capital trade into a general equilibrium model does, however, run into other difficulties which have to do with the individual and indivisible nature of human capital. To account for the fact that you are only interested in buying and consuming your own leisure time, not the time of some equally worthy individual, and that your incentives to utilize training opportunities depend on your reaping the benefits, each person or each household would have to be identified as a separate kind of human capital. Equally obvious is the fact that while you may part-train a man, you cannot train just part of him - from the point of

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x) Most people are not hired on a purely temporary basis - on "cotton-picking contracts". Employment contracts may assume a long-term view and various employment benefits may be credited to the employer on this assumption. This in fact is what is usually meant when current discussions refer to the trend towards "Japanized labor". A corresponding assumption of long-term utilization can often be traced - and is sometimes explicitly stated - in the conditions for various public employment benefits and training programs. One way of modelling these actual conditions would undoubtedly be to allow for the possibility of selling part of one's own human capital. Contracts can of course always be broken and tacit assumptions can be proved wrong - but usually there is some penalty involved. This then would merely mean - in model terms - that you can always buy back full control of your own labor even if you have partially "hocked" it, as it were, to your employer or some public agency - but this usually involves raising some extra money to make up the part of your capital value you had already mortgaged.

view of developing human capabilities, man is indivisible. If you therefore sell part of him, any change in his capabilities occasioned by his use will introduce externalities between the various agents involved. It thus seems that to assume trade in human capital in an attempt to model real life conditions creates more problem than it solves.

In terms of the kind of generalized activity analysis used here, the treatment of durable capital presents no special problem as long as capital ownership is kept within the production firms themselves. The capital goods before usage are accounted for as inputs and the used and possibly also changed capital goods are registered as outputs in the production technology. There is then no need for explicitly accounting for either use or the accompaning change brought about in the capital goods. When ownership and resource management is placed outside the production firms as in our model here, a new need arises to identify the flow of services between the resource users - the firms - and the resource owners - the holding companies. We have to consider not only the fact that the resources are used but also how they are used, since this affects the shape of the resources when they come back from their use in production. One reason for using holding companies in the model is indeed to make room for this kind of separate service accounting. When it comes to human capital we are not only concerned with what it can do in production, but almost equally interested in identifying what various productive processes can do to develop human capabilities.

The process of accounting for service flows can be modelled in various way. We have chosen to assume that two identifiable "capital services" are involved in any use of a capital resource of a certain kind and quality in production. One is the resource user service, an input in production, which depends only on the resource hired, not on the purpose for which the resource is rented. The user charges can be assumed to be positive throughout. The other is the resource development service, an output in production which, like any other output, depends on the kind of productive activity carried on. The development charges may be either positive - reflecting an improvement in the resource through production usage - or negative - compensating a depreciation in the capital good. The sum of user charges and development charges is the total rental paid for the capital good. For production capital we have made the simplifying assumption that these goods are affected in an identical way by all productive uses, which means that only one rental service and one positive, rental price have to be identified.

The holding companies' transformation possibilities thus in general involve transforming a capital good and a development service into another capital good and a user service. As long as there are a finite number of ways in which a capital good can be changed, the activity analysis can obviously always, in a formal sense, be transformed into this kind of description by a suitable increase in the still finite number of goods in the model. Whether it is also a practically convenient way of modelling real life will depend on the number or standardization of the development services for human and real estate resources that production gives rise to.

It is natural although by no means necessary to think of the transformation possibilities for a holding company as additive, i.e. as being the sum of the separate possibilities for various kinds of capital goods. This would mean that there are no "external effects" between different kinds of resource management. If the transformation possibilities concerning real estate and production capital differ between households this must be interpreted as due to differences and gaps in the mercantile knowledge necessary for handling special kinds of resources. Some households may e.g. not be equipped to handle certain complex kinds of real estate.

We also assume production technology to be such that the volume and character of development services produced with certain resources can be varied and furthermore that these services can be assigned in various ways to the resource units involved in the production process. Specifically we assume that for each relevant kind of resource there is one productive employment actually used which leaves the resource - real estate or human - unchanged.

The household, however, must buy the development service from the same firm which utilizes the unit to be developed. This kind of "tied sales" usually introduces an element of arbitrariness into market pricing. Under the above-mentioned assumptions, however, this will not be the case. The working of the model will be the same regardless of whether we account for the two kinds of services separately or not. There will always be one unique way of splitting the total rental into a user charge and a development charge.

After these introductary remarks we can continue specifying the model by dealing in turn with the three factors of production handled by the holding companies.

1.4 Factors of production

There are three kinds of factors of production in the model: real estate, human resources and production capital.

Real estate in this context refers to more than just land. It covers everything implied by the French word "immobilier", i.e. it includes all immovable properties or objects on land, e.g. roads, various cultivations and land improvements, houses and fixed machinery. The use of such a broad definition of real estate is needed once we want to remove the ownership of the factors of production from the production firms and establish instead a short-term leasing market. There is e.g. no acceptable way of establishing housing rents without involving land use, etc.

A piece of real estate can be described in terms of two different kinds of characteristics, unchangable and changable, respectively. Unchangable characteristics are the various locational properties, geographical coordinates, geological and climatological conditions, etc., that is, everything that determines its latent use and development possibilities. We assume that all existing real estate can be partitioned into a finite number of groups or types, each homogeneous as to these unchangable characteristics, and that the number of units in each such type is given once and for all. Real estate is thus non-reproducible - and eternal - as far as type is concerned.

Through various kinds of land improvements and construction activities the real estate acquires new changable characteristics, which together define the actual state of a certain piece of real estate. We assume here that there are a finite number of such possible actual states and that current real estate rentals depend only on these states. Real estate is thus in a certain sense reproducible when it comes to the actual state of the land. A piece of land will over time pass through a certain state-cycle. The responsable holding company can, by buying different real estate development services, determine each step in this cycle.

Each unit of real estate belonging to a certain holding company at time t can thus be identified as, $e_{it}^{\alpha\beta}$, where α gives the type while β in a corresponding manner refers to the state. We will let $e_{it} \varepsilon R_t^U$ denote a vector with zero for those components that do not correspond to some particular kind of real estate owned by the i'th holding company at time t. By $e'_{it} \varepsilon R_t^U$ we denote correspondingly the one period use of real estate in different states. Given the assumptions made, the number of non-zero components in e'_{it} will at most equal the number of different states.

Finally we use $\dot{e}_{jt} \varepsilon R_t^U$ to denote the real estate development services rendered at time t by the j'th production firm. The number of possible services of this kind - forms of construction, land improvements, etc. - is assumed finite but may be smaller, or bigger, than the number of different kinds of real estate.

One consequence of our definition above may be worth pointing out. In removing all "immovables" to outside ownership we have also eliminated the most common explanation for differences in production possibilities between firms. The task of these firms is now restricted to combining current available services in the most profitable way. The remaining differences in technological possibilities in a certain period must then wholly be ascribed to differences in technological knowledge between the firms. However, the change is more semantic than factual. Instead of saying that a firm can use certain processes because it already has a certain plant built we now say that although it does not itself own the plant it is the one who knows how to operate it.

When we pass to the second factor, human resources, we use definitions and concepts, analogous to those utilized above for real estate. There is one aspect of human resources that is especially troublesome in a model with infinite horizon. People, in contrast to land, die sooner or later. To avoid the complexities arising from this, we make the following simplifying assumptions. Each household reproduces itself indefinitely in such a way that its size and structure remains unchanged. The household may therefore calculate as if each individual of a certain type, i.e. with certain unchangable or inate characteristics, was immortal but necessarily passing through a predetermined life-cycle of aging before "passing into its second childhood".

By various kinds of training, i.e. by being exposed to various kinds of human development services in production firms, an individual may acquire different sorts of skills and capabilities, new states, and thereby pass into new phases of a state-cycle.

Analogous to what was said above about real estate, we assume that these possibilities for transformation or cycling depend on the type or inate characteristics.x) While types of individuals are non-reproducible, states are in this sense reproducible. The model thus encompasses all sorts of "life-long education", while all training is defined as "training on the job", even for the case where the job is just training.

Corresponding to our real estate definitions, we let $h_{it} \varepsilon R^U_t$ denote the vector measuring the human resources of the i'th holding company at time t, with non-zero components indicating its holdings of the various combinations of type and state of labor.

Likewise we let $h'_{it} \varepsilon R^U_t$ denote the use of the different states of human resources, assuming again that the number of non-zero components will at most equal the number of different states. While the individual type determines his development possibilities we thus let his current usefulness in production depend solely on his state, i.e. his actual skill or capabilities.

Finally $\dot{h}_{jt} \varepsilon R^U_t$ represents the various human development services or training (or detraining) opportunities produced at time t by the j'th firm. Nothing is assumed about the number of such opportunities except that it is finite.

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x) A similar treatment of training as being produced jointly with commodities and "sold" jointly with employment, has been presented by Rosen (1972). He starts off, however, at the point where this paper ends i.e. with homogeneous human capital. Since his purpose is rather to explore some implications of "training on the job" for the choices of employers and employees, his concepts are furthermore not framed within a general equilibrium context. Our MGE-model would seem to meet - and pass beyond - the suggestions for generalization of the Rosen-model put forward by Rosen himself in his note 9.

The third kind of factor is production capital, i.e. all reproducible inputs that are not incorporated into real estate or human resources. Although even in this instance one may talk about recycling in the sense used in industrial and environmental economics, there are no longer a limited number of non-reproducible individuals of certain given types. For simplifying reasons we assume here that for each commodity there is just one standardized change brought about by its use in any production process. To each unit of a capital item thus corresponds a unit of one capital service. Analogous to the definitions above, we denote the production capital owned by the i'th holding company at time t by $\bar{q}_{it} \varepsilon R^U_t$, and the production capital services produced by the i'th company $q_{xt} \varepsilon R^U_t$.

For "current inputs" or "circulating capital" the transformation possibilities for the holding companies will show the capital services only as output, the capital having been consumed in the process. Should one also wish to allow for "consumer's durable", this simply means that production capital services can be bought from the holding companies also by households.

1.5 Completing the specification

As a general representation of the capital goods bought as input by the holding company i at time t, we use k^-_{it}, and for the corresponding output k^+_{it}, defined as follows:

$$(5.1) \qquad k^-_{it} = e^-_{it} + h^-_{it} + \bar{q}^-_{it}$$

$$(5.2) \qquad k^+_{it} = e^+_{it} + h^+_{it} + \bar{q}^+_{it}$$

In the same way we define the capital use services produced by the holding company as k'_{it} :

$$(5.3) \qquad k'_{it} = e'_{it} + h'_{it} + \bar{q}'_{it}$$

The only difference between the capital use services produced and those consumed by the production firms is the consumption of leisure time and of real estate services by the household. We denote these by c_{ist}:x)

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x) To take into account the household's purchase of leisure and of real estate services, to be paid afterwards like all capital services, the budget restriction for the household given above in (2.1) must be rewritten as:

$$(2.1b) \qquad p_t c_{igt} + (1+r_{t-1})p_{t-1} c_{is,t-1} + c_{ibt} + v_t (\nu_{it} - \nu_{i,t-1}) \leq$$
$$\leq (1+r_{t-1})c_{ib,t-1} + \nu_{i,t-1} d_t$$

where $c_{igt} = c_{it} - c_{ist}$, i.e. all consumption except capital services.

(5.4) $\sum_i (k'_{it} - c_{ist}) = \sum_j z_{jt}$

Capital development services were defined separately only for real estate and human resources, which leads to the following definition for the purchases of the i'th company:

(5.5) $\dot{k}_{it} = \dot{e}_{it} + \dot{h}_{it}$

The utilization or transformation possibilities for holding company i in period t can then be represented by set

$T \subset R^U \times R^U \times R^U \times R^U$ where $(k^-_{it}, k'_{it}, \dot{k}_{i,t+1}, k_{t,t+1}) \in T$.

k^-_{it}, and $\dot{k}_{i,t+1}$, are both inputs although one period apart and the outputs, k'_{it}, and $k_{t,t+1}$, are separated in the same way.x)

Apart from transformation possibilities for all periods the holding company i will also be characterized by the initial capital, capital leasing and debt, i.e. k_{io}, k'_{io} and k_{ibo}.

The dividend for the holding company i at time t is defined so as to include the imputed value of leisure time. It can then be written as:

(5.6) $$d_{it} = (1 + r_{t-1})p_t k'_{i,t-1} + p_t(e^+_{it} + \bar{q}^+_{it}) + k_{ibt} - p_t\dot{k}_{it} - p_t(e^-_{it} + \bar{q}^-_{it}) - (1 + r_{t-1})k_{ib,t-1}$$

The first three terms on the right-hand side represent the company's sources of income, i.e. user charges with interest, sales of capital goods and new bonded loans. The last three terms correspondingly denote the various outlays, i.e. development charges, capital goods purchases and repayment with interest of old bond loans.

The holding company will try to find such time sequences of

e^-_{it}, $\bar{q}^-_{it}$, $\dot{k}_{it}$, k'_{it} and k_{ibt}

that will maximize the discounted value of expected future dividends, $\sum_{t=1}^{\infty} \beta_t d_{it}$, with the restrictions given by the period transformation possibilities and the initial values.

If we utilize the assumption mentioned earlier of additive transformation possibilities, dividends could be defined separately and maximized for each of the three sections within the holding company: the real estate department, the labor department and the material department. Since operations in real estate and material are financed separately by bond loans in each period and the transformation possibilities at each time t depend only on the decisions taken at the preceding time, t - 1, we immediately see that the myopic quality, which was found to characterize decisions in the production firms, would also be true of operations in real estate and material. Each of these departments could equally well maximize its dividends separately for each period. Since we do not have trade in human capital, this possibility of myopic decision-making does not exist where human resources are concerned. The company must then itself take into account the effects of current decisions concerning employment and training on future labor dividends, instead of having this done by the capital evaluation of a market.

To complete the specification of the equilibrium model, all we need now are the equilibrium conditions. There are three of them. For each $t \geq 1$ the following should hold:

(5.7) $$\sum_j q_{jt} + \sum_i (e^+_{it} + \bar{q}^+_{it}) + \sum_i k'_{it} = \sum_i c_{it} + \sum_i (e^-_{it} + \bar{q}^-_{it}) + \sum_j z_{jt} + \sum_i \dot{k}_{it}$$

(5.8) $$\sum_i c_{ibt} = \sum_i k_{ibt}$$

(5.9) $$\sum_i v_{ijt} = 1$$

The first equilibrium condition, (5.7), establishes equilibrium on all non-financial markets. The three kinds of supply on the left-hand side are, respectively: production of firms, capital goods sales and capital leasing by companies. On the right-hand side are the four kinds of corresponding demand: household consumption including leisure time, company purchases of real

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x) The representation of transformation possibilities is simplified here by not explicitly stating the restriction to tied purchases of development and user services.

The net product of the holding company at time t can be written as: $k^+_{it} + k'_{it} - \dot{k}_{it} - k^-_{it}$

If this net product is integrated over the whole economy with the corresponding net product for the production firms, the flows between companies and firms will no longer appear in the final term. Apart from household consumption including leisure time, the total net product will evidently only show the difference

$\sum_i (k^+_{it} - k^-_{it})$

denoting the demand for investment in new capital goods. In such an integrated model we no longer get a separate representation of the demand for investment in old capital goods, $\sum_i \dot{k}_{it}$.

estate and material goods, firm purchases of inputs and finally company purchases of development services.

The second condition, (5.8), simply states that supply should equal demand also in the bond-market.

(5.9) states that conditions on the share market are such that for every share of every firm there is always someone willing to hold it at the going price.

This ends the specification of the model. Apart from stating the equilibrium conditions we have characterized and stated behavior rules for the three kinds of agents, consumers, firms and companies.

Perhaps a word should be added on the v_{jt}, the value of the j'th firm's stock of shares at time t, since the determination of these market values may not be apparent from the specification. In equilibrium, any change of stock value over a period can only be due to the expected dividend. This means that if we assume, as we already have done above, that the discounted value of the sum of future dividends converges, then the value it will converge towards is the discounted stock value, i.e.:

$$\beta_t \, v_{jt} = \sum_{\tau=t}^{\infty} \beta_{\tau+1} \, d_{j\tau+1}$$

A picture of the structure of the assembled model for period t with aggregated sectors is presented in fig. 1. For the production firms collectively and individually the choice in each period is, as shown, simply that of picking an input-output pair that maximizes the dividend. The households taken together must distribute their dividend income in an optimal fashion between consumption and bond investments. The task for the holding companies, the truely "capitalistic" task, is somewhat more complex. They must find combinations of on one hand capital utilization and on the other hand investments in new capital goods or in existing real estate and human capital, that will maximize the discounted value of future dividends.

1.6 Income, wealth and the rate of return on human investments

Within the model, as specified above, some concepts of social accounting can easily be defined.

Gross national product (GNP) in period t can be directly defined as:

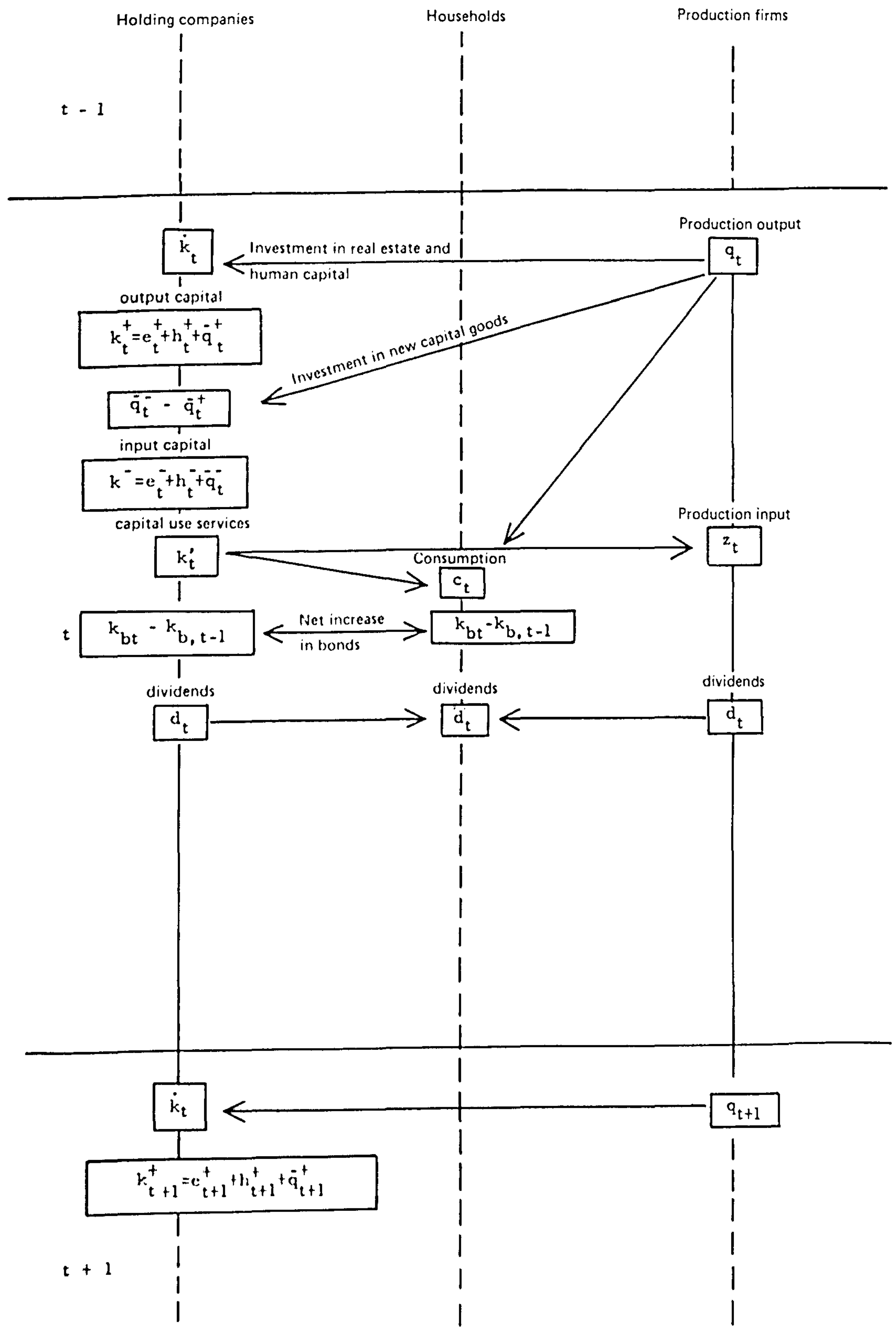
$$(6.1) \qquad GNP_t = \sum_i p_t\{\dot{k}_{it} + (\bar{q}^{-}_{it} - \bar{q}^{+}_{it})\} + \sum_i p_t c_{it}$$

This simply means that:

$$GNP_t = \text{Gross Investment} + \text{Consumption}$$

There are two kinds of investments involved. The first term on the right-hand side of (6.1) denotes the investments in non-

Figure 1. The structure of the model in period t with aggregated sectors



reproducible factors, i.e. the total value of development services. The second term represents the gross investments in new material goods and reproducible factors. The third term, denoting consumption, also includes consumption of leisure time. Although this is against social accounting practice, it is motivated here by the importance of leisure value as part of the yield of human capital.

Viewed from the production side the GNP_t could equally well be written as:

$$(6.2) \qquad GNP_t = \sum_j q_{jt} + \sum_i c_{ist}$$

where $c_{ist} = c_{it} - c_{igt}$, as earlier introduced, stands for the household consumption of real estate services and leisure. These services are thus added to the current output of the production firms to give the GNP. In these definitions we abstract from the fact that, in the model, service consumption is paid with interest in the succeeding period, as are service inputs to production. The internal loan transactions between households and their own holding companies are of no concern for the social accounting purposes indicated here.

To go from GNP to national income concepts we have to use capital values to measure changes in wealth. Since there are no market prices for human capital - or formally since p_t is defined with zero components for human capital - we must somehow directly define human capital values. This can be done analogous to the way stock values of firms were defined (see section 5 above), i.e. as discounted values of future expected yields.

Let us make the following simplifying assumptions. We assume that the transformation possibilities of the holding firms are additive also down to each individual unit of human capital, i.e. each labor unit can equally well be managed and accounted for separately. This means that for each individual at each time, we can identify in equilibrium the most profitable sequence of future yields. We further assume that holding companies have equal opportunities when it comes to managing labor. Since holding companies are managed competitively, maximizing the discounted value of the sequence of future yields, i.e. the capital value of the human resources, this means that in equilibrium each unit of human resource of a certain type and in a certain state will have the same capital value. One such unit, let us call it h^* (a vector with only one non-zero component) will undergo a cycle of transformations over time although retaining its individual identity. Its capital value at time t can be defined as:

$$(6.3) \qquad v_{h^*t} = \frac{1}{\beta_t} \sum_{\tau=t}^{\infty} \beta_{\tau+1} \{ (1 + r_\tau) p_\tau h^{*\prime}_\tau - p_{\tau+1} \dot{h}^*_{\tau+1} \}$$

The expression on the right-hand side of (7.3) is simply the discounted value of the future rentals or yields of the human

capital through its various developments, made up of sales of user services, paid with interest after use, and development costs, respectively.

We can now use these imputed human capital values to complement the price vector so as to get an imputed price vector, $\bar{p}_t$, with non-zero prices also for human capital.

With the help of this price vector we can now define a net national income in period t, NNI_t as follows:

$$(6.4) \qquad NNI_t = \sum_j (v_{jt} - v_{jt-1}) + \sum_i (\bar{p}_t \bar{k}_{it} - \bar{p}_{t-1} \bar{k}_{i,t-1}) + \sum_i p_t c_{it}$$

The meaning of this can also be expressed as:

NNI_t = Wealth formation + Consumption

The first term on the right-hand side of (6.4) denotes the change in total stock values of the production firms, while the second represents the corresponding change in value of the total stock of capital goods. This last expression can also be developed in the following way:

$$(6.5) \qquad \sum_i (\bar{p}_t \bar{k}_{it} - \bar{p}_{t-1} \bar{k}_{i,t-1}) = \sum_i \bar{p}_t (\bar{k}_{it} - \bar{k}_{t-1}) + \sum_i (\bar{p}_t - \bar{p}_{t-1}) \bar{k}_{t-1}$$

The wealth formation in capital goods can thus be split up into two parts: change in the physical volume of capital goods and change in capital prices, i.e. what is usually called capital gains.

Contrary to usual accounting practices, the income concept defined here includes wealth formation also in human capital and capital gains for all capital goods. This seems natural when, as here, we are especially interested in tracing the effects of changes in human capital. It means, however, that part of the measured income changes will materialize as actual purchasing power for the households and their holding companies only when future yields are realized and future dividends paid out. The social wealth concept used above, made up of the stock value of firms and the value of capital goods including human capital, can be derived as the discounted value of all future dividends, all future net incomes or all future consumption. This has already been demonstrated for stock values and dividends and has been defined for human capital. That it is also true for other capital goods depends on the fact that their services in the model are assumed to be sold at a given market price, so that any "producer's surplus" will not affect the value of capital goods for the owners, but will instead be registered in the firm dividends. The net national income can therefore alternatively be introduced as the interest on this social wealth.

If we compare the definitions of NNI_t and GNP_t we see that gross investment in the product is replaced by wealth formation in the income concept.

To simplify the subsequent discussion let us assume constant returns to scale in production, which means zero dividends for the production firms. This is indeed a very natural assumption in this model, where the firms have no durable capital of their own.

The difference between gross investment and wealth formation in the holding companies is not entirely due to physical consumption or potential deterioration of capital goods used in production. There are two other factors involved, one of which is capital gains. The other is the fact that when it comes to real estate and human resources there is no given relation between the cost of a certain development service and the value of its impact on the capital employed. Training opportunities offered by a production process may e.g. be of great value to employers of a certain type and state although they are only charged the common market price, which may be insignificant.

We can go on to define gross profits in a similar way. Profits in the production firms will equal dividends, i.e. are zero according to the assumption made earlier. If we consolidate the rest of the economy into a household sector to eliminate loan transactions imputed gross profit for this sector in period t, $\bar{\Pi}_t$, is then defined as:

$$(6.6) \qquad \bar{\Pi}_t = \sum_i \{ (1+r_t)p_t\, k'_{it} - p_{t+1}\, \dot{k}_{i,t+1} \} + \sum_i \bar{p}_{t+1}\, k^+_{t+1} - \sum_i \bar{p}_t\, k^-_t$$

The expression on the right-hand side is made up of three terms, each summed over all households. The first profit term designates rentals, i.e. the net payment received from the firms for services plus the value of household consumption of leisure and real estate services. The second is the value of output capital and the third denotes the value of input capital. The word "imputed" refers to the fact that imputed value changes in human capital have also been included. If we take into account the interest cost involved in holding the input capital over one period we get the imputed net profit, $\bar{\pi}_t$:

$$(6.7) \qquad \bar{\pi}_t = \bar{\Pi}_t - r_t \sum_i \bar{p}_t\, k^-_t$$

The imputed gross profit rate in period t, ρ_t, can be written as:

$$(6.8) \qquad \rho_t = \frac{\bar{\Pi}_t}{\sum_i \bar{p}_t\, k^-_t} = \frac{r_t \sum_i \bar{p}_t\, k^-_t + \bar{\pi}_t}{\sum_i \bar{p}_t\, k^-_t}$$

It follows directly from this definition that if net imputed profit is zero, $\rho_t = r_t$.

Applied separately to human capital this simply expresses the trivial relation that if income from human resources including increases in capital values is just enough to cover current interest on incoming human capital then the imputed gross profit rate for human capital will also be equal to this current rate of interest.

The concepts discussed so far are all total or average. Marginal concepts can be dealt with more conveniently if we assume for the time being that the transformation possibilities of the holding companies can be expressed by differentiable functions. If we also add the harmless assumption that capital services supplied have as their unit of measurement the services of one unit of the corresponding capital good, the transformation possibilities can be written as:

$$(6.9) \qquad k'_t = Mk^-_t$$

$$(6.10) \qquad k^+_{t+1} = f(\dot{k}_{t+1}, k^-_t)$$

(6.9) states that user services supplied in period t constitute a linear function of incoming stocks of capital goods. The quantity of user services of a capital good in state β is simply the total number of capital goods in this state summed over all types α. M here thus stands for a quadratic matrice of order U x U, where each row corresponding to a component of k'_t measuring user services of goods of a certain state has 1:s in all the places which measure capital goods of this state, while the rest of the matrice is made up of zeros.

(6.10) says that outgoing capital is a function of incoming capital and development services. Both these functions are then assumed to be differentiable. This is certainly not strictly the case, since it is a fundamental property of capital goods as defined here - and particularly of human beings - that they can only change as a unit. Our excuse for using this assumption here is the usual one employed by economists; aggregates may be taken to be large enough and, anyhow, discontinuities do not seem pertinent to the analysis.

We also abstract from the fact that the second function can be multi-valued due to the possibilities of different assignments of development services to the various capital units, i.e. we assume a given assignment rule that makes it possible to trace an expansion in a unique fashion.

If we differentiate (6.9) and (6.10) we get:

$$(6.11) \qquad dk'_t = Mdk^-_t .$$

$$(6.12) \qquad dk^+_{t+1} = f_{\dot{k}}\, d\dot{k}_{t+1} + f_k dk^-_t$$

If we take the price vectors $\bar{p}_t$ and $\bar{p}_{t+1}$ as given there are then two possibilities of marginally affecting profits and profit rates - by changing incoming capital and by changing the investment in development of real estate and human resources. Let us look at these in turn.

Marginal changes in incoming capital cannot be made in human resources, which at each point in time are given and non-tradable. With this restriction in mind we can define the marginal change in imputed gross profit resulting from a marginal change in incoming capital as:

$$(6.13) \qquad d\bar{\Pi}_t = \sum_i \left[(1+r_t)\, p_t\, M + (f_k \bar{p}_{t+1} - \bar{p}_t) \right] dk_t^-$$

If no development service is reassigned to the new capital we can neglect f_k and gross profits will change due to change in user charges and in the volume of capital gains. But we also know from differentiating (6.7) that:

$$(6.14) \qquad d\dot{\bar{\Pi}}_t = d\bar{\pi}_t + r_t \sum_i \bar{p}_t\, dk_t^-$$

Since we are studying an equilibrium point where net profits will be maximized along with dividends, marginal net profit is zero which obviously means that the marginal profit rate will equal the rate of interest. This is what we usually expect to find in equilibrium models, that is:

$$(6.15) \qquad d\rho_t = \frac{d\bar{\Pi}_t}{\sum_i \bar{p}_t\, dk_t^-} = r_t$$

If instead we study a change in development investments in real estate and human resources, the corresponding change in imputed gross profits will be:

$$(6.16) \qquad d\bar{\Pi}_t = \sum_i \{ \bar{p}_{t+1} f_{\dot{k}} - p_{t+1} \}\, d\dot{k}_{i,t+1} =$$

$$= \sum_i (\bar{p}_{t+1}\, dk^+_{i,t+1} - p_{t+1}\, d\dot{k}_{i,t+1}) = 0$$

That the change in profit must, in equilibrium, equal zero follows from (6.14) above. Not only must marginal net profit again be zero but this is now also true of the second term in (6.14), the change in interest on incoming capital. Applied to human investments, (6.16) states that the value of a marginal investment in equilibrium will equal its cost. It follows that the marginal profit rate is also zero:

$$(6.17) \qquad \frac{d\bar{\Pi}_t}{\sum_i p_{t+1}\, d\dot{k}_{i,t+1}} = 0$$

This may at first seem more surprising, as it means that e.g.

the marginal rate of profit on investments in human capital is zero. This, however, is merely a consequence of the fact that in the model we assume such investments to change the human capital immediately without any costly delays. Here we have also let these investments affect the imputed gross profit directly by way of the imputed capitalization of future increases in human rentals. If we rewrite (6.16), substituting from (6.3) above the full definition of imputed capital value, we get:

$$(6.18) \qquad \sum_i p_{t+1}\, d\dot{h}_{i,t+1} = \sum_i dh^+_{i,t+1} \left\{ \frac{1}{\beta_{t+1}} \sum_{\tau=t}^{\infty} \beta_{\tau+1} (1+r_\tau) p_\tau\, h^{*\prime}_\tau - p_{\tau+1} \dot{h}^*_{\tau+1} \right\}$$

where h^* as before traces the various states of each original unit of human capital. The expression within brackets on the right-hand side simply measures the discounted value of future human rentals or, what amounts to the same thing, the discounted value of future profits on human capital - without imputation of human capital values. The meaning of (6.18) can therefore be written simply as:

The cost of a marginal human investment =

The discounted value of the future marginal profits which result.

The rate of return of marginal human investments, defined as the internal yield rate, will then in equilibrium depend on the spacing of future marginal profits and on the sequence of interest rates involved. Specifically, if the rate of interest remains constant over the future, the marginal rate of return on human investments will equal this interest rate just as it will for other kinds of investment; see (6.15) above.

2 Homogeneity of human capital

2.1 The need for further assumptions

What we have shown in the first chapter is simply that the human capital concept can be incorporated in a general equlibrium context in a way similar to what is done with other factors of production and that it can also be fitted into a social accounting matrix.

We have traced the accounting relations between the financial concept of human capital value and the rentals or earnings of human resources. Human capital in this sense may be a conveniance for social accounting and a shorthand notion for discussing individual expectations.

The reason for using a "human capital approach", however, is usually much more ambitious. One hopes in this way to arrive at some testable relation between the volume of human investments in an individual and his or her earnings, possibly with some capability factor as an intermediate variable.

From this point of view our general equilibrium model is far too general. All we can generally say about the wage of an individual, i.e. his user charge in the model, is that it depends on his state, which in turn depends both on what type of person he is and on how much has been invested in him - assuming that the investments have been optimal. His human value is thus partly accounted for by investments done and partly by his type rent, i.e. the value of his development potential. In principle we could try to separate the two components by measuring at birth the discounted value of all future rentals until death - his individual type rent value which then also incorporates various surpluses in human investments. But we cannot measure either total rent or differential rent separately later on in life. Since people are assumed to be fundamentally different - to belong to different types and not just more or less capable - they will usually be found chosing different careers. The relative remuneration of these careers may well change with demand and supply conditions over time. In the same way the relative costs of various forms of human investments will vary and will not generally be proportionate to their relative value. In one respect, however, the model is better tailored to the needs of human capital theories than real life. In the model pure wages and human investments are accounted for separately, while in real life, we can usually only determine the total rentals for various kinds of jobs.

In order to derive the kind of simple relations between the volume of human capital in some sense and the individual earnings that human capital theories often aim at, the model must obviously be restricted and further specified in several ways.

By stepwise restricting and modifying our model, we will in the following try to approach the kind of human capital theory model needed to "explain" the relative structure of current earnings.

2.2 The first assumption: Physical human capital and the possibility of aggregation

One main assumption inherent in a "pure" human capital theory is that relative earnings depend only on the physical capital, the "earning potential", embodied in the individuals. Let us for the time being keep this assumption as general as possible, still allowing e.g. for a possible heterogeneity of this physical capital. If we call the price component for the i'th state of human capital w_i, which then measures the wage or user charge of an individual in state i, the assumption could be formally expressed as:

$$\frac{w_i}{w_j} = f(h) \tag{2.1}$$

where we for notational conveniance assume that the relation is a differentiable function.

What (2.1) states is then that the relative wage is a function only of the vector of human resources used. The relative wage is thus unaffected by changes e.g. in production capital, in cooperating real estate, in output mix or in the rest of the equilibrium price vector.

Let us reflect for a moment on what this requirement means in terms of our MGE-model. There we had human resources of various types and states and we assumed that earnings were only related to states. To fit into the assumption discussed here we must then first of all accept that all human resources in the same state, regardless of type, embody the same physical human capital. The distribution between states of embodied capital then determines relative earnings.

Even if the human capital is heterogeneous we cannot explain changes in relative earnings by reference to changes in relative capital prices, as we do in dealing e.g. with machines. There are no capital markets for human resources and any attempt to bring in changing imputed capital prices for various kinds of labor would obviously rob the physical,human capital concept of any explanatory power and bring us back to the starting point, i.e. to the financial concept of human capital value as a discounted sum of future earnings.

The quotient of wages in (2.1) expresses in equilibrium the marginal rate of substitution between the two kinds of factors, i.e. between human resource units of states i and j. The assumption of (2.1) can therefore equally well be expressed in the following manner. <u>The marginal rate of substitution between two kinds of labor should be independent of everything except the amounts of labor inputs.</u>

If we simplify the model to account only for a vector of production capital q, two kinds of labor inputs h_i and h_j and a homogeneous output C, the condition can be more directly stated as follows. "The marginal rate of substitution between the two labor inputs is independent of the output level and the vector of production capital".

In this form the condition is known as the Leontief condition (Leontief, 1947) and expresses a necessary condition for aggregating the two inputs. We can then call our condition above a "generalized Leontief condition" and have as an hypothesis that this condition also expresses a necessary aggregation condition. Let us try briefly to follow up this idea.

In our model we have represented production possibilities as point sets. Let us for illustrative purposes use a simplified form of such a set T with elements of the form (h, q, c) where h still stands for a vector of labor inputs, q for a vector of production capital and c for a vector of output goods.

Aggregation of h in terms of such a model simply means that there exists a well-defined function ϕ, which relates h to a scalar, an aggregate, H: $H = \phi(h)$, and which possesses the following property:

There exists a set T_a with elements (H, q, c), such that $(H, q, c) \in T_a$ if and only if $(h, q, c) \in T$.

(For a more extensive discussion of meanings of aggregation the reader is refered to Fisher (1965), Morishima (1961) and Bliss (1975)).[x]

That the Leontief condition is a necessary condition for aggregation can easily be seen in the following way. Let us write the differentiable production function as: $C = P(q, h)$. If we

x) The meaning of this can perhaps be better grasped intuitively if expressed in an alternative way.

For every possible collection of production capital, q, and output, c, in T there is paired a set of possible combinations of labor inputs, $\{h \mid (h, q, c) \in T = S_h$. Seeking an aggregating function then means we are looking for a way of ordering these sets S_h in a linear way so that each set can be assigned a unique number H. This is obviously only possible if a complete (and continuous) order is already established between the sets by inclusion, i.e. if the sets are such that either $S_{h_i} \subseteq S_{h_j}$ or $S_{h_i} \supseteq S_{h_j}$. A collection of sets S_h ordered in this way is said to be nested.

The definition of aggregation given above can also be shown to be equivalent to the following condition (for proof cf e.g. Bliss (1975)):

The sets S_h are nested.

substitute the aggregate $H = \phi(h)$ in this function we get $C = P(q, H)$. The marginal rate of substitution between two kinds of labor becomes:

$$\frac{w_i}{w_j} = \frac{P_H \cdot \phi_i}{P_H \cdot \phi_j} = f(h) \tag{2.2}$$

where P_H and ϕ_i represent partial derivatives with respect to H and an i state component of h, respectively.

As long as we can reformulate our model in terms of differentiable functions this demonstration can obviously easily be extended to encompass our generalized version of the Leontief condition.[x]

The concrete meaning of our generalized Leontief condition is easily spelled out. It is obviously a very strong assumption that is only fulfilled in some very special theoretical cases and almost certainly never in real life.

One such case is of course the case of perfectly homogeneous labor, an infinite elasticity of substitution between any two states of human capital.

Another case exemplifies the opposite, with fixed coefficients for different kinds of labor - an elasticity of substitution equal to zero.

A possible third theoretical case would be when labor is already aggregated in real life by being organized in "labor companies" that offer for sale a homogeneous intermediate labor service.

These examples suffice to indicate the strength - and lack of realism - of the assumption.[xx]

x) It should perhaps be pointed out that here we have treated aggregation conditions in the most general form, which is motivated by our aim of applying the conclusions also to our very general MGE-model. Had we narrowed our attention only to stationary equilibria the aggregating conditions would have been formulated in a different way, stressing not only the difficulties in establishing a linear order but also the problem of doing so in such a way as to preserve functional relations in the aggregate. This is best exemplified by the discussion, wellknown in capital theory, of the possibility of establishing meaningful chain-indices for capital goods between different stationary equilibria. (See e.g. Champernowne (1953-54) and Bliss (1975)).

xx) If we want to restate the assumptions in terms of a stationary model version (see note at the end of the chapter), it should be observed that the assumption does not require the wage structure to be constant between stationary equilibria. This would be the case if human capital were really "non-reproducible". As it is the wage structure may change between equilibria, not as a direct consequence of other changing equilibrium prices, but indirectly via induced changes in the composition of different states of human capital.

2.3 The second assumption: Homogeneous human capital

The first step towards restricting our model only established that the earning capacity for an individual in a certain state is somehow physically embodied in the individual. This obviously does not take us much further when it comes to finding a uniform explanation for the relative earnings. We have only as it were moved the problem to be explained one step backwards, into the physical character and productivity of the various individuals.

Since it is not the aim of human capital theories to provide explanations of physical differences in productivity between different individuals, the next step to be taken is obvious. We must also assume that human capital is homogeneous, i.e. that any two units of human capital have an infinite elasticity of substitution. This means that the human capital of different individuals, h, can not only be aggregated to H but also measured with the same measuring rod, i.e. in units of H.

When all human capital is one and the same, its distribution between individuals does not matter, by definition (we abstract here from indivisibilities). In (2.1) the wage quotient depended on the variable composition of the human capital stock. With homogeneous capital this functional relation must be constant, determined by the given human capital values, H_i and H_j, of the respective states of human resources.

$$(3.1) \qquad \frac{w_i}{w_j} = f(h) = g\left(\frac{H_i}{H_j}\right)^{x)}$$

We have so far only dealt with what homogeneous human capital is, without explicitly saying anything about how it is formed. Our MGE-model provided a very general description of human investments in terms of "development services", $\dot{h}$. If we assume that the possible transformations of human capital can be represented separately in functional form, the process described in the model for human capital formation can be written as:

$$(3.2) \qquad H_t = \psi^{\alpha} (H_{t-1}, \dot{h}_t) = H_o^{\alpha} + \psi^{\alpha} (\dot{h}_1, \ldots\ldots, \dot{h}_t)$$

x) In studying a linear stationary equilibria we can make this statement more precise. We then know that net rentals on all capital will just pay the interest costs on the capital values. (3.1) thus becomes:

$$\frac{w_i}{w_j} = \frac{raH_i}{raH_j} = \frac{H_i}{H_j}$$

where a represents some given unit price of human capital.

(3.2) says that for an individual of type α his amount of human capital at time t is a function of both his human capital in the preceding period and the development service, the human investment, that he has absorbed since then. This recursive formula can equally well be rewritten, as is done at the end of (3.2), in terms of an initial type capital, H_o^{α}, and a function of the series of investments up to time t, that measures what we can call the investment-induced capital.

2.4 The third and fourth assumptions: Homogeneity of human investments and the ordering of human types

By means of our two successive restrictions we have now arrived at the concept of a homogeneous human capital, whose actual amount determines the state and through that, also the earnings of an individual. Is this then enough on which to build a general and applicable economic explanation of relative earnings? To see that the answer to this question is still no, it suffices to look at the expression in (3.2.) above.

To determine the amount of human capital and, in turn, the earnings of an individual we must obviously know a) what type of individual he or she is b) what the exact form of the functional relationship in (3.2) is for this type and c) what kind of experiences the individual has had and in what order. This is a rather big order, certainly too big for anyone aspiring to reach a simple and unified explanation in economic terms. Since we assume here that this is the aim of the pure human capital theory we must introduce further restrictions.

One first such restriction must deal with the various kinds of human investments. It does not help us to know that the human capital, which is inaccessible for direct measurement, is homogeneous if there are any number of heterogeneous inputs that can produce this capital. We would then need exact knowledge of the effect of each of these on human capital production. The only assumption that will let us escape from these difficulties is the assumption of homogeneous human investments. We assume in other words that we can treat the $\dot{h}$ as homogeneous, measurable and aggregable in terms of scalar $\dot{H}$, in the same way as we assumed human capital, the h, to be homogeneous. It must be fully homogeneous, not just possible to aggregate in the production function, since we also need to be sure that the order of investments over time can be neglected.

With this assumption (3.2) can be rewritten as:

$$(4.1) \qquad H_t = \psi^{\alpha} (\dot{H}_t)$$

where $\dot{H}_t$ measures the sum of human investments for an individual of type α up until time t.

The usual empirical interpretation of the assumption is of course that the collection of investments in an individual can be measured by the sum of discounted investment costs.

However, there is still a major obstacle left before we can arrive at a unified explanation. As long as the functional relations, ψ^{α}, are left wholly unspecified, we could still have heterogeneous people even though both human capital and human investments are assumed homogeneous. If the set of functions, ψ^{α}, differ between themselves in many ways, e.g. in many functional constants, people of different types will be heterogeneous in that their differences cannot be measured by any one scalar. We would then still have to make specific estimations of the function for each type, which would usually require more information and other kinds of information than we have available to us.

To explicitly exclude this possibility we must assume that the functional relations, the "production functions for human capital", are the same for all types only differing in one "type constant", say α. This means we now assume that (4.1) can be written:

$$H_t = \psi(\dot{H}, \alpha) \qquad (4.2)$$

α can be said to represent some uni-dimensional measure of capability. People may be more or less capable but they are otherwise the same. To give capability an unambiguous meaning one should also add the further assumption that given the same volume of human investments, a more capable person is always a better prospect for further investments. This is analogous to the natural assumption that human investments always have a non-negative yield. These two assumptions may be written as:

$$\psi'_{\alpha} \geq 0 \qquad (4.3)$$

$$\psi'_{\dot{H}} \geq 0$$

By means of these last assumptions we have finally restricted our model to the point where it can be said to yield a unified explanation of relative earnings in terms of embodied human capital. "All" we have to do - if we believe in the explanation - is to try to estimate the ψ-function and measure the capability of various individuals on some scalar scale.

We certainly cannot claim that all so-called human capital theories are based on these assumptions. What we can claim is that those theories that have the stated explanatory goal and are conceived within the context of a general equilibrium must make these, or analogous, assumptions. Our guess is that this covers a major part of the literature on human capital theory.

A note on stationary equilibrium

The MGE-model is somewhat unwieldly for serving as an example and frame of reference. Instead of using a general intertemporal equilibrium it is sometimes more convenient to formulate the problems in terms of a comparison between different stationary equilibria, all possible within the given technology but with different equilibrium prices.

This also makes the empirical interpretation easier since in a stationary model you can avoid the difficulty of having to separate wages from net human investments.

In principle you could attain the same advantages by using, instead of a stationary model, a model with semi-stationary growth, i.e. a model where all inputs and outputs keep expanding proportionately at a given rate.

However, this procedure is not open to us since we have explicitly incorporated real estate into our model and have defined types of real estate in terms of given and unchangeable characteristics of land. With our broad interpretation of real estate it would make even less sense than usual to make use of economists' worn-out excuse that land expansion is to be interpreted as a land-augmenting but otherwise neutral technical progress.

Since we are no longer interested in tracing individual investments in human resources we can simplify by reverting to the more traditional institutional arrangements, splitting up the holding companies so that produced capital goods are managed by the production firms while the human resources and real estate are held directly by the households. We may also treat the collection of firms as if it were only one maximizing unit, since the production technologies of the firms add up to the total production plan for the economy, which will maximize profits only if this is true of each component firm.

The equilibrium being stationary there will be no net investment in human resources or in any other kind of resources, which means that in the aggregate, from the firm's point of view, we can treat human resources and real estate as if they were unaffected by production and compensated only by user charges paid when production matures after one period.

For notational convenience we can further partition the price vector p into separate vectors for consumption, and for various factors of production. This means that we treat the same commodity appearing both in consumption and as production input as two separate commodities.

With these modifications the transformation taking place in the firms can then be expressed as the following pair of inputs and outputs, being one of the many possible transformations included in the, once and for all, given set of production possibilities, T:

(1) $\{(0, q, e, h), (c, q, 0, 0)\} \in T$

where q stands for producer inputs, e for real estate, h for human capital and c for consumption.

What (1) says is then simply that, with the help of the "non-producible" factors, real estate and human resources, a given vector of production capital reproduces itself and also leaves a consumption surplus. This is the production cycle that is assumed to repeat itself during each period in the stationary equilibrium.

Another common way of characterizing the stationary equilibrium is to write down the expression for aggregate firm profit. With the use of the partitioned price vector the profit expression will appear in the following well-known format:

(2) $\Pi = p_c c - rp_q q - p_a e - p_w h$

$p_c c$ here represents the income from consumption sales, $rp_q q$ is the interest cost of holding stocks of production capital, while $p_a e$ and $p_w h$ represent rentals for real estate of different kinds and for human resources, respectively, both measured at the time of payment.

This all sounds so familiar that it may be well to remind the reader of some aspects of (2), due to its derivation from the more general model, which make it still somewhat special.

Real estate and human resources here are "non-producible" only as to type but not as to state. Since state is what matters in production, there is still a large, although restricted, choice between alternative distributions between states, reachable within the given technology by way of investments in development.

Secondly we cannot claim here that equilibrium prices will be uniquely determined by the chosen technological transformation. Much of the recent discussion around stationary models has assumed that prices are not dependent on demand and these assumptions have been formalized into various so-called non-substitution theorems. However, there can never be a unique equilibrium price independent of demand if there is joint production of goods that have to face different demands. This is certainly the case in our MGE-model, where development services were specifically defined in terms of a joint production process.

(2) can be still further simplified. Although linearity or constant returns to scale is not a necessary concomitant of stationarity, it is rather hard to find plausible excuses for

not making this assumption. Therefore in the following we assume $\Pi = 0$.

The competitive conditions in equilibrium can now be expressed in a straightforward way.

Since we are dealing with a competitive equilibrium we know that the chosen transformation is more profitable than any other possible transformation at the given equilibrium prices. If we call this transformation number one and number the variables accordingly, we have

(3) $\Pi^1 = p_c^1 - r^1 p_q^1 q^1 - p_a^1 e^1 - p_w^1 h^1 = 0$

If we compare this with an alternative transformation called number two, the condition of competitive equilibrium ensures that:

(4) $p_c^1 c^2 - r^1 p_q^1 q^2 - p_a^1 e^2 - p_w^1 h^2 \leq 0$

If we subtract (3) from (4) and define $\Delta y = y^2 - y^1$ where y is any vector of values we get:

(5) $p_c^1 \Delta c - r^1 p_q^1 \Delta q - p_a^1 \Delta c - p_w^1 \Delta h \leq 0$

Already from this restatement of competitive conditions we notice one main difference from the usual discussions in capital theory. There it is assumed that non-produced factors are strictly non-producible which means that Δe and Δh in (5) are both equal to zero and that we can continue to study in isolation e.g. the impact of capital intensity on consumption standards. When investments in human resources and real estate are brought in as here, the discussion has to be broadened to deal also with changing patterns in these kinds of capital.

When human capital is homogeneous we can use the aggregate directly in reformulating the profit expression for our stationary equilibrium (cf (2) above).

(6) $p_c c - rp_q q - raH - p_a e = 0$

If we want to illustrate how human capital theory impinges on the controversial issues in capital theory, this can easily be done with (6) as a starting point.

Let us make all the simplifying assumptions that were used earlier in the more "naive" discussions of capital theory. This means that on top of our earlier assumptions we now also assume that a) consumption and produced capital can also be treated as aggregates and represented by C and Q and b) no land is needed in production, i.e. all components of e are equal to zero. (6) then reduces to:

(7) $C - rbQ - raH = 0$

where b represents the unit price of produced capital. If we now compare two alternative equilibria, using the competitive condition, as in (5) above, we arrive by successive subtracting to the following condition:

(8) $\Delta (rb)\ \Delta Q + \Delta (ra)\ \Delta H \leq 0$

where, as before, $\Delta(rb) = r^2b^2 - r^1b^1$, etc.

In the usual capital theory discussions, labor resources were assumed given and unchangeable, i.e. $\Delta H \equiv 0$. (8) can then be interpreted as expressing the well-known notion that "a stationary state with a higher level of capital relative to non-produced factors cannot have a higher rental of capital in terms of consumption".

Here, with $\Delta H \neq 0$, i.e. with possibilities for human investments, the interpretation of (8) becomes somewhat more complex. The condition now states that "if a stationary state with higher level of produced capital also has a higher rental for this capital, then the resulting cost increase must be at least compensated by either a decrease in human capital or a lowering of the rental on this capital".

Since this whole line of reasoning presupposes uninhibited aggregation throughout it does not really merit much interest. As is well known one outcome of recent controversies in capital theory has been a rather general agreement that an over-all-aggregation of produced capital is never theoretically justifiable. There is an irony in the fact that human capital theory, building on aggregating conditions, matured and was turned into applications at the very time that agreement was reached on the impossibility of capital aggregation in general.

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