Redispatch in Zonal Pricing Electricity Markets

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Abstract

Zonal pricing electricity markets operate sequentially. First, the suppliers compete in a spot market. Second, to alleviate the congestion in the transmission line, in a redispatch market, the suppliers in the importing node are called into operation to increase their production, and the suppliers in the exporting node are compensated to reduce their production. I characterize the equilibrium in a zonal market when the competition is imperfect and the spot and redispatch markets operate sequentially. I also work out the equilibrium when the transmission line is taken into account in the spot market, i.e., it is not necessary to introduce a redispatch market to alleviate the congestion in the transmission line. I find that the consumers' welfare and suppliers' profits depend crucially on the type of redispatch design implemented by the auctioneer, and that could introduce long term investment distortions.

KEYWORDS: electricity auctions, redispatch design, transmission constraint, zonal pricing electricity markets.
JEL codes: D43, D44, L13, L94

1 Introduction

Electricity markets are organized as nodes, where a node is a market with consumers and suppliers. Those nodes are connected through transmission lines. In the presence of transmission constraints, electricity markets can be organized as nodal or as zonal pricing electricity markets. In a nodal pricing electricity market, the equilibrium price differs across nodes when the transmission line is congested. In contrast, in a zonal pricing electricity market, the equilibrium price is the same in all the nodes that belong to the same zone. When the transmission line is congested, in a zonal pricing electricity market a redispatch mechanism has to be introduced to alleviate the congestion in the transmission line.

Zonal pricing electricity markets is one of the most salient designs to organize electricity markets in the presence of transmission constraints, and the European Commission proposes that design for the integration of the European electricity markets.[1] However, our knowledge of zonal pricing electricity markets when the competition in those markets is imperfect is still limited. I contribute to improve our understanding of those markets by characterizing the equilibrium in a zonal pricing electricity market when the competition is imperfect and when four different redispatch designs are implemented by the auctioneer to manage the congestion in the transmission line.

I use a duopoly model similar to Fabra et al. (2006). In the basic set up, there are two suppliers with positive production capacities and nil marginal costs that are located in two different nodes ("North" and "South"). Each supplier faces a perfectly inelastic demand in each node that is known with certainty when the suppliers submit their offer prices. The aim of this paper is to focus on the design of redispatch mechanisms to manage the congestion in zonal pricing electricity markets. The introduction of elastic demand does not change the results, but it complicates the analysis. The assumption that suppliers have perfect information concerning node demand is reasonable when applied to markets where offers are "short lived", such as in Spain, where there are 24 hourly day-ahead markets each day. The introduction of demand and supply in both nodes allows the model to understand the economic forces that determine that flow of electricity. The two nodes are connected through a transmission line with a limited transmission capacity.[2]

The spot electricity market is organized as a zonal pricing electricity market, i.e., the equilibrium price in both nodes is the same. In the spot electricity market, each supplier submits a single price offer for its entire capacity[3] in a discriminatory price auction such as those used in the UK wholesale electricity market, or in a uniform price auction such as those used in the majority of European countries (e.g., Nord Pool or Italy). When the auction is uniform, the price received by the suppliers and the price paid by the consumers is the same. In contrast, when the auction is discriminatory, those prices differ. In particular, in the discriminatory price auction, I assume that the price paid by the consumers is the average of the price received by the suppliers.[4]

As soon as the transmission line is congested, some type of redispatch mechanism must be introduced to avoid the congestion in the transmission line. The supplier located in the exporting node must reduce its production and the supplier located in the importing node must increase its production to alleviate the congestion in the transmission line. The redispatch can follow a market based mechanism (Holmberg and Lazarczyk, 2015)

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[1]For a complete literature review of bidding zones in zonal pricing electricity markets, see Ofgem (2014) and ENTSO-E (2014). For a complete analysis of the delineation of regions for Transmission System Operators coordination and the governance of System Operators in the integration of European electricity markets, see ENTSO-E (2015), and European Commission (2015).

[2] The term "transmission capacity constraint" is used throughout this article in the electrical engineering sense: a transmission line is constrained when the flow of power is equal to the capacity of the line, as determined by engineering standards.

[3]Fabra et al. (2006) show that the equilibrium outcome allocation does not change when firms submit single price offers for their entire capacity and when they submit a set of price-quantity offers.

[4]To my knowledge, England is the unique country that work out the equilibrium in the wholesale electricity market by using a discriminatory price auction. For a complete review of the way in which the prices paid by the consumers are worked out, and the transition from the fixed-term contracts to variable tariffs in England read Ofgem (2013), and Ofgem (2017).

or a command and control mechanism (Krause, 2005). When a redispatch market based mechanism is introduced, the redispatch market is organized as a discriminatory price auction.

I characterize the equilibrium by using four different redispatch designs. First, I work out the equilibrium when the auction in the spot electricity market is uniform and when an ex-ante redispatch mechanism is introduced by the auctioneer, i.e., it is not necessary to implement a redispatch market to alleviate the congestion in the transmission line. Second, I work out the equilibrium when the auction in the spot electricity market is discriminatory and when an ex-ante redispatch mechanism is introduced by the auctioneer. Third, I work out the equilibrium when the auction in the spot electricity market is uniform, an ex-post redispatch mechanism is introduced and the suppliers submit the same bid in the spot and in the redispatch market. Fourth, I work out the equilibrium when the auction in the spot electricity market is uniform, an ex-post redispatch mechanism is introduced and the suppliers submit different bids in both markets.

I find that the profits of the supplier located in the importing node are the same with independence of the redispatch design. The supplier located in the importing node can always satisfy the residual demand at the maximum price allowed by the auctioneer. Therefore, its profits are the same with independence of the redispatch design.

The profits of the supplier located in the exporting node depend crucially on the type of redispatch design. When the auction in the spot electricity market is discriminatory and an ex-ante redispatch mechanism is introduced by the auctioneer, the profits of the supplier located in the importing node are larger than the profits of the supplier located in the exporting node. In that case, the suppliers sell their production at their own bid, and the profits of the supplier located in the importing node are larger, since that supplier faces a larger demand. When the auction in the spot electricity market is uniform and an ex-ante redispatch mechanism is introduced by the auctioneer, the profits of the supplier located in the importing node are lower than the profits of the supplier located in the exporting node. In that case, the supplier located in the importing node sets the price, and the supplier located in the exporting node satisfies the demand in its own node and the demand in the importing node up to the transmission capacity at the maximum price allowed by the auctioneer. The introduction of an ex-post redispatch mechanism increases the profits of the supplier located in the exporting node, since in that case, that supplier is compensated in the redispatch market for the electricity that it cannot sell in the importing node.

The change on suppliers' profits described above can introduce long term investment distortions. In particular, when the auction is discriminatory and an ex-ante redispatch mechanism is introduced by the auctioneer, the suppliers prefer to invest in the importing node since the expected equilibrium profits are larger in that node. In contrast, when the auction is uniform and an ex-ante redispatch mechanism is introduced by the auctioneer, the suppliers prefer to invest in the exporting node. The introduction of an ex-post redispatch mechanism makes even more attractive to invest in the exporting node. Therefore, the introduction of different redispatch designs affect crucially long-term investment incentives.

The consumers' surplus also depend on the type of redispatch design. In particular, when the auction in the spot electricity market is uniform and for any type of redispatch design, the consumers' surplus is nil, since the equilibrium price is equal to consumers' reserve price. In contrast, when the auction in the spot electricity market is discriminatory and an ex-ante redispatch mechanism is introduced by the auctioneer, the consumers' surplus is positive, since the equilibrium price is lower than consumers' reserve price.

In their seminal papers, Bohn et al. (1984), Hogan (1992), Wu et al. (1996) Chao and Peck (1996) characterize the equilibrium in a perfect competitive nodal pricing electricity market. Adding more structure to those models, Bjørndal and Jørnsten (2001), and Bjørndal and Jørnsten (2007) characterize the equilibrium in a zonal pricing electricity market and analyze the effect of the configurations of different bidding zones on equilibrium outcome allocations. Following Bjørndal and Jørnsten (2001), and Bjørndal and Jørnsten (2007), I assume that an ex-ante redispatch mechanism is introduced by the auctioneer. I complement their analysis by characterizing the equilibrium when the competition is imperfect and when the spot electricity market is cleared by using a uniform and a discriminatory price auction.

Holmberg and Lazarczyk (2015) compare the equilibrium performance between nodal pricing electricity markets, zonal pricing electricity markets with counter-trading, and discriminatory pricing in large games with many producers and certain information. They conclude that the three market designs result in the same efficient dispatch, but that the zonal pricing with counter-trading results in additional payments to producers in export-constrained nodes, which leads to inefficient investments in the long run. I follow their approach, characterizing the equilibrium using three different redispatch designs: First, I work out the equilibrium when the auction in the spot electricity market is discriminatory and when an ex-ante redispatch mechanism is introduced by the auctioneer. Second, I work out the equilibrium when the auction in the spot electricity market is uniform, an ex-post redispatch mechanism is introduced and the suppliers submit the same bid in the spot and in the redispatch market. Third, I work out the equilibrium when the auction in the spot electricity market is uniform, an ex-post redispatch mechanism is introduced and the suppliers submit different bids in both markets. I complement their analysis by introducing imperfect competition.

In a context of imperfect competition, there exists also a wide literature that characterizes the equilibrium in an electricity market in the presence of transmission constraints. Borenstein et al. (2000) characterize the equilibrium in an electricity network where suppliers compete in quantities as in a Cournot game. Holmberg and Philpott (2012) solve for symmetric supply function equilibria in electricity networks when demand is uncertain ex-ante. Escobar and Jofré (2010) analyze the effect of transmission losses and transmission costs on equilibrium outcome allocations, but they neglect transmission constraints.

Dijk and Willems (2011) work out the equilibrium in a zonal pricing electricity market taking into account the redispatching strategic effect. Following Joskow and Tirole's (2000) approach, they consider that the demand is concentrated to only one node, i.e., the electricity always flow in the same direction. I extend their analysis by introducing supply and demand in both nodes and I characterize the equilibrium when the electricity flows in one direction, and when it flows in the other direction.

The article proceeds as follows. Section 2 describes the model when the transmission line is congested. Section 3 characterizes the equilibrium . Section 4 compares suppliers' profits and consumers' welfare for the four redispatch designs. Section 5 concludes the paper. In annex 1, I describe the model when the transmission line is congested and when it is not congested. In annex 2 are all the proofs of the paper.

2   The model

Set up of the model. There exist two electricity nodes, node North and node South, that are connected by a transmission line with capacity T. Both nodes belong to the same zone, i.e., the equilibrium price in both nodes is the same even when the transmission line is congested (zonal pricing).

There exist two duopolists with capacities k_n and k_s, where subscript n means that the supplier is located in node North and subscript s means that the supplier is located in node South. The suppliers' marginal costs of production are c_n and c_s for production levels less than the capacity, while production above the capacity is impossible (i.e., infinitely costly). Suppliers are symmetric in production costs $c_n = c_s = c = 0$.[5] The level of demand in any period, θ_n in node North and θ_s in node South, is independent of the node price, i.e., perfectly inelastic.[6] I introduce two more assumptions on demand levels. First, $\theta_i \in [\underline{\theta}_i, \overline{\theta}_i] \subseteq [0, k_i + T]$, $i = n, s$, i.e., the installed production capacity in each node plus the electricity that flows from the other node is enough to satisfy the peak demand in each node. Second, $\overline{\theta}_i + \overline{\theta}_j < k_i + k_j$, i.e., the total installed production capacity is enough to satisfy the peak demand in both nodes.

The capacity of the transmission line can be lower than the installed capacity in each node $T \leq min\{k_s, k_n\}$, i.e., the transmission line could be congested for some realization of demands (θ_s, θ_n). When $T > min\{k_s, k_n\}$, the transmission line is not congested and the equilibrium is as in Fabra et al. (2006). In the rest of this section, I describe the model when the transmission line is congested. In annex 1, I present a complete description of the model also when the transmission line is not congested.

Timing of the game. Having observed the realization of demands $\theta \equiv (\theta_s, \theta_n)$, each supplier simultaneously and independently submits a bid in the spot electricity market specifying the minimum price at which it is willing to supply up to its capacity, $b_i^S \leq P$, $i = n, s$, where the super script S denotes the spot electricity market, and P denotes the "market reserve price", possibly determined by regulation. P can be interpreted as the price at which all consumers are indifferent between consuming and not consuming, or a price cap imposed by the regulatory authorities (von der Fehr and Harbord, 1993). Moreover, when the auction in the spot electricity market is discriminatory, the equilibrium is in mixed strategies. In that case, when the demand is inelastic, the introduction of a price cap guarantees the existence of the upper bound of the support in a mixed-strategy

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[5]In this paper, I analyze the performance of different redispatch designs in zonal markets. In order to focus on this effect, I assume that suppliers are symmetric in production costs. To facilitate the design of the figures and the intuitions behind the results, in all the figures and examples, I also assume that the suppliers are symmetric in production capacity $k_n = k_s = k > 0$.

[6]The introduction of elastic demand does not change the results, but it complicates the analysis and the intuition. Therefore, to keep the analysis as simple as possible, I assume inelastic demand.

equilibrium (Baye et al., 1992; Deneckere et al., 1996; Fabra et al., 2006).

Let $b^S \equiv (b_s^S, b_n^S)$ denote a bid profile in the spot electricity market. On basis of this profile, the auctioneer calls suppliers into operation and works out suppliers' outcomes and profits. If suppliers submit different bids, the capacity of the lower-bidding supplier is dispatched first. If the capacity of the lower-bidding supplier is not sufficient to satisfy total demand, the higher-bidding supplier's capacity is then dispatched to serve residual demand. If the two suppliers submit equal bids, then supplier i is ranked first with probability ρ_i, where $\rho_n + \rho_s = 1$, $\rho_i = 1$ if $\theta_i > \theta_j$, and $\rho_i = \dfrac{1}{2}$ if $\theta_i = \theta_j$, $i = n, s$, $i \neq j$.[7]

When the auction in the spot electricity market is uniform and an ex-ante redispatch mechanism is introduced by the auctioneer, the output allocated to supplier n in the spot electricity market (supplier s's output function is symmetric), denoted by $q_n^{u1}(b^S; \theta)$, is given by:[8]

$$q_n^{u1}(b^S; \theta) = \begin{cases} \theta_n + T & \text{if } b_n^S \leq b_s^S \text{ and } \theta_s - T > \theta_s + \theta_n - k_n \\ \theta_n - T & \text{if } b_n^S > b_s^S \text{ and } \theta_n - T > \theta_s + \theta_n - k_s \end{cases} \quad (1)$$

When supplier n submits the lower bid in the spot electricity market and the transmission line is congested, supplier n cannot satisfy the demand in node South, even when it has enough production capacity. Therefore, the total demand that supplier n can satisfy is $(\theta_n + T)$ (top left-hand panel, figure 2, annex 1). When supplier n submits the higher bid in the spot electricity market, and the transmission line is congested, supplier n's residual demand is defined by $(\theta_n - T)$ (top right-hand panel, figure 2, annex 1).

When the auction in the spot electricity market is discriminatory and an ex-ante redispatch mechanism is introduced by the auctioneer, the output allocated to supplier n in the spot electricity market, denoted by $q_n^d(b^S; \theta)$, is as when the auction is uniform and an ex-ante redispatch mechanism is introduced by the auctioneer (top panels, figure 2, annex 1).

When the auction in the spot electricity market is uniform and an ex-post redispatch mechanism is introduced by the auctioneer, the output allocated to supplier n in the spot electricity market, denoted by $q_n^{u2}(b^S; \theta) = q_n^{u3}(b^S; \theta)$, is given by:

$$q_n^{u2}(b^S; \theta) = q_n^{u3}(b^S; \theta) = \begin{cases} \min\{\theta_s + \theta_n, k_n\} & \text{if } b_n^S \leq b_s^S \text{ and } \theta_s - T > \theta_s + \theta_n - k_n \\ \theta_s + \theta_n - k_s & \text{if } b_n^S > b_s^S \text{ and } \theta_n - T > \theta_s + \theta_n - k_s \end{cases} \quad (2)$$

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[7]The implemented tie-break rule is such that if the bids of both suppliers are equal and demand in node i is larger than demand in node j, the auctioneer first dispatches the supplier located in node i. Moreover, when the auction in the spot electricity market is discriminatory, the equilibrium in this model is in mixed strategies. In that case, the tie-breaking rule ensures the existence of a mixed strategies equilibrium (Dasgupta and Maskin, 1986).

[8]I use the super script $u1$ to denote the design where a uniform price auction is implemented in the spot electricity market and an ex-ante redispatch mechanism is introduced by the auctioneer; the super script d to denote the design where a discriminatory price auction is implemented in the spot electricity market and an ex-ante redispatch mechanism is introduced by the auctioneer; the super script $u2$ to denote the design where a uniform price auction is implemented in the spot electricity market, an ex-post redispatch mechanism is introduced by the auctioneer and the suppliers submit the same bid in the spot and in the redispatch market; finally, I use the super script $u3$ to denote the design where a uniform price auction is implemented in the spot electricity market, an ex-post redispatch mechanism is introduced by the auctioneer and the suppliers submit different bids in the spot and in the redispatch market.

When an ex-post redispatch mechanism is introduced, the congestion is not taken into account when the spot electricity market is cleared. Therefore, when supplier n submits the lower bid in the spot electricity market, it satisfies the total demand $(\theta_s + \theta_n)$ up to its production capacity (k_n) (bottom left-hand panel, figure 2, annex 1). When it submits the higher bid, it satisfies the residual demand $(\theta_s + \theta_n - k_s)$ (bottom right-hand panel, figure 2, annex 1).

When the transmission line is congested and an ex-post redispatch mechanism is introduced by the auctioneer to alleviate the congestion in the line, the outcome allocated to supplier n in the redispatch market is denoted by (I use the super script R to denote the redispatch market):

$$
q_n^R(b^S; \theta) = \begin{cases} min\left\{\theta_s + \theta_n, k_n\right\} - (\theta_n + T) & \text{if } b_n^S \leq b_s^S \text{ and } \theta_s - T > \theta_s + \theta_n - k_n \\ (\theta_n - T) - (\theta_s + \theta_n - k_s) & \text{if } b_n^S > b_s^S \text{ and } \theta_n - T > \theta_s + \theta_n - k_s \end{cases} \tag{3}
$$

When supplier n submits the lower bid in the spot electricity market, it is dispatched first, but due to the transmission constraint it cannot satisfy the total demand or sell its entire production capacity $(min\left\{\theta_s + \theta_n, k_n\right\})$, but only $(\theta_n + T)$. Therefore, in the redispatch market it has to buy back the difference between what it wants to sell and what it can sell $(min\left\{\theta_s + \theta_n, k_n\right\} - (\theta_n + T))$. When supplier n submits the higher bid in the spot electricity market, it is dispatched last. Due to the transmission constraint, it faces a high residual demand and it can sell more electricity $(\theta_n - T)$ that what it sells in the spot electricity market $(\theta_s + \theta_n - k_s)$. Therefore, in the redispatch market it can sell all the electricity that it could not sell in the spot electricity market $((\theta_n - T) - (\theta_s + \theta_n - k_s))$.

Finally, the payments are worked out by the auctioneer. When the auction in the spot electricity market is uniform and an ex-ante redispatch mechanism is introduced by the auctioneer, the price received by a supplier in the spot electricity market for any positive quantity dispatched by the auctioneer is equal to the higher bid accepted in the auction. Hence, for a given realization of $\theta \equiv (\theta_s, \theta_n)$ and a bid profile $b \equiv (b_s, b_n)$, supplier n's profits can be expressed as $\pi_n^{u1}(b^S; \theta)$:

$$
\begin{cases} b_s^S(\theta_n + T) & \text{if } b_n^S \leq b_s^S \text{ and } \theta_s - T > \theta_s + \theta_n - k_n \\ b_n^S(\theta_n - T) & \text{if } b_n^S > b_s^S \text{ and } \theta_n - T > \theta_s + \theta_n - k_s \end{cases} \tag{4}
$$

When supplier n submits the lower bid in the spot electricity market and the transmission line is congested, supplier s sets the price and supplier n's profits are defined by $(b_s^S(\theta_n + T))$ (top-left panel, figure 3, annex 1). When supplier n submits the higher bid in the spot electricity market and the transmission line is congested, it sets the price and its profits are defined as $(b_n^S(\theta_n - T))$ (top-right panel, figure 3, annex 1).

When the auctioneer runs a discriminatory price auction in the spot electricity market and an ex-ante redispatch mechanism is introduced by the auctioneer, the price received by a supplier in the spot electricity market for any positive quantity dispatched by the auctioneer is equal to its own offer price, whenever a bid is wholly or partly accepted. Hence, for a given realization of demands $\theta \equiv (\theta_s, \theta_n)$ and a bid profile $b \equiv (b_s, b_n)$, supplier n's profits can be expressed as $\pi_n^d(b^S; \theta)$:

$$\begin{cases} b_n^S(\theta_n + T) & \text{if } b_n^S \leq b_s^S \text{ and } \theta_s - T > \theta_s + \theta_n - k_n \\ b_n^S(\theta_n - T) & \text{if } b_n^S > b_s^S \text{ and } \theta_n - T > \theta_s + \theta_n - k_s \end{cases} \tag{5}$$

When the auction in the spot electricity market is discriminatory and an ex-ante re-dispatch mechanism is introduced by the auctioneer, supplier n's profits are represented in the top panels in figure 3, annex 1.

When the auction in the spot electricity market is uniform, an ex-post redispatch mechanism is introduced by the auctioneer and supplier n submits the same bid in the spot and in the redispatch market, supplier n's profits can be expressed as $\pi_n^{u2}(b^S; \theta)$:

$$\begin{cases} b_s^S \min\{\theta_s + \theta_n, k_n\} - ... \\ \quad b_n^S(\min\{\theta_s + \theta_n, k_n\} - (\theta_n + T)) & \text{if } b_n^S \leq b_s^S \text{ and } \theta_s - T \leq \theta_s + \theta_n - k_n \\ b_n^S(\theta_s + \theta_n - k_s) + ... \\ \quad b_n^S((\theta_n - T) - (\theta_s + \theta_n - k_s)) & \text{if } b_n^S > b_s^S \text{ and } \theta_n - T > \theta_s + \theta_n - k_s \end{cases} \tag{6}$$

When supplier n submits the lower bid in the spot electricity market and the trans-mission line is congested, supplier s sets the price and supplier n's profits in that mar-ket are defined as $(b_s^S \min\{\theta_s + \theta_n, k_n\})$. However, due to the transmission constraint, supplier n cannot satisfy the demand in both nodes, and it has to use the redispatch market to buy back the capacity that cannot be sold in the spot electricity market $(\min\{\theta_s + \theta_n, k_n\} - (\theta_n + T))$. Given that the redispatch market is designed as a dis-criminatory price auction, supplier s's expenses in the redispatch market are determined by $b_n^S (\min\{\theta_s + \theta_n, k_n\} - (\theta_n + T))$ (bottom-right panel, figure 3, annex 1). By sum-ming and subtracting the term $b_s^S(\theta + T)$ in the first equation in 6, we can rewrite it as $b_s^S(\theta_n + T) + (b_s^S - b_n^S)(\min\{\theta_s + \theta_n, k_n\} - (\theta_n + T))$. This last expression has an useful economic interpretation, the first term represents supplier n's profits in the spot electric-ity when the transmission constraint is taken into account, the second term represents supplier n's compensation for not being able to satisfy the demand in both nodes.

When supplier n submits the higher bid in the spot electricity market and the trans-mission line is congested, supplier n's profits in that market are defined as $(b_n^S(\theta_s + \theta_n - k_s))$. However, due to the transmission constraint, supplier n can sell more electricity that what it sells in the spot electricity market. Therefore, in the redispatch market it can sell all the electricity that it could not sell in the spot electricity market $((\theta_n - T) - (\theta_s + \theta_n - k_s))$. Given that the redispatch market is organized as a discriminatory price auction, supplier n' profits in that market are defined as $b_n^S((\theta_n - T) - (\theta_s + \theta_n - k_s))$ (bottom-left panel, figure 3, annex 1).

After the algebra transformations described above, equation 6 can be rewritten as $\pi_n^{u2}(b^S; \theta)$:

$$\begin{cases} b_s^S(\theta_n + T) + ... \\ \quad (b_s^S - b_n^S)(\min\{\theta_s + \theta_n, k_n\} - (\theta_n + T)) & \text{if } b_n^S \leq b_s^S \text{ and } \theta_s - T \leq \theta_s + \theta_n - k_n \\ b_n^S(\theta_s + \theta_n - k_s) + ... \\ \quad b_n^S((\theta_n - T) - (\theta_s + \theta_n - k_s)) & \text{if } b_n^S > b_s^S \text{ and } \theta_n - T > \theta_s + \theta_n - k_s \end{cases} \tag{7}$$

When the auction in the spot electricity market is uniform, an ex-post redispatch mechanism is introduced by the auctioneer and supplier n submits different bids in the spot and in the redispatch market, supplier n's profits can be expressed as $\pi_n^{u3}(b^S, b^R; \theta)$:

$$
\begin{cases}
b_s^S \min\{\theta_s + \theta_n, k_n\} - ... \\
\quad b_n^R (\min\{\theta_s + \theta_n, k_n\} - (\theta_n + T)) & \text{if } b_n^S \le b_s^S \text{ and } \theta_s - T \le \theta_s + \theta_n - k_n \\
b_n^S(\theta_s + \theta_n - k_s) + ... \\
\quad b_n^R((\theta_n - T) - (\theta_s + \theta_n - k_s)) & \text{if } b_n^S > b_s^S \text{ and } \theta_n - T > \theta_s + \theta_n - k_s
\end{cases}
\tag{8}
$$

The unique difference between equations 6 and equation 8 is that in equation 8, the suppliers that are redispatched participate actively in the redispatch market by submitting their own bid. Therefore, in equation 6, supplier n's bid in the redispatch market is b_n^S, but in equation 8 is b_n^R.

As with equation 6, after doing some algebra equation 8 can be rewritten as $\pi_n^{u3}(b^S, b^R; \theta)$:

$$
\begin{cases}
b_s^S(\theta_n + T) + ... \\
\quad (b_s^S - b_n^R)(\min\{\theta_s + \theta_n, k_n\} - (\theta_n + T)) & \text{if } b_n^S \le b_s^S \text{ and } \theta_s - T \le \theta_s + \theta_n - k_n \\
b_n^S(\theta_s + \theta_n - k_s) + ... \\
\quad b_n^R((\theta_n - T) - (\theta_s + \theta_n - k_s)) & \text{if } b_n^S > b_s^S \text{ and } \theta_n - T > \theta_s + \theta_n - k_s
\end{cases}
\tag{9}
$$

As can be observed by comparing equations 4, 5, 7 and 9, the introduction of different redispatch designs in the zonal market change suppliers' profits functions. These equations present a lot of similarities, but also important differences that will affect the characterization of the equilibrium. In the rest of the paper, I work out the equilibrium when different redispatch designs are implemented and I compare them in terms of consumers' welfare and suppliers' profits.

3 Equilibrium

In this section I characterize the equilibrium for each of the four redispatch designs presented in the model section. As in the model section, I assume that node N is the importing node and node S is the exporting node.

In lemma 1, I study the type of equilibrium in the spot electricity market when a uniform and discriminatory price auction are implemented by the auctioneer.

Lemma 1. When the transmission line is congested, the equilibrium price in the spot electricity market is in pure strategies when the auction is uniform, but a pure strategies equilibrium does not exist when the auction is discriminatory.

Proof. When the transmission line is congested, the supplier located in the importing node faces a positive residual demand. In that case, when the auction is uniform, the supplier located in the importing node submits the maximum bid allowed by the auctioneer, and the supplier located in the low-demand node submits a bid that makes undercutting unprofitable.

In contrast, when the auction is discriminatory, a pure strategies equilibrium does not exist, since the suppliers have incentives to undercut each other to be dispatched first in the auction. □

In lemma 2, I study the equilibrium in the redispatch market.

Lemma 2. When the auction in the spot electricity market is uniform, an ex-post redispatch mechanism is introduced by the auctioneer and the suppliers submit different bids in the spot and in the redispatch market, in the redispatch market, the supplier located in the exporting node submits a bid equal to zero, and the supplier located in the importing node submits the maximum bid allowed by the auctioneer.

Proof. I can proof lemma 2 by using either equation 8 or equation 9.

According with equation 8, the profits of the supplier located in the exporting node are defined by $b_n^S \min\{\theta_s + \theta_n, k_s\} - b_s^R(\min\{\theta_s + \theta_n, k_s\} - (\theta_s + T))$, where $b_s^R(\min\{\theta_s + \theta_n, k_s\} - (\theta_s + T))$ represents supplier s' expenses when it buys back the electricity that it could not sell in the spot electricity market. Supplier s minimizes those expenses by submitting a bid equal to zero in the redispatch market.

According with equation 8, the profits of the supplier located in the importing node are defined by $b_n^S(\theta_s + \theta_n - k_s) + b_n^R((\theta_n - T) - (\theta_s + \theta_n - k_s))$, where $b_n^R((\theta_n - T) - (\theta_s + \theta_n - k_s))$ represents supplier n's profits in the redispatch market. Supplier n maximizes those profits by submitting the maximum bid allowed by the auctioneer.

If instead of using equation 8, I use equation 9, the profits of the supplier located in the exporting node are defined by $b_n^S(\theta_s + T) + (b_n^S - b_s^R)(\min\{\theta_s + \theta_n, k_s\} - (\theta_s + T))$, where $(b_n^S - b_s^R)(\min\{\theta_s + \theta_n, k_s\} - (\theta_s + T))$ represents the compensation for the electricity that supplier s wants to sell in the spot electricity market, but that it cannot sell because of the transmission constraint. Supplier s maximizes that compensation by submitting a bib equal to zero in the redispatch market.

The profits of the supplier located in the importing node are the same in equations 8 and 9.

Therefore, by using either equation 8 or equation 9, in the redispatch market, the supplier located in the importing node maximizes its profits by submitting the maximum bid allowed by the auctioneer, and the supplier located in the exporting node maximizes its profits by submitting a bid equal to zero. □

The suppliers can participate actively in the spot and in the redispatch markets, only when they can submit independent bids in both markets. When the supplier submits the same bid in the spot and in the redispatch market, the suppliers cannot participate actively in the redispatch market, but the same logic described in lemma 2 can be applied to understand the incentives of the suppliers in that case. In particular, by studying equations 6 and 7 and by using the same arguments that in lemma 2, it is easy to prove that if the supplier located in the importing node could participate in the redispatch market it would maximize its profits by submitting a bid equal to zero, and the supplier located in

the exporting node would maximize its profits by submitting the maximum bid allowed by the auctioneer.

Based on the ancillary results presented in lemmas 1, and 2, I present the main result of this section.

Proposition 1. When the transmission line is congested, depending on the redispatch design, the characterization of the equilibrium falls in one of the next four categories:

  i. When the auction in the spot electricity market is uniform and an ex-ante redispatch mechanism is introduced by the auctioneer, there are multiplicity of Nash equilibria in the spot electricity market.

 ii. When the auction in the spot electricity market is discriminatory and an ex-ante redispatch mechanism is introduced by the auctioneer, the equilibrium is in mixed strategies.

iii. When the auction in the spot electricity market is uniform, an ex-post redispatch mechanism is introduced by the auctioneer and the suppliers submit the same bid in the spot and in the redispatch market, there is an unique Nash equilibrium in the spot and in the redispatch market.

 iv. When the auction in the spot electricity market is uniform, an ex-post redispatch mechanism is introduced by the auctioneer and the suppliers submit different bids in the spot and in the redispatch market, there are multiplicity of Nash equilibria in the spot electricity market, but an unique Nash equilibrium in the redispatch market.

When the auction in the spot electricity market is uniform and an ex-ante redispatch mechanism is introduced by the auctioneer, the supplier located in the high-demand node faces a high residual demand. In that case, the supplier located in the high-demand node maximizes its profits by submitting the maximum bid allowed by the auctioneer and the supplier located in the low-demand node submits a bid that makes undercutting unprofitable (equations 18 and 21, annex 2).

When the auction in the spot electricity market is discriminatory and an ex-ante redispatch mechanism is introduced by the auctioneer, as I prove in lemma 1, the equilibrium is in mixed strategies. In that case the supplier located in the high-demand node faces a high residual demand and it submits in expectation higher bids than the supplier located in the low-demand node, i.e., the cumulative distribution of the supplier located in the high-demand node stochastic dominates the one of the supplier located in the low-demand node (equation 31, annex 2).

When the auction in the spot electricity market is uniform, an ex-post redispatch mechanism is introduced by the auctioneer and the suppliers submit the same bid in the spot and in the redispatch market, that bid has to be an equilibrium simultaneously in both markets. In that case, the supplier located in the high-demand node submits the maximum bid allowed by the auctioneer and the supplier located in the low-demand node submits a bid equal to zero, i.e., the equilibrium is unique (equation 37, annex 2).

The pair of strategies defined in equation 37 maximizes suppliers' profits simultaneously in both markets. The supplier located in the high-demand node maximizes its profits in the spot electricity market, since it satisfies the residual demand at the maximum price allowed by the auctioneer ($b_n^S(\theta_s + \theta_n - k_s)$, equation 7) By submitting the maximum bid allowed by the auctioneer, the supplier located in the high-demand node also maximizes its profits in the redispatch market, since when it is called into operation in that market, it sells its production capacity at the maximum bid allowed by the auctioneer ($b_n^S((\theta_n - T) - (\theta_s + \theta_n - k_s))$, equation 7). The supplier located in the low-demand node maximizes its profits in the spot electricity market, since it satisfies the demand in its own node and the demand in the importing node up to the transmission capacity at the price sets by the supplier located in the high-demand node ($b_n^S(\theta_s + T)$, equation 7). By submitting a bid equal to zero, the supplier located in the low-demand node also maximizes the compensation that it receives by the capacity that it cannot sell in the spot electricity market ($(b_n^S - b_s^S)(\min\{\theta_s + \theta_n, k_s\} - (\theta_s + T))$, equation 7).[9]
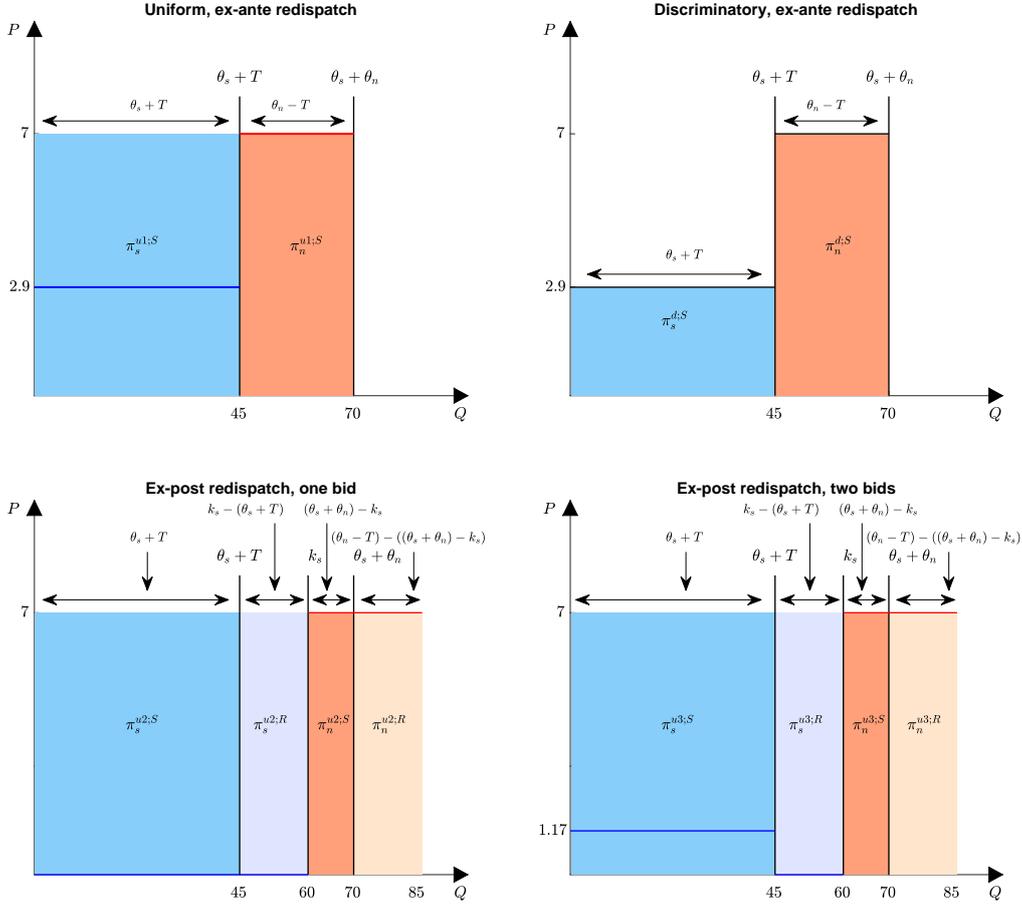
When the auction in the spot electricity market is uniform, an ex-post redispatch mechanism is introduced by the auctioneer and the suppliers submits different bids in the spot and in the redispatch market, the suppliers participate actively in both markets. In that case, the supplier located in the high-demand node submits the maximum bid allowed by the auctioneer in both markets. In contrast, the supplier located in the low-demand node submits a bid that makes undercutting unprofitable in the spot electricity market, and a bid equal to zero in the redispatch market, i.e., there are multiplicity of pure strategies equilibria in the spot electricity market, but an unique pure strategies equilibrium in the redispatch market (equation 41, annex 2).

The strategies defined in equation 41 maximizes suppliers' profits in both markets. The supplier located in the high-demand node maximizes its profits in the spot electricity market by satisfying the residual demand at the maximum price allowed by the auctioneer, since when it is called into operation in that market it sells its production capacity at the maximum bid allowed by the auctioneer ($b_n^S(\theta_s + \theta_n - k_s)$, equation 9). The supplier located in the high-demand node also maximizes its profits in the redispatch market by submitting the maximum bid allowed by the auctioneer ($b_n^R((\theta_n - T) - (\theta_s + \theta_n - k_s))$, equation 9). The supplier located in the low-demand node maximizes its profits in the spot electricity market by submitting a bid that makes undercutting unprofitable. In that case, it satisfies the demand in its own node and the demand in the importing node up to the transmission capacity at the price set by the supplier located in the high-demand node ($b_n^S(\theta_s + T)$, equation 9). The supplier located in the low-demand node maximizes its profits in the redispatch market by submitting a bid equal to zero, since by doing that, it maximizes the compensation for the capacity that it cannot sell in the spot electricity market ($(b_n^S - b_s^R)(\min\{\theta_s + \theta_n, k_s\} - (\theta_s + T))$, equation 9).

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[9]The equilibrium behaviour of the supplier located in the low-demand node can be also analyzed by using equation 6. In that case, the supplier located in the low-demand node maximizes its profits in the spot electricity market by submitting a bid equal to zero, since it sells its entire production capacity at the price set by the supplier located in the high-demand node ($b_n^S \min\{\theta_s + \theta_n, k_s\}$, equation 6). By submitting a bid equal to zero, the supplier located in the low-demand node also minimizes its expenses in the redispatch market when it has to buy back the production capacity that it cannot sell in the spot electricity market ($b_s^S(\min\{\theta_s + \theta_n, k_s\} - (\theta_s + T)$, equation 6).

Figure 1: Model comparison. Suppliers' profits functions ($\theta_s = 5, \theta_n = 65, k_s = k_n = 60,$ $T = 40, c_s = c_n = 0, P = 7$)

4   Model Comparison and Welfare Analysis

In the previous section, I characterize the equilibrium, and I explain the economic forces that determine suppliers' strategies. In this section, by using an example and the equations that characterize the equilibrium, I compare the performance of the four redispatch designs based on suppliers' profits, consumers' surplus and long-term investment incentives.

When the auction in the spot electricity market is uniform and an ex-ante redispatch mechanism is introduced by the auctioneer, the supplier located in the high-demand node submits the maximum bid allowed by the auctioneer setting the price in the zonal market (columns 3 and 4, table 1). The supplier located in the low-demand node submits a bid that makes undercutting unprofitable (column 2, table 1). In that case, the supplier located in the high demand node satisfies the residual demand, the supplier in the low-demand node satisfies the demand in its own node and the demand in the other node up to the transmission capacity, and suppliers' profits are defined by equations 19 and 22 in annex 2 (columns 11 and 12, table 1; top left-hand panel, figure 1). Finally, given that the equilibrium price is equal to the reserve price, consumers' surplus is zero (equations 20 and 23, annex 2; column 13, table 1).

When the auction in the spot electricity market is discriminatory and an ex-ante redispatch mechanism is introduced by the auctioneer, the equilibrium is in mixed strategies and the supplier located in the high-demand node submits higher bids in expectation (equation 33, annex 2; columns 9 and 10, table 1). Suppliers' profits are defined by equation 35 in annex 2 (column 13, table 1; top right-hand panel, figure 1). Consumers' surplus is positive, since the expected equilibrium price is lower than the reserve price (equations 34 and 36, annex 2; columns 10 and 13, table 1).

When the auction is uniform, an ex-post redispatch mechanism is introduced by the auctioneer and the suppliers submit the same bid in the spot and in the redispatch market, the supplier located in the high-demand node submits the maximum bid allowed by the auctioneer setting the price in the zonal market (columns 3 and 4, table 1). Supplier n satisfies the demand in the spot electricity market and its profits in that market are defined as $\pi_n^{u2;S} = 7(65 + 5 - 60) = 70$ (dark-red area, bottom left-hand panel, figure 1). Due to the transmission constraint, supplier s cannot sell its entire production capacity, and in the redispatch market, supplier n is called into operation again to sell the production capacity that it could not sell in the spot electricity market, and its profit in that market are defined as $\pi_n^{u2;R} = 7((65 - 40) - (65 + 5 - 60)) = 105$ (light-red area, bottom left-hand panel, figure 1). Supplier n's profits in the spot and in the redispatch market are defined in equation 38, annex 2. The sum of supplier n's profits in both markets is $\pi_n^{u2;S} + \pi_n^{u2;R} = 175$ (column 11, table 1).

The supplier located in the low-demand node submits a bid equal to zero in the spot electricity market to make undercutting unprofitable (column 2, table 1). By using equation 7 to work out the equilibrium profits, I obtain an useful economic interpretation of supplier s's profits. Supplier s' profits can be calculated as the profits that it obtains by selling its production capacity up to transmission capacity in the spot electricity market $\pi_s^{u2;S} = 7(5+40) = 315$, plus the compensation that it receives for not being able to sell its entire production capacity in the spot electricity market $\pi_s^{u2;R} = (7-0)(60-(5+40)) = 105$ (dark-blue area and light-blue area, bottom left-hand panel, figure 1). By summing the profits in both markets, I obtain supplier s's profits $\pi_s^{u2;S} + \pi_s^{u2;R} = 315 + 105 = 420$ (equation 38, annex 2; column 12, table 1).

By using equation 6 instead of equation 7, supplier s it is dispatched first in the spot electricity market, selling its entire production capacity at the price set by supplier n, and its profits are defined as $\pi_s^{u2;S} + \pi_s^{u2;R} = 7(60) = 420$ (sum of the dark-blue and light-blue areas, bottom left-hand panel, figure 1).[10] Due to the transmission constraint, supplier s cannot sell its entire production capacity in the high-demand node, and it has to buy back the production capacity that it cannot sell in that node. Given that supplier s has to submit the same bid in both markets, and that the auction in the redispatch market is discriminatory, supplier s's expenses in that market are defined as $e_s^{u2;R} = 0(60-(5+40)) = 0$. By subtracting the expenses in the redispatch market from the profits in the spot electricity market, I obtain supplier s's profits $\pi_s^{u2;S} + \pi_s^{u2;R} - e_s^{u2;R} = 420 - 0 = 420$ (equation 38, annex 2; column 12, table 1).

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[10]It is important to notice that supplier n's profits in the spot electricity market should be a single area that covers the dark-blue and the light-blue areas. I explain supplier n's profits by summing $\pi_s^{u2;S} + \pi_s^{u2;R}$ to avoid introducing more graphs.

As when a redispatch mechanism is introduced ex-ante by the auctioneer, consumers' surplus is zero since the equilibrium price is equal to consumers' reserve price (equation 39, annex 2; column 13, table 1).

When the auction is uniform, an ex-post redispatch mechanism is introduced by the auctioneer and the suppliers submit different bids in the spot and in the redispatch market, the supplier located in the high-demand node submits the maximum bid allowed by the auctioneer in both markets (equation 41, annex 2; columns 3 and 5, table 1). In contrast, the supplier located in the low-demand node submits a bid that makes under-cutting unprofitable in the spot electricity market (41, annex 2; column 2, table 1). The supplier located in the low-demand node submits a bid equal to zero in the redispatch market (41, annex 2; column 5, table 1). Despite the changes on equilibrium strategies, suppliers' profits and consumers' surplus do not change (equations 42 and 43, annex 2; columns 11, 12, 13, table 1; bottom right-hand panel, figure 1).

As can be observed in table 1 supplier n's profits are the same with independence of the market design ($\pi_n^{u1;S} = \pi_n^{d;S} = \pi_n^{u2;S} + \pi_n^{u2;R} = \pi_n^{u3;S} + \pi_n^{u3;R}$, figure 1). However, supplier s's profits change substantially depending on the type of auction. When an ex-ante redispatch mechanism is introduced by the auctioneer and the auction is discriminatory, supplier s's profits are lower than when the auction is uniform ($\pi_s^{u1;S} > \pi_s^{d;S}$, figure 1). The introduction of an ex-post redispatch mechanism increases supplier s's profits, since in that case, it is compensated for the electricity that it cannot sell in the spot electricity market ($\pi_s^{u2;S} + \pi_s^{u2;R} = \pi_s^{u3;S} + \pi_s^{u3;R} > \pi_s^{u1;S} > \pi_s^{d;S}$, figure 1).

The change on supplier s's profits induced by the changes on the design could induce distortions on investment decisions. In particular, when the auction in the spot electricity market is discriminatory and an ex-ante redispatch mechanism is introduced by the auctioneer, the suppliers want to invest in the high-demand node, since the equilibrium profits in that node are larger $\pi_n^{d;S} \geq \pi_s^{d;S}$. When the auction in the spot electricity market is uniform and an ex-ante redispatch mechanism is introduced by the auctioneer, the suppliers want to invest in the low-demand node since $\pi_s^{u1;S} \geq \pi_n^{u1;S}$. The introduction of an ex-post redispatch market makes even more attractive to invest in the low-demand node ($\pi_s^{u2;S} + \pi_s^{u2;R} = \pi_s^{u3;S} + \pi_s^{u3;R} > \pi_s^{u1;S} > \pi_n^{u1;S}$). Therefore, the introduction of different redispatch designs change suppliers' profits and that could have important investment implications in the long-term.

The introduction of different redispatch designs also affect consumers' surplus. In particular, when the auction is uniform and for any type of redispatch mechanism, consumers' surplus is zero, since the equilibrium price is equal to consumers' reserve price. In contrast, when the auction in the spot electricity market is discriminatory and an ex-ante redispatch mechanism is introduced by the auctioneer, the equilibrium price is lower than consumers' reserve price and consumers' surplus is positive.

5   Conclusion

In the presence of transmission constraints, electricity markets can be organized as nodal or as zonal pricing electricity markets. In a nodal pricing electricity market, the equilib-

Table 1: Model comparison ($\theta_s = 5, \theta_n = 65$, $k_s = k_n = 60$, $T = 40$, $c_s = c_n = 0$, $P = 7$)

Design	b_s^S	b_n^S	P^S	b_s^R	b_n^R	$\underline{b}$	$E(b_s^S)$	$E(b_n^S)$	$E(b^S)$	π_n	π_s	CS
I	$[0, 2.9]$	7	7	–	–	–	–	–	–	175	315	0
II	–	–	–	–	–	2.9	4.4	5.03	4.98	175	131.2	140.9
III	0	7	7	–	–	–	–	–	–	175	420	0
IV	$[0, 1.17]$	7	7	0	7	–	–	–	–	175	420	0

I: Ex-ante redispatch, uniform. II: Ex-ante redispatch, discriminatory. III: Ex-post redisptach, one bid.

IV: Ex-post redispatch, two bids.

rium price is the same in all the nodes even when the transmission line is congested. In contrast, in a zonal pricing electricity market, the equilibrium is the same in all the nodes even when the transmission line is congested.

In this paper I characterize the equilibrium in a zonal pricing electricity market when the competition is imperfect by using four different redispatch designs. First, I work out the equilibrium when the auction in the spot electricity market is uniform and an ex-ante redispatch mechanism is introduced by the auctioneer. Second, I work out the equilibrium when the auction in the spot electricity market is discriminatory and an ex-ante redispatch mechanism is introduced by the auctioneer. Third, I work out the equilibrium when the auction in the spot electricity market is uniform, an ex-post redispatch mechanism is introduced by the auctioneer and the suppliers submit the same bid in the spot and in the redispatch market. Finally, I work out the equilibrium when the auction in the spot electricity market is uniform, an ex-post redispatch mechanism is introduced by the auctioneer and the suppliers submit different bids in the spot and in the redispatch market.

I find that the profits of the supplier located in the importing node are the same with independence of the redispatch design. However, the profits of the supplier located in the exporting node depend crucially on the type of redispatch design, and that can introduce long term investment distortions. In particular, when the auction in the spot electricity market is discriminatory and an ex-ante redispatch mechanism is introduced by the auctioneer, the equilibrium profits of the supplier located in the importing node are larger than the ones of the supplier located in the exporting node. In contrast, when the auction in the spot electricity market is uniform and an ex-ante redispatch mechanism is introduced by the auctioneer, the equilibrium profits of the supplier located in the exporting node are higher than the ones of the supplier located in the importing node. The introduction of an ex-post redispatch mechanism increases the profits of the supplier located in the exporting node.

The consumers' surplus also depend on the type of redispatch. In particular, when the auction in the spot electricity market is uniform and for any type of redispatch design, the consumers' surplus is nil, since the equilibrium price is equal to consumers' reserve price. In contrast, when the auction in the spot electricity market is discriminatory and an ex-ante redispatch mechanism is introduced by the auctioneer, the consumers' surplus is positive, since the equilibrium price is lower than consumers' reserve price.

In this paper, I focus my analysis in the performance of different redispatch designs

in terms of consumers' welfare and suppliers' profits. To make the analysis as simple as possible, I assume that the suppliers submits a single bid for their entire production capacity and their productions costs are zero. Fabra et al. (2006) prove that those assumptions do not affect the equilibrium price. However, in the next future, I would like to generalize the equilibrium by introducing step biding functions and positive production costs. I also would like to characterize the equilibrium by introducing more suppliers in each node.

Annex 1. The model

In the model section, I explain suppliers' outcomes and profits functions when the transmission line is congested. In this annex, I extend that analysis to the cases in which the transmission line is not congested. I also introduce different figures to facilitate the understanding of the formulas.

Timing of the game. After the supplier observe the demand and submit their bids. The auctioneer works out suppliers' outcomes and profits functions.

When the auction in the spot electricity market is uniform and an ex-ante redispatch mechanism is introduced by the auctioneer, the output allocated to supplier n in the spot electricity market (supplier s's output function is symmetric), denoted by $q_n^{u1}(b^S; \theta)$, is given by:

$$q_n^{u1}(b^S; \theta) = \begin{cases} \min\{\theta_s + \theta_n, \theta_n + T, k_n\} & \text{if } b_n^S \leq b_s^S \\ \max\{0, \theta_n - T, \theta_s + \theta_n - k_s\} & \text{if } b_n^S > b_s^S \end{cases} \tag{10}$$
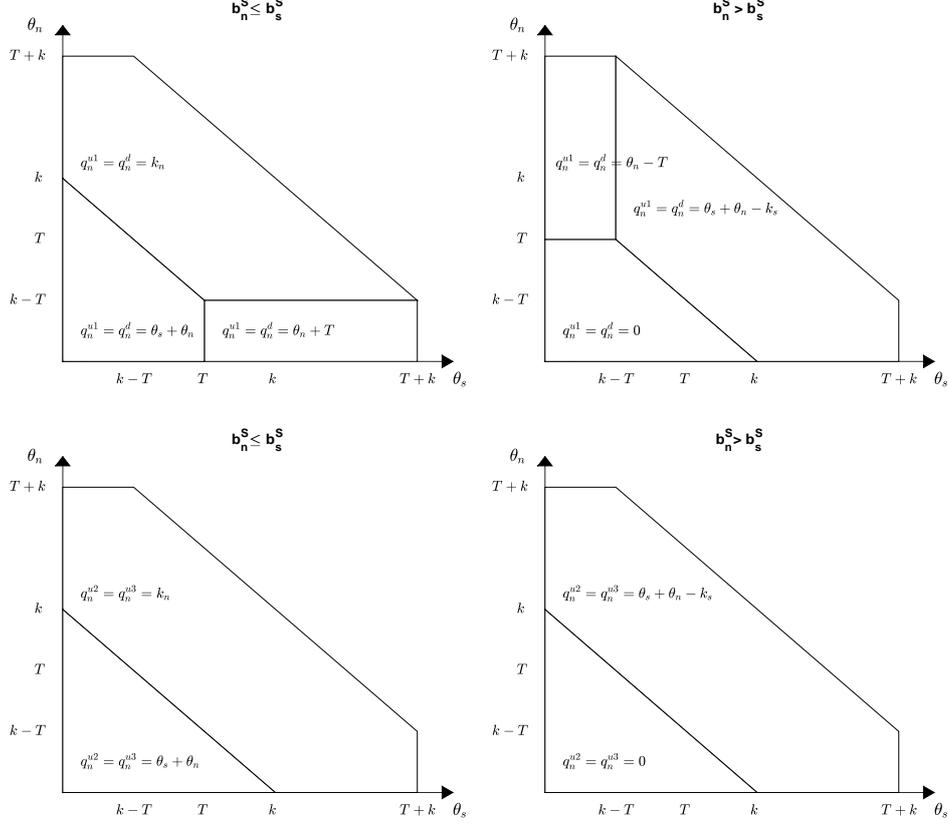
When supplier n submits the lower bid in the spot electricity market ($b_n^S \leq b_s^S$), supplier n's total demand is represented in the top left-hand panel, figure 2. In that case, when demand in both nodes is low and the transmission line is not congested, supplier n can satisfy total demand ($\theta_s + \theta_n$). If the demand in node South is larger than the transmission capacity $\theta_s > T$, supplier n cannot satisfy the demand in node South, even when it has enough production capacity; therefore, the total demand that supplier n can satisfy is ($\theta_n + T$). Finally, if the demand is large enough, the total demand that supplier n can satisfy is its own production capacity (k_n).

When supplier n submits the higher bid in the spot electricity market ($b_n^S > b_s^S$), supplier n's residual demand is represented in the top right-hand panel, figure 2. In that case, when demand in both nodes is low and the transmission line is not congested, supplier s satisfies total demand and therefore, the residual demand that remains for supplier n is zero. The total demand that supplier s can satisfy diminishes due to the transmission constraint. As soon as the demand in node North is larger than the transmission capacity ($\theta_n > T$), the demand in that node cannot be satisfied by supplier s and thus, some residual demand ($\theta_n - T$) remains for supplier n. When total demand is large enough, supplier s cannot satisfy total demand and some residual demand ($\theta_s + \theta_n - k_s$) remains for supplier n.

When the auction in the spot electricity market is discriminatory and an ex-ante redispatch mechanism is introduced by the auctioneer, the output allocated to supplier n in the spot electricity market, denoted by $q_n^d(b^S; \theta)$, is as when the auction in the spot electricity market is uniform and an ex-ante redispatch mechanism is introduced by the auctioneer (top panels, figure 2)

When the auction in the spot electricity market is uniform and an ex-post redispatch mechanism is introduced by the auctioneer, the output allocated to supplier n, denoted by $q_n^{u2}(b^S; \theta) = q_n^{u3}(b^S; \theta)$, is given by

Figure 2: Supplier n's output function ($k_s = k_n = k = 60$, $T = 40$)

$$q_n^{u2}(b^S; \theta) = q_n^{u3}(b^S; \theta) = \begin{cases} \min\{\theta_s + \theta_n, k_n\} & \text{if } b_n^S \le b_s^S \\ \max\{0, \theta_s + \theta_n - k_s\} & \text{if } b_n^S > b_s^S \end{cases} \qquad (11)$$

When an ex-post redispatch mechanism is introduced, the congestion is not taken into account when the spot electricity market is cleared. Therefore, when supplier n submits the lower bid in the spot electricity market, it satisfies the total demand ($\theta_s + \theta_n$) up to its production capacity (k_n) (bottom left-hand panel, figure 2). When it submits the higher bid, it satisfies the residual demand ($\theta_s + \theta_n - k_s$) (bottom right-hand panel, figure 2).

When the transmission line is congested and an ex-post redispatch mechanism is introduced by the auctioneer to alleviate the congestion in the line, the outcome allocated to supplier n in the redispatch market is denoted by

$$q_n^R(b^S; \theta) = \begin{cases} \min\{\theta_s + \theta_n, k_n\} - (\theta_n + T) & \text{if } b_n^S \le b_s^S \text{ and } \theta_s - T < \theta_s + \theta_n - k_n \\ (\theta_n - T) - (\theta_s + \theta_n - k_s) & \text{if } b_n^S > b_s^S \text{ and } \theta_n - T > \theta_s + \theta_n - k_s \end{cases} \qquad (12)$$

When supplier n submits the lower bid in the spot electricity market ($b_n^S \le b_s^S$), it is dispatched first, but due to the transmission constraint it cannot satisfy the total demand up to its production capacity ($\min\{\theta_s + \theta_n, k_n\}$), but only ($\theta_n + T$) . Therefore, in the redispatch market it has to buy back the difference between what it wants to sell and what it can sell ($\min\{\theta_s + \theta_n, k_n\} - (\theta_n + T)$).

When supplier n submits the higher bid in the spot electricity market ($b_n^S > b_s^S$), it is dispatched last. Due to the transmission constraint it can sell more electricity ($\theta_n - T$) that what it sells in the spot electricity market ($\theta_s + \theta_n - k_s$). Therefore, in the redispatch market it can sell all the electricity that it could not sell in the spot electricity market (($\theta_n - T) - (\theta_s + \theta_n - k_s)$).

Finally, the payments are worked out by the auctioneer. When the auction in the spot electricity market is uniform and an ex-ante redispatch mechanism is introduced by the auctioneer, the price received by a supplier for any positive quantity dispatched by the auctioneer is equal to the higher bid accepted in the auction. Hence, for a given realization of $\theta \equiv (\theta_s, \theta_n)$ and a bid profile $b \equiv (b_s, b_n)$, supplier n's profits can be expressed as $\pi_n^{u1}(b^S; \theta)$:

$$\begin{cases} b_n^S\ (\theta_i + \theta_j) & \text{if } b_n^S \leq b_s^S \text{ and } \theta_s \leq T \text{ and } \theta_s + \theta_n \leq k_n \\ b_s^S \min\{\theta_n + T, k_n\} & \text{if } b_n^S \leq b_s^S \text{ and } \theta_s > T \text{ or } \theta_s + \theta_n > k_s \\ b_n^S \max\{0, \theta_n - T, \theta_s + \theta_n - k_s\} & \text{otherwise} \end{cases} \tag{13}$$

When supplier n submits the lower bid in the spot electricity market ($b_n^S \leq b_s^S$), supplier n's profits are represented in the top-left panel, figure 3. In that case, when the transmission line is not congested and it has enough production capacity to satisfy the total demand, it sets the equilibrium price and its profits are defined as ($b_n^S(\theta_s + \theta_n)$). When the transmission or the production capacity is binding, supplier s sets the price and supplier n's profits are defined as $\left(b_s^S \min\{\theta_n + T, k_n\}\right)$.

When supplier n submits the higher bid in the spot electricity market ($b_n^S > b_s^S$), supplier n's profits are represented in the top-right panel, figure 3. In that case, when the transmission line is not congested and supplier s has enough production capacity to satisfy the total demand, supplier n is not dispatched and its profits are (0). When the transmission line is congested or when supplier s does not have enough production capacity to satisfy the total demand, supplier n sets the price and its profits are defined as $\left(b_n^S \max\{0, \theta_n - T, \theta_s + \theta_n - k\}\right)$.
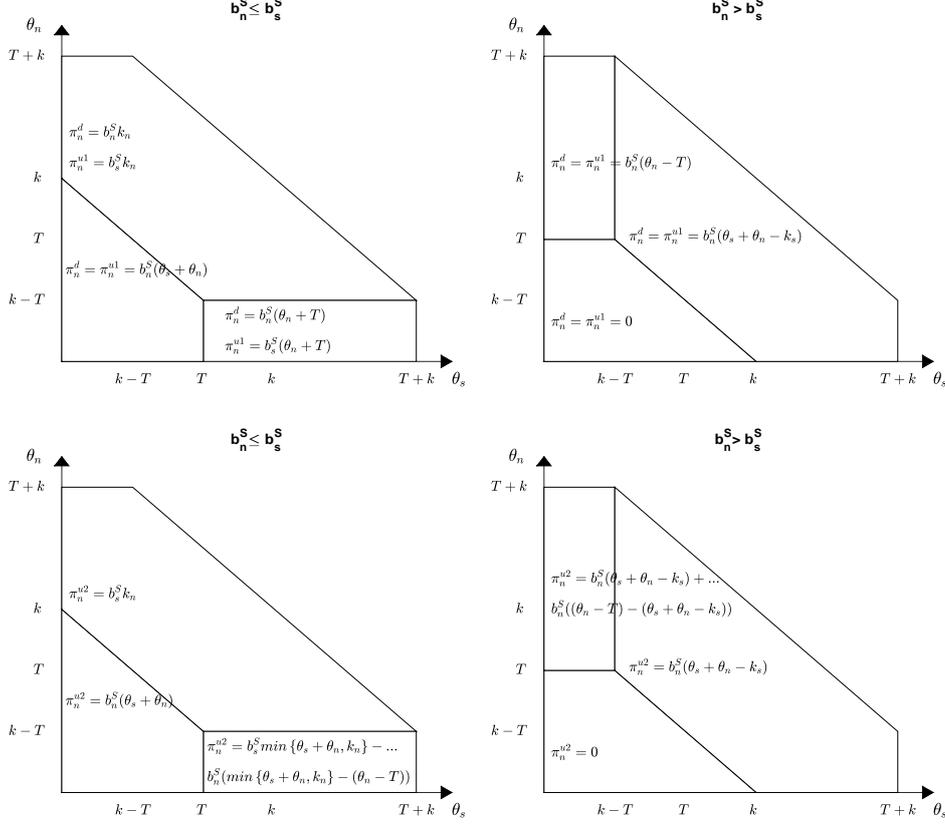
When the auction in the spot electricity market is discriminatory and an ex-ante redispatch mechanism is introduced by the auctioneer, the price received by a supplier in the spot electricity market for any positive quantity dispatched by the auctioneer is equal to its own offer price, whenever a bid is wholly or partly accepted. Hence, for a given realization of demands $\theta \equiv (\theta_s, \theta_n)$ and a bid profile $b \equiv (b_s, b_n)$, supplier n's profits can be expressed as $\pi_n^d(b^S; \theta)$:

$$\begin{cases} b_n^S \min\{\theta_s + \theta_n, \theta_n + T, k_n\} & \text{if } b_n^S \leq b_s^S \\ b_n^S \max\{0, \theta_n - T, \theta_s + \theta_n - k_s\} & \text{if } b_n^S > b_s^S \end{cases} \tag{14}$$

When the auction in the spot electricity market is discriminatory and an ex-ante redispatch mechanism is introduced by the auctioneer, supplier n's profits are represented in the top panels in figure 3.

When the auction in the spot electricity market is uniform and an ex-post redispatch mechanism is introduced by the auctioneer, supplier n's profits can be expressed as $\pi_i^{u2}(b^S; \theta)$:

Figure 3: Supplier n's profit function ($k_n = k_s = k = 60$, $T = 40$)

$b_n^S \leq b_s^S$

θ_n

$T+k$

k

$\pi_n^d = b_n^S k_n$

$\pi_n^{u1} = b_s^S k_n$

T

$k - T$

$\pi_n^d = \pi_n^{u1} = b_n^S(\theta_s + \theta_n)$

$\pi_n^d = b_n^S(\theta_n + T)$

$\pi_n^{u1} = b_s^S(\theta_n + T)$

$k - T$ T k $T+k$ θ_s

$b_n^S > b_s^S$

θ_n

$T+k$

k

$\pi_n^d = \pi_n^{u1} = b_n^S(\theta_n - T)$

T

$\pi_n^d = \pi_n^{u1} = b_n^S(\theta_s + \theta_n - k_s)$

$k - T$

$\pi_n^d = \pi_n^{u1} = 0$

$k - T$ T k $T+k$ θ_s

$b_n^S \leq b_s^S$

θ_n

$T+k$

k

$\pi_n^{u2} = b_s^S k_n$

T

$\pi_n^{u2} = b_n^S(\theta_s + \theta_n)$

$k - T$

$\pi_n^{u2} = b_s^S \min\{\theta_s + \theta_n, k_n\} - ...$

$b_n^S(\min\{\theta_s + \theta_n, k_n\} - (\theta_n - T))$

$k - T$ T k $T+k$ θ_s

$b_n^S > b_s^S$

θ_n

$T+k$

k

$\pi_n^{u2} = b_n^S(\theta_s + \theta_n - k_s) + ...$

$b_n^S((\theta_n - T) - (\theta_s + \theta_n - k_s))$

T

$\pi_n^{u2} = b_n^S(\theta_s + \theta_n - k_s)$

$k - T$

$\pi_n^{u2} = 0$

$k - T$ T k $T+k$ θ_s

$$
\begin{cases}
b_n^S(\theta_s + \theta_n) & \text{if } b_n^S \leq b_s^S \text{ and } \theta_s + \theta_n < k_n \text{ and } \theta_s < T \\
b_s^S \min\{\theta_s + \theta_n, k_n\} - ... \\
\quad b_n^S\left(\min\{\theta_s + \theta_n, k_n\} - (\theta_n + T)\right) & \text{if } b_n^S \leq b_s^S \text{ and } \theta_s - T \leq \theta_s + \theta_n - k_n \\
b_s^S k_n & \text{if } b_n^S \leq b_s^S \text{ and } \theta_s - T > \theta_s + \theta_n - k_n \\
b_n^S 0 & \text{if } b_n^S > b_s^S \text{ and } \theta_s + \theta_n < k_s \text{ and } \theta_n < T \\
b_n^S(\theta_s + \theta_n - k_s) + ... \\
\quad b_n^S\left((\theta_n - T) - (\theta_s + \theta_n - k_s)\right) & \text{if } b_n^S > b_s^S \text{ and } \theta_n - T > \theta_s + \theta_n - k_s \\
b_n^S(\theta_s + \theta_n - k_s) & \text{if } b_n^S > b_s^S \text{ and } \theta_n - T \leq \theta_s + \theta_n - k_s
\end{cases} \tag{15}
$$

When supplier n submits the lower bid in the spot electricity market ($b_n^S \leq b_s^S$), supplier n's profit is represented in the bottom-left panel, figure 3. In that case, when the transmission line is not congested and it has enough production capacity to satisfy the total demand, it sets the equilibrium price and its profits are defined as ($b_n^S(\theta_s + \theta_n)$). When the transmission capacity is binding, supplier n's profits are as I describe in the model section. Finally, when the production capacity is binding, but the transmission capacity is not binding, supplier s sets the price and supplier n's profits are defined as ($b_s^S k_n$).

When supplier n submits the higher bid in the spot electricity market ($b_n^S > b_s^S$), supplier n's profits are represented in the bottom-right panel, figure 3. In that case, when the transmission line is not congested and supplier s has enough production capacity to

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satisfy the total demand, supplier n is not dispatched and its profits are (0). When the transmission capacity is binding, supplier n's profits are as I describe in the model section. Finally, when supplier s does not have enough production capacity to satisfy the total demand, supplier n sets the price and its profits are defined as $(b_n^S(\theta_s + \theta_n - k_s))$.

When the auction in the spot electricity market is uniform, an ex-post redispatch mechanism is introduced by the auctioneer, but the bid that supplier n submits in the spot electricity market is different that the one in the redispatch market, supplier n's profits are as in equation 15, but taking into account that supplier n participates in the redispatch market by submitting its own bid (b_n^R).

Annex 2. Equilibrium

The intersection of the areas in figure 3 generates three different equilibrium areas: low-demand area (area A, figure 4), intermediate demand area (areas $A1$ and $B1$, figure 4), and high-demand area (area $B2$, figure 4). In the main text, I present the results only when the transmission line is congested (intermediate demand area). However, in this annex, I characterize the equilibrium also when the transmission line is not congested (low-demand and high-demand areas).

Proposition 1.

Uniform price auction in the spot electricity market, and ex-ante redispatch mechanism introduced by the auctioneer. By using lemma 1, the proof is as follows:

When the *demand is low* (area A, figure 4) any supplier has enough production capacity to satisfy the total demand, and the transmission line is not congested. Therefore, the suppliers compete fiercely to be dispatched first in the auction and the equilibrium bids are

$$b_n^S = b_s^S = c = 0. \tag{16}$$

The equilibrium profits are zero for both suppliers, and the electricity flows from the high-demand node to the low-demand node, since the tie-breaking rule establishes that when both suppliers submits the same bid, the supplier located in the high-demand node is dispatched first in the auction.

Consumers' surplus is defined by:

$$CS = (P - 0)(\theta_s + \theta_n) \tag{17}$$

When the *demand is intermediate* and the transmission capacity is binding, but the production capacity is not binding (area $A1$), only the supplier located in the high-demand node faces a positive residual demand. Therefore, the pure strategies equilibrium is defined by

$$b_s^S \in \left[0, \frac{P(\theta_n - T)}{\theta_s + \theta_n}\right]; \ \ b_n^S = P. \tag{18}$$

The equilibrium price in the spot electricity market is P.

The profits are defined by:

$$\overline{\pi}_s = P(\theta_s + T); \quad \overline{\pi}_n = P(\theta_n - T). \tag{19}$$

The electricity flows from the low-demand node to the high-demand node, and the transmission line is congested.

Consumers' surplus is defined by:

$$CS = (P - P)(\theta_s + \theta_n) = 0 \tag{20}$$

When the *demand is intermediate* and the transmission capacity and the production capacity are binding (area $B1$), both suppliers face a positive residual demand. Therefore, there are two possible types of equilibria. In each of them, one of the suppliers submits the maximum bid allowed by the auctioneer and the other submits a bid that makes undercutting unprofitable.

When the supplier located in the high-demand node sets the price in the auction, the pure strategies equilibrium is defined by:

$$b_s^S \in \left[0, \frac{P(\theta_n - T)}{k_n}\right]; \quad b_n^S = P. \tag{21}$$

The equilibrium price is P.

The profits are defined by:

$$\overline{\pi}_s = P(\theta_s + T); \quad \overline{\pi}_n = P(\theta_n - T). \tag{22}$$

The electricity flows from the low-demand node to the high-demand node, and the transmission line is congested.

Consumers' surplus is defined by:

$$CS = (P - P)(\theta_s + \theta_n) = 0 \tag{23}$$

When the supplier located in the low-demand node sets the price in the auction, the pure strategies equilibrium is defined by:

$$b_n^S \in \left[0, \frac{P(\theta_s + \theta_n - k_n)}{\theta_s + T}\right]; \quad b_s^S = P. \tag{24}$$

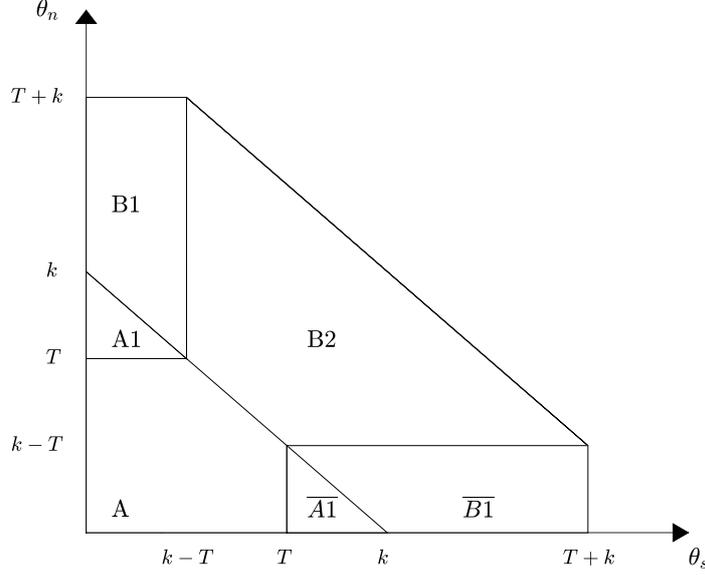The equilibrium price in the spot electricity market is P.

The profits are defined by:

$$\overline{\pi}_s = P(\theta_s + \theta_n - k_n); \quad \overline{\pi}_n = P(k_n). \tag{25}$$

The electricity flows from the high-demand node to the low-demand node, and the transmission line is not congested.

Consumers' surplus is defined by:

$$CS = (P - P)(\theta_s + \theta_n) = 0 \tag{26}$$

Figure 4: Equilibrium areas. $(k_i = k_j = k = 60, c = 0, T = 40)$

When the *demand is high* (area $B2$, figure 4) the transmission capacity is not binding, but the production capacity is binding and both suppliers face a positive residual demand. The pure strategies equilibrium is defined by

$$b_i^S = P; \ \ b_j^S \in \left[0, \frac{P \max\{\theta_j + \theta_i - k_j\}}{k_i}\right] \ \ \forall i,j = s,n. \tag{27}$$

The equilibrium price in the spot electricity market is P.

The profits are defined by either:

$$\overline{\pi}_i = P(\theta_j + \theta_i - k_j); or \ \overline{\pi}_j = Pk_j \ \ \forall i,j = s,n. \tag{28}$$

The electricity flows from one node to the other depending on which equilibrium is selected by the suppliers, but the transmission line is not congested.

Consumers' surplus is defined by:

$$CS = (P - P)(\theta_s + \theta_n) = 0 \tag{29}$$

Discriminatory price auction in the spot electricity market, and ex-ante redispatch mechanism introduced by the auctioneer. The equilibrium is as in Blázquez (2018). However, I present the main equations that characterize the equilibrium in area $B1$ in figure 4 to facilitate the comparison with the other three redispatch designs.

First, the lower bound of the support is defined by:

$$\underline{b}_s^S = \underline{b}_n^S = \frac{P(\theta_n - T)}{k_n} \tag{30}$$

Second, I work out the cumulative distribution functions.

$$F_s(b^S) = \begin{cases} 0 & \text{if } b^S < \underline{b}^S \\ \dfrac{k_n}{k_n - (\theta_n - T)} \dfrac{b^S - \underline{b}^S}{b^S} & \text{if } b^S \in \left(\underline{b}^S, P\right) \\ 1 & \text{if } b^S = P \end{cases}$$

$$F_n(b^S) = \begin{cases} 0 & \text{if } b^S < \underline{b}^S \\ \dfrac{\theta_s + T}{(\theta_s + T) - (\theta_s + \theta_n - k_n)} \dfrac{b^S - \underline{b}^S}{b^S} & \text{if } b^S \in \left(\underline{b}, P\right) \\ 1 & \text{if } b^S = P \end{cases} \qquad (31)$$

Given that $\underline{b}_n^S > \underline{b}_s^S$, it is easy to show that $F_s(P)$ is continuous in the upper bound of the support, and that $F_n(P)$ is discontinuous in the upper bound of the support:

$$F_s(P) = \frac{k_n}{k_n - (\theta_n - T)} \frac{P - \dfrac{P(\theta_n - T)}{k_n}}{P} = 1$$

$$F_n(P) = \frac{\theta_s + T}{(\theta_s + T) - (\theta_s + \theta_n - k_n)} \frac{P - \dfrac{P(\theta_n - T)}{k_n}}{P} < 1$$

Third, the probability distribution function is equal to:

$$f_s(b^S) = \frac{\partial F_s(b^S)}{\partial b^S} = \frac{k_n}{k_n - (\theta_n - T)} \frac{\underline{b}^S}{(b^S)^2}$$

$$f_n(b^S) = \frac{\partial F_n(b^S)}{\partial b^S} = \frac{\theta_s + T}{(\theta_s + T) - (\theta_s + \theta_n - k_n)} \frac{\underline{b}^S}{(b^S)^2} \qquad (32)$$

Fourth, the expected bid is determined by:

$$E_s(b^S) = \int_{\underline{b}^S}^{P} b^S f_s(b_s^S) \partial b^S = \int_{\underline{b}^S}^{P} \frac{k_n}{k_n - (\theta_n - T)} \frac{\underline{b}^S}{b^S} \partial b^S = \frac{k_n}{k_n - (\theta_n - T)} \underline{b}^S \left[ln(b^S) \right]_{\underline{b}^S}^{P}$$

$$E_n(b^S) = \int_{\underline{b}^S}^{P} b^S f_n(b_n^S) \partial b^S = \int_{\underline{b}^S}^{P} \frac{\theta_s + T}{(\theta_s + T) - (\theta_s + \theta_n - k_n)} \frac{\underline{b}^S}{(b^S)^2} \partial b^S =$$

$$= \frac{\theta_s + T}{(\theta_s + T) - (\theta_s + \theta_n - k_n)} \underline{b}^S \left[ln(b^S) \right]_{\underline{b}^S}^{P} + (1 - F_n(P)) P \qquad (33)$$

Given that $F_n(b^S)$ is discontinuous in the upper bound of the support, to work out supplier n's expected bid is necessary to multiply the maximum bid allowed by the auctioneer by the probability that supplier n assigns to that bid $(1 - F_n(P)) P$, where $F_n(P) = F_n(b^S)$, when $b^S \longrightarrow P$.

When the auction is discriminatory, the expected equilibrium price in the spot market is defined by:

$$E(b^S) = \frac{E(b_s^S)\theta_s}{(\theta_s + \theta_n)} + \frac{E(b_n^S)\theta_n}{(\theta_s + \theta_n)} \qquad (34)$$

Fifth, the expected profit is defined by:

$$
\begin{aligned}
\overline{\pi}_n &= \underline{b}^S(\theta_s + \theta_n) \\
\overline{\pi}_s &= \underline{b}^S(\theta_s + T)
\end{aligned}
\tag{35}
$$

The electricity flows in expectation from the low-demand node to the high-demand node, and the transmission line is congested.

Consumers' surplus are defined by:

$$
CS = (P - E(b^S))(\theta_s + \theta_n) \geq 0
\tag{36}
$$

Uniform price auction in the spot electricity market, ex-post redispatch mechanism introduced by the auctioneer, and the suppliers submits the same bid in the spot and in the redispatch market. By using lemmas 1 and 2, the proof is as follows:

When the transmission line is not congested (areas A and $B2$), the equilibrium is as when an ex-ante redispatch mechanism is introduced by the auctioneer. When the transmission line is congested, I assume that the electricity flows from the low-demand node (node S) to the high-demand node (node N), i.e., S is the exporting node and N is the importing node.

Solving by backward induction, I characterize the equilibrium in the redispatch market. According with equation 7, supplier s's profits are given by $b_n^S(\theta_n + T) + (b_s^S - b_n^S)(\min\{\theta_s + \theta_n, k_s\} - (\theta_s + T))$, where $(b_s^S - b_n^S)(\min\{\theta_s + \theta_n, k_s\} - (\theta_s + T))$ represents the compensation for the electricity that supplier s wants to sell in the spot electricity market, but that it cannot sell because of the transmission constraint. If supplier s could participate in the redisptach market it would submit a bib equal to zero to maximize that compensation.

According with equation 7, supplier n's profits are given by $b_n^S(\theta_s + \theta_n - k_s) + b_n^S((\theta_n - T) - (\theta_s + \theta_n - k_s))$, where $(b_n^S((\theta_n - T) - (\theta_s + \theta_n - k_s)))$ represents supplier n's profits in the redispatch market. If supplier n could participate in the redisptach market it would submit the maximum bid allowed by the auctioneer to maximize those profits.

Given that the bid submitted by the suppliers in the redispatch market has to be the same as the one in the spot electricity market, it is necessary to check that the bid that the suppliers want to submit in the redispatch market is also the one that they want to submit in the spot electricity market. Otherwise, it does not exist a pair of strategies that clear both markets simultaneously.

When the transmission line is congested, the unique possible equilibrium is the spot electricity market is the one in which supplier n submits the maximum bid, and supplier s submits a bid that makes undercutting unprofitable. Therefore, the unique pair of strategies that makes compatible an equilibrium in the spot and in the redispatch market simultaneously is defined by:

$$
b_s^S = 0; \quad b_n^S = P,
\tag{37}
$$

The equilibrium price in the spot electricity market is P.

By plugging those values in equation 7, the profits are defined by:

$$
\begin{aligned}
\overline{\pi}_s &= P(\theta_s + T) + (P - 0)\left(\min\{\theta_s + \theta_n, k_n\} - (\theta_s + T)\right); \\
\overline{\pi}_n &= P(\theta_s + \theta_n - k_s) + P((\theta_n - T) - (\theta_s + \theta_n - k_s)).
\end{aligned}
\tag{38}
$$

The electricity flows from the low-demand node to the high-demand node, and the transmission line is congested.

Consumers' surplus is defined by:

$$
CS = (P - P)(\theta_s + \theta_n) = 0
\tag{39}
$$

Uniform price auction in the spot electricity market, ex-post redispatch mechanism introduced by the auctioneer, and the suppliers submits different bids in the spot and in the redispatch market. By using lemmas 1 and 2, the proof is as follows:

When the transmission line is not congested, the equilibrium price is as in the previous two cases. When the transmission line is congested, given that the bid submitted by the suppliers in the spot and in the redispatch market can be different, I have to work out the equilibrium in both markets. I characterize the equilibrium proceeding by backward induction, first by working out the equilibrium in the redispatch market and then in the spot electricity market. According with equation 9, the profits of the supplier located in the exporting node are defined by $b_n^S(\theta_n + T) + (b_n^S - b_s^R)(\min\{\theta_s + \theta_n, k_s\} - (\theta_s + T))$, where $(b_n^S - b_s^R)(\min\{\theta_s + \theta_n, k_s\} - (\theta_s + T))$ represents the compensation for the electricity that supplier s wants to sell in the spot electricity market, but that it cannot sell because of the transmission constraint. Supplier s maximizes that compensation by submitting a bib equal to zero in the redispatch market.

According with equation 9, the profits of the supplier located in the importing node are defined by $b_n^S(\theta_s + \theta_n - k_s) + b_n^R((\theta_n - T) - (\theta_s + \theta_n - k_s))$, where $b_n^R((\theta_n - T) - (\theta_s + \theta_n - k_s))$ represents supplier n's profits in the redispatch market. Supplier n maximizes those profits by submitting the maximum bid allowed by the auctioneer.

When the transmission line is congested, the unique possible equilibrium in the spot electricity market is the one in which supplier n submits the maximum bid, and supplier s submits a bid that makes undercutting unprofitable. Therefore, the equilibrium bids in the spot electricity market are defined by:

$$
b_s^S \in \left[0, \frac{P(\theta_s + \theta_n - k_s)}{k_n}\right]; \quad b_n^S = P,
\tag{40}
$$

Summarizing, the equilibrium bids in the spot and in the redispatch market are defined by:

$$
\begin{aligned}
b_s^S &\in \left[0, \frac{P(\theta_s + \theta_n - k_s)}{k_n}\right]; \quad b_n^S = P, \\
b_s^R &= 0; \quad b_n^R = P.
\end{aligned}
\tag{41}
$$

The equilibrium price in both markets is P.

By plugging those values in equation 9, the profits are defined by

$$
\begin{aligned}
\overline{\pi}_s &= P(\theta_s + T) + (P - 0)\left(\min\{\theta_s + \theta_n, k_n\} - (\theta_s + T)\right); \\
\overline{\pi}_n &= P(\theta_s + \theta_n - k_s) + P((\theta_n - T) - (\theta_s + \theta_n - k_s)).
\end{aligned}
\tag{42}
$$

The electricity flows from the low-demand node to the high-demand node, and the transmission line is congested.

Consumers' surplus are defined by:

$$
CS = (P - P)(\theta_s + \theta_n) = 0 \tag{43}
$$

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