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Obfuscation and Rational Inattention in Digitalized Markets

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Abstract

This paper studies the behavior of competing firms in a duopoly with rational inattentive consumers. Firms play a sequential game in which they decide to obfuscate their individual prices before competing on price. Probabilistic demand functions are endogenously determined by the consumers' optimal information strategy, which depends on the firms' obfuscation choice and the consumers' unrestricted prior beliefs. We show that the game may result in an obfuscation equilibrium with high prices where both firms obfuscate and a transparency equilibrium with low prices and no obfuscation, providing an argument for market regulation. Lower information costs and asymmetric prior beliefs about prices reduce the probability of an obfuscation equilibrium. Using data on Sweden, we document a decrease in price complexity and corresponding prices in the market for mobile phone subscriptions in the last two decades. Our model rationalizes these changes and explains why complexity and high prices persist in some but not all digitalized markets.

Keywords: Rational Inattention, Obfuscation, Price Competition, Digitalized Markets

JEL Codes: D11, D21, D43

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1 Introduction

Consumers need to exert costly effort to learn about the price and quality of different products. In many markets, the complexity of price-quality assessment is a result of obfuscation practices by firms that make it harder for consumers to learn about their offers. Examples of such practices include add-on pricing (e.g., Ellison, 2005; Gabaix and Laibson, 2006), hidden shipping and handling costs (e.g., Hossain and Morgan, 2006; Brown et al., 2010), and complicated or confusing product descriptions (e.g., Ellison and Ellison, 2009; Chioveanu and Zhou, 2013). The optimal degree of obfuscation is a strategic decision by firms that is subject to consumers' information acquisition process, which largely depends on exogenous factors, such as market and search structure, information costs, or prior beliefs of consumers. In recent years, digital technology has reshaped many of these factors. In particular, it has markedly reduced information costs, which makes it easier for consumers to compare prices (see Goldfarb and Tucker, 2019, for an overview).

Empirically, this reduction in information costs has led to decreasing prices in highly digitalized markets compared with traditional ones (e.g., Brynjolfsson and Smith, 2000; Brown and Goolsbee, 2002; Orlov, 2011). In the empirical part of this study, we document this trend in the Swedish mobile phone subscription market. Furthermore, the complexity of pricing schemes for mobile subscriptions has decreased. Mobile subscriptions were previously characterized by a variable add-on pricing structure over different dimensions. Today, most subscription plans are based on fixed, postpaid pricing, making it much easier for the consumer to assess mobile services' "final" price per month or year.³ In comparison, service contracts in the financial and health industries are still based on complex pricing schemes.⁴ In this article, we discuss a model in which these diverging developments can be explained by equilibrium pricing and obfuscation behavior of profit-maximizing firms when consumers are rational inattentive. Our model has important implication for effective market regulation as complex pricing schemes due to obfuscation makes it more costly for consumers to learn about firms' offers and therefore decreases competition.

We consider a duopoly where firms compete on price and decide beforehand whether to obfuscate their prices. Obfuscation increases consumers' information costs to study prices. Consequently, obfuscation affects the consumers' optimal information strategy and thus the endogenous demands for the products. The resulting demands are probabilistic, following a generalized multinomial logit function, and depend on consumers' information costs, the prices set by firms, and the prior beliefs about the price-quality

¹Our notion of obfuscation is very general and refers to practices by firms to make their offers less transparent to (some) consumers. Other related terms used in the literature include shrouding attributes, strategic price complexity, and price frame dispersion.

²Several studies examine a variety of firms' actions that aim at manipulating consumers' information acquisition process in different market settings and with diverging implications for the resulting complexity of price-quality assessment in equilibrium. For example, see Gabaix and Laibson (2006), Carlin (2009), Ellison and Ellison (2009), Ellison and Wolitzky (2012), Chioveanu and Zhou (2013), and Heidhues et al. (2016).

³The importance of customer confusion in the early times of mobile subscriptions in the UK is documented by Turnbull et al. (2000).

⁴For general evidence of inertia in insurance markets, see, for example, Abaluck and Gruber (2016), Handel (2013), Handel and Kolstad (2015), Marquis and Holmer (1996), or Marzilli Ericson (2014). Woodward and Hall (2010) provide evidence for information costs and confusion in the mortgage market.

differentials of the products. Our framework makes use of recent advancements in rational inattention approaches to discrete choices (e.g., Matejka and McKay, 2014) that do not put any restriction on consumers' prior beliefs or on the type and extent of information processing. Digitalization is interpreted as a decrease in consumers' information costs to learn about obfuscated prices.

Our results first highlight that rational inattentive consumers could incentivize firms in duopolies to optimally obfuscate their true prices. If both firms obfuscate their price, competition between them is lower, and equilibrium prices and profits increase. Obfuscation may be a mutual best reply. This result extends findings from the standard search cost literature (e.g., Ellison and Wolitzky, 2012).

We show that prices and obfuscation behavior in equilibrium crucially depend on how much firms can increase consumers' information costs by obfuscating their price. An equilibrium with high prices and obfuscation only exists if the increase in information costs is large enough. Otherwise, the rents from decreased competition may be too low, giving firms an incentive to choose a transparent pricing scheme in the quest to secure the entire market. Eventually, this makes obfuscation a dominated strategy, leading to zero equilibrium profits as in Bertrand (1883) – a prisoner's dilemma for the firms. Following our theoretical results, the development of transparent pricing schemes for mobile services may be the result of digitalization. Digital technologies may have decreased individual information costs to the extent that a transparent pricing scheme has become the dominant strategy for mobile service providers. The difference to other markets could be attributed either to lower initial information costs or to a comparably larger relative change in information costs due to digitalization.

Additionally, we find that the equilibrium outcome depends on the shape of the consumers' prior beliefs. If, according to the prior beliefs, one firm seems more attractive than the other, i.e., has higher prominence,⁵ the consumer assigns a larger weight to the more attractive firm. The weight arises endogenously from the optimal information strategy of the consumer and can be directly related to a quality parameter in linear random utility models (see Anderson et al., 1992). In a scenario in which both firms obfuscate, a higher a priori prominence is connected to higher optimal prices, a higher market share, and higher profits. Importantly, a larger weight on one product is connected to a smaller weight on the other one.⁶ The less prominent firm will thus claim less of the rent arising from obfuscation, which increases the incentive to use a transparent pricing scheme in an attempt to claim the entire market. As a consequence, large differences in prominence are connected to less obfuscation in equilibrium. These results are in line with empirical observations. In the mobile subscription industry, switching costs and other reasons for habit persistence increase the differences in prominence (Reme et al., 2018). Therefore, decreasing information costs may have larger equilibrium consequences in the mobile market compared to other markets with more symmetric prior beliefs about prices.

⁵Prominence in our model implies that, according to their prior beliefs, consumers expect the price-quality differential of one firm to be higher than that of competing firms. Prominence, in general, is studied by, for example, Armstrong et al. (2009) and Chioveanu (2019).

⁶This depends on the fact that the exact weights will be a monotonic transformation of the unconditional choice probabilities of all possible actions, which have to sum up to 1. See the discussion in sections 3 and 4 for further details.

Finally, we find that competition in the form of new market entrants alters optimal obfuscation behavior and prices. First, if the new competitor is significantly less prominent than existing firms, obfuscation equilibria cease to exist. Second, if the new competitor is sufficiently prominent, two channels become important. The competition channel increases competition among obfuscating firms. As a result, the optimal prices and profits of existing firms decrease, thereby also decreasing the likelihood of obfuscation equilibria. The information channel affects the equilibrium outcome by changing the prior belief and thus the consumer's optimal information strategy. This may have opposing effects if a new market entry increases the incentive to process at least some information and if the new information strategy favors the less prominent firm, such that resulting profits under obfuscation are more balanced. The effect on obfuscation behavior and prices will largely depend on the consumer's prior belief about the new product and its correlation with existing options.

Our results have important implications for market regulation. Obfuscation in our model reduces competition among firms and leads to a decrease in consumer surplus. As a consequence, a policymaker may want to regulate markets to avoid obfuscation. According to our model, two main policy options exist. First, there is the option to decrease the information costs for consumers to learn about prices. This can be achieved either by increasing comparability between prices and facilitating search, for example, by digitalization, or by restricting product features that complicate consumers' assessment of the final price of the product, such as add-on pricing. Second, there is the option to increase the difference in the prominence of firms, for example, by decreasing market entry barriers for new and unknown competitors that are likely to be less prominent than existing firms. Both of these policies make obfuscation and high prices less likely in equilibrium and increase social welfare.

Our demand-side approach takes advantage of a seminal paper by Matejka and McKay (2014) that formalizes the idea of rational inattentive consumers, initiated by Sims (2003), in a discrete choice framework.⁸ Importantly, our framework inherits the idea that consumers have access to all information and freely choose – subject to a generally applicable information cost function – what kind of information to process.⁹ By incorporating this approach, we extend several strands of literature.

Imperfect information in markets has long been a central theme in industrial organization research (e.g., Smallwood and Conlisk, 1979; Shapiro, 1982; Stiglitz, 1989). Building upon this literature, several studies focus on optimal practices by firms that make it harder for some consumers to assess the true valuation of the product, i.e., make the firms' offers less transparent. We denote such practices as obfuscation. Some of these studies utilize exogenous information structures, for example by assuming that

⁷Heidhues et al. (forthcoming) reach a similar conclusion, studying secondary price features of products in a similar setting.

⁸Caplin et al. (2019) extend this framework by providing necessary and sufficient conditions that allow identification of products about which a rational inattentive consumer will process at least some information. For our paper, these conditions are particularly relevant for the cases with asymmetric obfuscation behavior. Huettner et al. (2019) further extend the framework of Matejka and McKay (2014) for heterogeneous information costs for different choices.

⁹Our convex cost function is based on Shannon entropy, commonly used in the literature (e.g., Sims, 2003; Matejka and McKay, 2014; Caplin et al., 2019).

a fraction of consumers are sophisticated and make optimal decisions while another fraction of consumers choose randomly. In these settings, the firms usually manipulate the relative number of sophisticated consumers by obfuscation. Examples include Gabaix and Laibson (2006), who study add-on pricing; Carlin (2009), who considers the retail financial industry; and Chioveanu and Zhou (2013) and Piccione and Spiegler (2012), who focus on the comparability of prices. While certainly helpful, such models do not model the endogenous character of consumers' information search, which has important implications for firms' optimal obfuscation behavior.

Studies that analyze optimal obfuscation behavior in a search-theoretic model, along the lines of Salop and Stiglitz (1977), Varian (1980), and Stahl (1989), are closely related to our approach. ¹⁰ Ellison and Wolitzky (2012) present a sequential search model, in which firms affect the time needed to assess their prices. In the model of Wilson (2010), firms can influence the consumers' order of search by obfuscation. Taylor (2017) adopt a search-theoretic model in which obfuscation helps firms to target the most valuable consumers. In these (sequential) search models, the information set is endogenous, as it depends on the consumers' search costs. In a sequential framework, after learning the valuation of one product, consumers decide, conditional on prior expectations and search costs, whether to continue their search. The crucial difference to our approach is that information about one product is binary because consumers either do not learn anything about a product or perfectly identify the relevant valuation. While this setup is a good fit for window-shopping or browsing of simply priced products, it fails to account for complex pricing structures and for highly digitalized markets in which individuals have almost unlimited access to all kinds of information. The order of search does not play a role in our model, and the optimal information strategy of the consumer is unrestricted in the sense that the consumer is free to choose any type of informative signals. As a result, the consumer is typically not perfectly informed about the products' true valuation at the time of purchase. Therefore, our search structure may offer a more realistic representation of digitalized markets with complexly priced products than classical (sequential) search models or models of exogenous information.

The industrial organization literature on the "prominence" of firms (e.g., Armstrong et al., 2009; Armstrong and Zhou, 2011; Rhodes, 2011; Chioveanu, 2019) deploys similar "binary" search models or exogenous information structures on the demand side. In our model, the prominence of firms arises due to unrestricted prior beliefs, which allows for a variety of "prominence" concepts, depending on the exact shape of the prior belief. Again, the order of search is subordinate.

The paper by Matejka and McKay (2014) triggered a number of recent studies in industrial organization that focus on different aspects, such as inattentive sellers (Matějka, 2015) or inattention to quality and optimal pricing (Martin, 2017). Two studies that incorporate rational inattentive demand in a model of industrial organization are particularly relevant. First, Matějka and McKay (2012) investigate equi-

 $^{^{10}}$ Grubb (2015), Spiegler (2016), and Armstrong (2017) provide overviews of recent relevant theoretical work on the intersection of industrial organization and consumer search.

libria in an oligopoly when consumers are rational inattentive. Second, Boyacı and Akçay (2017) study the optimal pricing behavior of a monopolistic firm that faces a rational inattentive consumer and can choose to buy or not to buy the product. We add to this literature by extending the models along the obfuscation dimension. This important dimension has a natural connection to rational inattention as the attractiveness of obfuscation practices strongly depends on how and which type of information consumers process.¹¹

We start by presenting some stylized facts about the Swedish mobile subscription market in section 2. In section 3, we present our model setup. Section 4 derives the equilibrium conditions, and section 5 provides numerical results of our model. Section 6 concludes the paper.

2 Background: The Swedish Mobile Subscription Market

The empirical part of this paper provides a descriptive analysis of the Swedish mobile subscription market to exemplify our motivation. We document stylized facts about the development of mobile subscriptions.

Data

We use yearly market share data for the Swedish mobile subscription market between 2011 and 2018 provided by the Swedish Post and Telecom Authority. In detail, we observe yearly revenues of firms fragmented by type of subscription (i.e., prepaid and postpaid) and fees (i.e., variable and fixed fees). We adjust revenues for inflation using general yearly consumer price indices. Additionally, we use the consumer price index (CPI) for the product group of telephone services and equipment from Statistics Sweden.

Prices and Subscriptions

We start by evaluating prices in the subscription market. Even though we do not observe prices in our data set, we use several measures that allow us to infer average price movements. Figure 1a shows the development of the CPI for telecommunication and telecommunication equipment between 2011 and 2015. In this time period, the yearly CPI dropped by 10%. In Figure A.1 of Appendix A we document that the producer prices, which do not include equipment expenses, show the same trend. Between 2014 and 2020, producer prices decreased by 13%. The same holds true for average revenues per subscription in Figure A.2, which also decreased over the last two decades. Furthermore, decreasing prices are not a unique phenomenon of the Swedish mobile subscription market. Indeed, average revenues per user in

¹¹Indeed, Grubb (2015) highlights the importance of "research [that] would be to consider firms investing in obfuscation that raises the cost of attention and hence increases the noise with which prices are evaluated." Similarly, Matějka and McKay (2012) note that "there is a natural connection between our model and the literature on obfuscation [...] which aims to understand practices by firms that serve to make the terms of their offers less transparent."

¹²Note that Statistics Sweden solely offers data on the CPI for telecommunication expenses until 2011.

the mobile subscription market decreased in most European countries (Commission, 2019), which leads us to our first stylized fact.

Stylized Fact 1. In recent years, prices in the mobile subscription market decreased.

Figure 1b illustrates aggregated yearly revenues between 2011 and 2017. Total revenues increase slightly. Looking at yearly revenues divided into fixed and variable fees subscriptions, we see that this increase can be attributed to a strong increase in fixed subscription fees. In comparison, the revenues that are due to variable fees significantly decreased. In 2011 54% of the yearly revenues could be attributed to fixed fees, while in 2017 fixed fees were responsible for 83% of the revenues. We observe a similar heterogeneous development in the number of subscriptions in Figure 1c. Overall, the market size of the mobile market increased slightly. Between 2011 and 2017, the total number of subscriptions increased from 13.4 to 14.4 million. As we saw above, the increasing number of subscriptions increases the aggregate revenues, while average revenues per subscriptions decrease in accordance with stylized fact 1 (see Figure A.2). If we divide the number of subscriptions into postpaid and prepaid subscriptions between 2011 and 2017, we see an increase in the share of postpaid subscriptions from 66% to 76%, while the share of prepaid subscriptions decreased. This is due to the fact that fixed fee subscriptions are usually postpaid, while variable fees are indicative for prepaid subscriptions. Looking at the characteristics of the subscriptions, we conclude:

Stylized Fact 2. Fixed-fee and postpaid subscriptions became more common, replacing variable-fee and prepaid subscriptions.

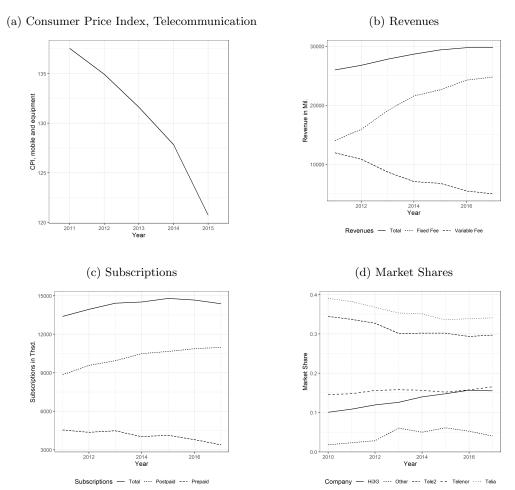
Intuitively, both the increase in fixed fees and the increase in postpaid contracts are in line with the reasoning that consumers purchase easier contracts. The complexity in pricing schemes of mobile subscriptions has decreased. At the same time, the average price decreased, while the number of products increased. We argue that the descriptive example of the Swedish market is representative of mobile subscription markets in general. Historically, we observed markets with variable pricing structures where it has been difficult to assess mobile services' final price per year or month. In recent years, we observe not only reduced prices but also less complex pricing schemes.

Competition

Additionally, we observe an increase in competition in the Swedish mobile subscription market. The source of competition in the market is manifold. On the one hand, the distribution of 3G licenses in 2000 led to the entry of a new competitor that gained market share relatively fast (OECD, 2015). On the other hand, network-sharing agreements that allow easier entry are common and frequently used (OECD, 2015). Figure 1d shows the market shares of the four biggest operators. After the entry of a fourth competitor in 2010, market shares of all companies remained largely unchanged. However, we

observe slight decreases in market shares of the two largest firms (around 5 percentage points each) while smaller firms increased their market shares.

Figure 1: The Swedish Mobile Subscription Market



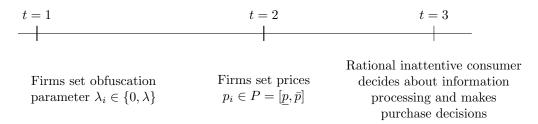
Note: The four parts of this figure illustrate key stylized facts of the Swedish mobile subscription market: (a) the yearly consumer price index for telecommunication and telecommunication equipment between 2011 and 2015; (b) aggregated revenue, divided into fixed and variable fees and adjusted for inflation, in 2011 prices; (c) the number of mobile subscriptions in the Swedish mobile subscription market between 2011 and 2017, divided into postpaid and prepaid subscriptions; and (d) the four biggest competitors' market shares between 2011 and 2017.

3 Model

3.1 Basic framework

There are two firms $i \in N = \{1, 2\}$ that produce a homogeneous good of quality $q \in \mathbb{R}$. The firms face two sequential strategic decisions. In t = 1, they decide simultaneously on the individual obfuscation parameter $\lambda_i \in \{0, \lambda\}$, where λ is fixed and given. Obfuscation comes at a fixed cost of $\xi > 0$. In a subsequent second step (t = 2), firms set their prices, $p_i \ge 0$. The timeline of the game is summarized in Figure 2.

Figure 2: Timeline of the game



The maximization problem of firm $i \in N$ in the second stage is given by:

$$\max_{p_i \ge 0} \eta_i(p_i, p_{-i}, \lambda_i, \lambda_{-i}, G_0(\cdot)) * (p_i - c) - \mathbf{1}_{\lambda_i = \lambda} * \xi,$$

where η_i represents the expected demand or equivalently the likelihood that the representative consumer buys the product of firm i. The market share for firm i is a result of the optimal choice and the consumer's information processing strategy, which depends on the prices of the respective firm i and its competitor -i, the firms' obfuscation choices λ_i and λ_{-i} , and the prior beliefs about prices held by the representative consumer, $G_0(\cdot)$.

Each obfuscation-price combination determines the conditional demands of the representative consumer for each firm, η_i . Firms decide about obfuscation as well as prices non-cooperatively and with complete information. The game has perfect recall. We assume zero marginal costs, c = 0, for simplicity. Formally, we describe the game in the following definition:

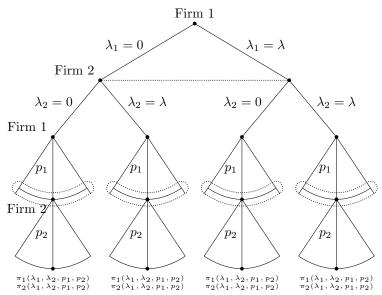
Definition 1. N is a set of two players. The play is ordered in a tree T, described in Figure 3. Thus, the game has two proper subgames. The tree has five decision nodes D as well as terminal nodes Z. In each initial node, players decide about their obfuscation strategy $\lambda_i \in \{0, \lambda\} \ \forall i \in N$. After the realization of obfuscation choices, players observe the outcome of the first stage. The combination of obfuscation outcomes leads to four decision nodes D. Firms set prices $p_i \geq 0 \ \forall i \in N$ simultaneously. Each combination of obfuscation and prices leads to a final node $z \in Z$. The payoff of each firm i can be described as a mapping from the terminal node, $\pi: Z \to \mathbb{R}$. S is the set of strategy profiles, i.e., the set of possible pairs of strategies, one for each player, i.e., $(s_1, s_2) \in S$.

In our results section, we consider subgame perfect equilibrium as our solution concept. A pure strategy profile $s \in S$ is subgame perfect if for each proper subgame the strategy profile constitutes a Nash equilibrium. A strategy profile $s \in S$ is a pure Nash equilibrium if $\forall i$ and $\forall s_i \in S_i$, $U_i(s_i, s_{-i}) \geq U_i(s_i', s_{-i})$.

3.2 Consumer Demand

The representative consumer can buy from either firm 1 (i = 1) or firm 2 (i = 2). The payoff of choosing a specific action is given by the price-quality differential: $k_i = q - p_i$, $\forall i \in \mathbb{N}$. The consumer

Figure 3: Game Tree



Note: The figure describes the game tree. Initially, both firms set the obfuscation parameter λ_i simultaneously. The decision of λ_i is binary as firms can choose a transparent pricing scheme ($\lambda_i = 0$) or obfuscation of their price ($\lambda_i = \lambda$). The dotted line refers to an information set. After observing both choices of obfuscation, the firms choose prices p_i simultaneously. The payoff functions for firm 1 and firm 2 are denoted as π_1 and π_2 .

does not perfectly observe the state of nature, i.e., the prices set by firms, but has a prior belief about their joint distribution $G_0(k_1, k_2) = G_0(\mathbf{k})$. We denote as the prior belief on all costless information \tilde{k} , $\Omega_0(\mathbf{k}) = G_0(\mathbf{k}|\tilde{k})$. Following the rational inattention literature, the consumer has the possibility to gather and process signals \mathbf{s} to decrease the uncertainty about the state of nature. The optimal precision of the processed signals is subject to a convex Shannon entropy-based information cost function, commonly used in the rational inattention literature (e.g., Matejka and McKay, 2014; Sims, 2003).

Processing information about both products with $\lambda_i < \infty$ results in a posterior belief, $F(\mathbf{k}|\mathbf{s})$. Given this posterior belief, the consumer chooses the product a that gives the highest expected payoff, in this case the product with the lowest expected price:

$$a(F(\mathbf{k}|\mathbf{s})) = \arg\max_{i \in N} E_{F(\mathbf{k}|\mathbf{s})}(q - p_i)$$

Choosing the optimal information strategy, more precisely the joint distribution of signals and states $F(\mathbf{s}, \mathbf{k})$, the consumer trades off the cost of information processing with its benefits.¹³ The cost function $\hat{c}(F) = \lambda(H(\Omega_0) - E_s[H(F(\mathbf{k}|\mathbf{s})])$ is the product of the obfuscation parameter λ and the expected Shannon entropy $H(\cdot)$ reduction by means of the processed signal.¹⁴ The benefits arise from the fact that processing more precise signals results in less uncertain posterior beliefs, which in turn translates into a higher

¹³This information strategy has to be consistent with the consumer's prior belief. The condition $\int_s F(ds, \mathbf{k}_j) = \Omega_0(\mathbf{k})$ assures this.

¹⁴This notation is only possible since we restrict the individual information costs to zero and λ . If the firms can set individual non-zero information costs different from each other, $0 < \lambda_1 < \lambda_2 < \infty$, the problem gets more pronounced (see Huettner et al., 2019).

expected value of the optimal choice. Formally, the consumer chooses an information strategy to solve the following maximization problem:

$$\max_{F \in \Delta(\mathbb{R}^{2N})} \int_{\mathbf{k}} \int_{\mathbf{s}} k_{a(F(\mathbf{k}|\mathbf{s}))} F(d\mathbf{s}|\mathbf{k}) \Omega_{0}(d\mathbf{k}) - \hat{c}(F)$$
s.t.
$$\int_{\mathbf{s}} F(d\mathbf{s}, \mathbf{k}) = \Omega_{0}(\mathbf{k})$$

One important feature of rational inattention is that each action strategy is associated with a particular signal (Matejka and McKay, 2014). This allows us to rewrite the maximization problem of the consumer in terms of choice probabilities. We show, in accordance with Matejka and McKay (2014), that solving this maximization problem gives us the respective conditional choice probabilities $\eta_i(\mathbf{k})$ that will follow a multinomial logit function:¹⁵

$$\eta_{i}(\mathbf{k}) = \begin{cases}
0 & \text{if } \eta_{i}^{0} = 0 \\
\frac{\eta_{i}^{0} e^{(q-p_{1})/\lambda}}{\eta_{1}^{0} e^{(q-p_{1})/\lambda} + \eta_{2}^{0} e^{(q-p_{2})/\lambda}} & \text{if } 0 < \eta_{i}^{0} < 1, \\
1 & \text{if } \eta_{i}^{0} = 1
\end{cases} \tag{1}$$

where η_i^0 are the unconditional choice probabilities, which are not equal to the prior beliefs $\Omega_0(\mathbf{k})$ but are rather a result of the maximization problem itself. It is the collection of all conditional choice probabilities $\eta_i(\mathbf{k})$ integrated over the respective prior belief $\Omega_0(\mathbf{k})$: $\eta_i^0 \equiv \int_{\mathbf{k}} \eta_i(\mathbf{k}) \Omega_0(d\mathbf{k})$. As a consequence, η_i^0 is independent of the realized state, i.e., the prices set by firms, but affected by the prior belief over different states and the price obfuscation parameter λ . $\eta_i^0 = 1$ implies that the consumer would not process any information and buys product i with a probability of one $(\eta_i(\mathbf{k}) = 1)$ irrespective of the prices set by obfuscating firms. Accordingly, the consumer never chooses a product with $\eta_i^0 = 0$. These non-considerations of products arise endogenously and will depend on a cutoff that depends on the normalized payoffs evaluated at prior beliefs (Caplin et al., 2019). This may happen if one of the products seems a priori much more attractive to the consumer than the other options and information costs are comparably large.¹⁶

The consumer's optimal probabilistic choice will follow a multinomial logit form irrespective of whether only one firm chooses to obfuscate or both firms choose to obfuscate.¹⁷ However, the cases differ with respect to the relevant (conditional) prior beliefs $\Omega_0(\mathbf{k})$. As a result, the corresponding unconditional choice probabilities η_i^0 differ.

$$\int_{\mathbf{k}} \left(\frac{e^{k_i/\lambda}}{\sum_{j=0}^2 \eta_j^0 e^{k_j/\lambda}} \right) \Omega_0(d\mathbf{k}) = 1, \quad \forall i \eta_i^0 > 0$$

¹⁵See Appendix B for derivations

¹⁶Caplin et al. (2019) provide the corresponding necessary and sufficient conditions for the unconditional choice probabilities that we apply in our simulations later. If $0 < \eta_i^0 < 1$, the normalization conditions laid out by Matejka and McKay (2014) apply:

 $^{^{17}\}mathrm{See}$ Appendix B for derivations

In the case in which both firms have zero information costs, the information strategy and the derivation of the consumer demands are trivial. The consumer will choose signals that perfectly identify the realized state, implying that the consumer will buy the product at a lower price. The conditional choice probabilities $\eta_i(\mathbf{k})$ will thus equal 1 if $p_i < p_j, \forall j \neq i$ and zero otherwise.¹⁸

4 Equilibrium Conditions

In the spirit of backward induction, we start by solving for subgame perfect equilibria of the competitive pricing game of the firms conditional on the obfuscation decision made in the first stage.

4.1 Second Stage

If both firms choose not to obfuscate, the pricing game is equal to a well-studied Bertrand duopoly. Both obfuscation parameters are equal to zero such that the rational inattentive consumer observes prices perfectly. The unique best reply of both firms is given by $p_1 = p_2 = 0$, the Bertrand paradox. The corresponding firm profits are zero.

Proposition 1. If both firms choose to obfuscate, the optimal prices and profits are uniquely determined by following set of equations:

$$p_i^* = c + \frac{\lambda}{(1 - \eta_i^*(\mathbf{k}))}.$$
(2)

for $i \in N$, where $\eta_i^*(\mathbf{k})$ denote the equilibrium market share, and the corresponding equilibrium profits of firm i are given by:

$$\pi_i^* = \frac{\lambda}{(1 - \eta_i^*(\mathbf{k}))} \eta_i^*(\mathbf{k}) \tag{3}$$

Proof. Given the multinomial logit demand function derived above, the derivation of equilibrium prices and profits in the case of simultaneous obfuscation follows a similar logic as in Anderson et al. (1992). The first-order conditions are given as follows:

$$\eta_i(\mathbf{k}) - \frac{\eta_i(\mathbf{k})(1 - \eta_i(\mathbf{k}))}{\lambda}(p_i^* - c) = 0,$$

For an interior solution, the optimal price of firm i is thus given by equation (2). Anderson et al. (1992) show that such a system of equations has a unique solution. Equilibrium profits are given by equation 3.

¹⁸If two or more actions inhibit the same state-contingent payout, a tie-breaking rule is applied, resulting in equally distributed conditional choice probabilities among these options.

¹⁹In Anderson et al. (1992) the multinomial logit demand arises from a representative consumer with linear random utility and double exponential distributed error terms. In contrast, under rational inattention, the randomness arises not from noise with respect to preferences, but from the signal the consumer receives.

As noted above, the unconditional choice probabilities are independent of firms' prices as they do not affect the consumer's prior belief. As a result, firms cannot influence the unconditional choice probabilities in the second stage if $\lambda_i = \lambda$.

If only one firm chooses to obfuscate, this changes, as the consumer will be perfectly informed about the transparent price set in the second stage. Consequently, the prior belief conditional on all costless information $\Omega_0(\mathbf{k}) = G_0(\mathbf{k}|\tilde{k})$ changes that is relevant for the consumer's optimal information strategy. As a result, the transparent firm can influence the unconditional choice probabilities.²⁰

Proposition 2. If only one firm chooses to obfuscate, the optimal price of the transparent firm $(\lambda_i = 0)$ is equal to \bar{p}_i , where $\bar{p}_i = \max p_i$ such that $\eta_i^0 = \eta_i(\mathbf{k}) = 1$.

Proof. First, note that the transparent firm will not set a price below \bar{p}_i , because, if $p_i < \bar{p}_i$, $\eta_i(\mathbf{k}) = 1$ and accordingly $\frac{\partial \pi_i}{\partial p_i} = 1 > 0$.

If $p_i \geq \bar{p_i}$, both firms will face a multinomial demand as derived above. The maximized profit function for a given unconditional choice probability is equal to $\pi_i^*(\eta_i^0) = \frac{\lambda}{(1 - \eta_i^*(\mathbf{k}))} \eta_i^*(\mathbf{k})$. Plugging in the corresponding demand function, the profit function can be rewritten to:

$$\pi_i^*(\eta_i^0) = \lambda \frac{\eta_i^0}{(1 - \eta_i^0)} \frac{e^{(q - p_1^*)/\lambda}}{e^{(q - p_2^*)/\lambda}} = \lambda \frac{\eta_i^0}{(1 - \eta_i^0)} e^{(p_2^* - p_1^*)/\lambda}$$

Applying the Envelope theorem, the derivative of the profit function with respect to the unconditional choice probability is given by:

$$\frac{\partial \pi_i^*}{\partial \eta_i^0} = \frac{\lambda}{(1 - \eta_i^0)^2} e^{(p_2^* - p_1^*)/\lambda} > 0,$$

which is strictly larger than zero if $\eta_i^0 < 1$.

As a consequence of Proposition 2, the transparent firm's profits and optimal price in equilibrium are given by $\pi_i = p_i^* = \bar{p}_i$. The obfuscating firm has a market share of zero and can set any p_2 in equilibrium. As a result, the equilibrium profits of the obfuscating firm are equal to $-\xi$. Proposition 2 further implies that the consumer will not process any information. \bar{p}_i is the transparent firm's cutoff price level at which it becomes optimal for the consumer to consider the offer of the obfuscating firm and process some information.²¹ The exact value of the cutoff level \bar{p}_i depends on the prior beliefs and the information costs parameter λ . A closed-form solution does not exist. We provide a numerical solution for the firms' optimal prices and profits below.

²⁰See Appendix C for further descriptions.

²¹This cutoff level directly relates to the consideration set's cutoff level discussed in Caplin et al. (2019).

Table 1: First Stage Payoff matrix

Firm 2
$$\lambda_{2} = 0 \qquad \lambda_{2} = \lambda$$
Firm 1
$$\lambda_{1} = 0 \qquad (0, 0) \qquad (\bar{p}_{1}(G_{0}(\cdot), \lambda), -\xi)$$

$$\lambda_{1} = \lambda \qquad (-\xi, \bar{p}_{2}(G_{0}(\cdot), \lambda)) \qquad \left(\frac{\lambda}{(1 - \eta_{1}^{*}(\mathbf{k}))} \eta_{1}^{*}(\mathbf{k}) - \xi, \frac{\lambda}{(1 - \eta_{2}^{*}(\mathbf{k}))} \eta_{2}^{*}(\mathbf{k}) - \xi\right)$$

Note: The table presents the payoff matrix of both firms in the first stage. Firms 1 and 2 set their obfuscation parameters simultaneously. Each combination of obfuscation parameters relates to a payoff function.

4.2 First Stage

In the first stage, both firms have the option to obfuscate or choose a transparent pricing structure. The simultaneous and discrete decision to obfuscate leads to four different nodes of the game. Using the Nash equilibria of the last subgame, we can represent the simultaneous move game in the first stage in a symmetric 2×2 payoff matrix presented in Figure 1.

If both firms choose a transparent pricing scheme, prices, as well as profits, are zero. Following Proposition 2, asymmetry in obfuscation results in an advantage for the transparent firm. While the transparent firm claims the entire market, the opponent has a negative profit due to obfuscation costs. If both firms decide to obfuscate, prices and profits in the second stage depend on consumers' prior beliefs and the size of information costs. Two subgame perfect equilibria may exist.

Proposition 3. There always exists a subgame perfect equilibrium that is defined by the following strategies:

$$s_i = (\lambda_i = 0, p_i = 0) \quad \forall i \in N$$

In the case that the opponent of a firm does not obfuscate, i.e., $\lambda_{-i} = 0$ where $-i \in N \setminus i$, it is always the best reply for firm i to be transparent and set $\lambda_i = 0$ as $0 > -\xi$. Thus, the equilibrium $(\lambda_1 = 0, \lambda_2 = 0)$ of the first subgame always exists independent of prior beliefs and information costs.

Proposition 4. There exists a subgame perfect equilibrium that is defined by the following strategies and conditions:

Strategies:

$$s_i = (\lambda_i = \lambda, p_i = c + \frac{\lambda}{(1 - \eta_i^*(\mathbf{k}))}) \quad \forall i \in N$$

Conditions:

$$\frac{\lambda}{(1 - \eta_i^*(\mathbf{k}))} \eta_i^*(\mathbf{k}) - \xi \ge \bar{p}_i(G_0(\cdot)) \quad \forall i \in N$$

The first stage may have another pure strategy equilibrium, as both firms may find it optimal to obfuscate: $\lambda_i = \lambda$ for $i \in N$. The existence of the equilibrium requires that the payoffs of the joint obfuscation case, $\frac{\lambda}{(1-\eta_i^*(\mathbf{k}))}\eta_i^*(\mathbf{k}) - \xi$, is larger than the payoff in the separating case for the respective firms, which is equal to the cutoff price level $\bar{p}_i(G_0(\cdot))$. The condition is dependent on the distribution of prior beliefs, $G_0(\cdot)$, as well as the size of λ and ensures that no firm has an incentive to deviate from $s_i = (\lambda_i = \lambda, p_i = c + \frac{\lambda}{(1-\eta_i^*(\mathbf{k}))})$.

Proposition 5. With non-zero obfuscation costs ($\xi > 0$), there is no separating equilibrium in which only one firm obfuscates.

If $\xi > 0$ and $\lambda_i \neq \lambda_{-i}$, only the transparent firm would make non-negative profits, while the obfuscating firm suffers a loss due to non-zero obfuscation costs ξ , given that an opponent chooses a transparent pricing scheme. Consequently, it is the dominant strategy to be transparent as well, yielding a payoff of zero, and a separating equilibrium does not exist.

Overall, the condition presented in Proposition 4 is crucial for interpreting the game in the first stage. If the condition holds, the first stage is a pure coordination game with two pooling equilibria. The payoff is strictly higher in the obfuscation equilibrium compared to the equilibrium in which both firms are transparent and thus have zero profits. If the condition does not hold, the game is equivalent to a prisoner's dilemma. Even though mutual obfuscation would result in a higher payoff, the game has a unique equilibrium in pure strategies, in which both firms set $\lambda_i = 0 \ \forall i \in \mathbb{N}$.

While no closed-form solution for the condition in Proposition 4 exists, the simulation results in the next section show that the size of λ and the distribution of consumers' prior beliefs, in particular the relative prominence of the firms, play a crucial role for the outcome of the game.

5 Results

5.1 Second Stage: The Effect of Information Cost Parameter λ

Symmetric Priors

To get an initial idea of how different information costs influence the equilibrium outcomes, we start by considering that consumers have symmetric prior beliefs about the attractiveness of the options. Both firms are equally prominent, implying that consumers do not distinguish between the different products before processing information.²² The unconditional choice probabilities will be identical across firms, $\eta_i^0 = 0.5$, making them completely homogeneous. The model reduces to a symmetric logit model. The optimal prices are given by $p_i^* = c + 2\lambda$ with corresponding profits of $\pi_i = \lambda - \xi$ in the case where both firms obfuscate. In the other obfuscation cases, firms' optimal behavior is symmetric as well and follows the optimality conditions derived above.

Figure 4 shows the payoffs for increasing information costs for different decisions in the first stage.²³ If both firms choose to obfuscate, the equilibrium profits and prices are linearly increasing in λ . Both firms benefit from increasing information costs, as this decreases competition among firms. Symmetric prior beliefs ensure that the market is shared equally.

For the transparent firm in the separating case ($\lambda_1 = 0, \lambda_2 = \lambda$), profits are also increasing in λ , while marginal profits are decreasing in λ . This is because the profits for the transparent firm mirror the cutoff price level \bar{p}_1 , for which the consumer finds it optimal to process at least some information. If processing information is costly, the transparent firm can set a higher price and still claim the entire market, as the benefit for the consumer of potentially learning about a lower price of the obfuscating firm is offset by the higher information costs, making the consumer more likely to choose the transparent ("safe") option. If the transparent firm sets a price higher than the expected value of the obfuscated price, according to the consumer's prior belief, the consumer will always choose the uncertain option as it gives the consumer a higher expected value. As a result, \bar{p}_1 is always below the expected price of the obfuscating firm.

If the variance of the obfuscated price according to the prior belief, σ_{ii}^2 , where i is the obfuscating firm, is smaller, the consumer prefers the transparent option for lower values of λ , as the consumer's expected value from learning about the obfuscated price is lower. As a result, the transparent firm's profits in the separating case are higher for the same values of λ and lower variance of the prior belief, given that the mean of the prior belief does not change. If both firms do not obfuscate, profits are always zero. In a separating equilibrium, the obfuscating firm $(\lambda_1 = \lambda, \lambda_2 = 0)$ has a negative profit of $-\xi$.

Asymmetric Priors

With asymmetric priors, the consumer prefers one firm, say the more prominent one, before any information is processed. In our simulation, we assume that $G_0(p_1) \sim N(4.4, 2)$ and $G_0(p_2) \sim N(4, 2)$, implying that, before processing information, the consumer expects the product of firm 1 to be more expensive, i.e., firm 2 seems more attractive. The prominence translates into different unconditional choice probabilities that can be interpreted as different product qualities in the standard logit-demand model (see Matějka and McKay, 2012). Accordingly, the ex ante preferred firm (firm 2 in our example) sets a higher price

²³Note that, as the game is symmetric and the consumer does not distinguish between the two products before processing information, the graph is equivalent for both firms.

Both firms obtuscate (\(\lambda/\lambda\)
No obtuscation (0/0)
Firm 1 transparent/ Firm 2 obtuscates (0/\lambda)
Firm 1 obfuscates/ Firm 2 transparent (\(\lambda/\lambda\))

5

4

1

2

Figure 4: Profit Functions for Symmetric Prior Beliefs

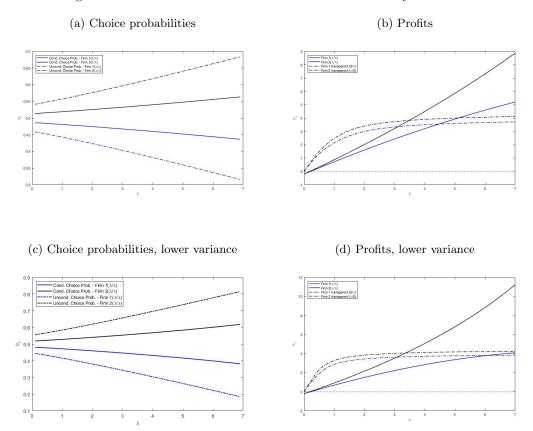
Note: The graph describes the profits of firm 1 for increasing obfuscation (λ) and different obfuscation choices when consumers have symmetric prior beliefs across all prices. The graph for firm 2 is identical.

0

than firm 1 if both firms obfuscate $\lambda_i = \lambda \, \forall i \in N$. As market shares, i.e., conditional choice probabilities, are also larger for more prominent firms, this directly translates into higher equilibrium profits. The difference in the unconditional choice probabilities (see Figure 5a), and accordingly profits (see Figure 5b), will be higher for larger information costs (λ). If information processing is more costly, the rational inattentive consumer will optimally choose less precise signals, relying more on prior beliefs. As in the symmetric case, the competition among firms decreases as a result, which benefits both firms as long as the consumer processes some information about both products ($\eta_i^0 > 0$). Besides, the a priori difference in prominence gets more important for increasing values of λ , as the consumer puts more weight onto the prior belief and the likelihood that the consumer will discover the lower price of the less prominent firm decreases. Furthermore, if there is lower variance of the prior beliefs about both prices σ_{ii}^2 , i = 1, 2), the importance of the difference in prominence will play a bigger role, as the consumer's expected value of learning about the actual price difference decreases. Accordingly, the difference between the unconditional choice probabilities (see Figure 5c) and thus profits (Figure 5d) becomes larger for the same values of λ .

For the separating case, the same logic applies as in the symmetric case. Again, the obfuscating firm $(\lambda_1 = \lambda, \lambda_2 = 0)$ has a negative profit of $-\xi$. The transparent firm will set $\bar{p_i}$ and has a market share of one. The difference for the asymmetric case is, however, that the relevant expected price, constituting the asymptote for profits of the transparent firm, varies conditional on which firm obfuscates. The consumer

Figure 5: Choice Probabilities and Profit Functions for Asymmetric Prior Beliefs



Note: The four parts of this figure illustrate conditional $\eta_i(\mathbf{k})$ and unconditional choice probabilities η_i^0 of the consumer and profits of firms for increasing obfuscation parameter (λ) and different obfuscation choices. We simulate asymmetric prior beliefs as an independent multinomial normal distribution with a mean of $\mu = (4.4, 4)$. Parts (a) and (b) of this figure present choice probabilities with a variance of $\sigma^2 = (2, 2)$. The blue line refers to the choice unconditional and conditional choice probabilities as well as the profit functions for firm 1 while the black line shows profits for firm 2. Parts (c) and (d) of the figure present results for a lower variance of prior beliefs, i.e., $\sigma^2 = (1.5, 1.5)$.

finds it less beneficial to learn something about an obfuscated price if the consumer expects the price to be comparably high. This implies that the more prominent firm, as compared to the less prominent firm, can set a higher \bar{p}_i , yielding higher profits, for the same values of λ .

These simulation results for the subgame perfect equilibria in the second stage have important implications for the obfuscation choice in the first stage. These implications are analyzed in more detail in the next section.

5.2 First Stage

We have shown that two equilibria may exist in the first stage. First, there always exists a pure transparency equilibrium, in which both firms do not obfuscate. This equilibrium is independent of λ and consumers' prior beliefs. Second, there exists an obfuscation equilibrium in which both firms obfuscate. The condition for the existence of the obfuscation equilibrium is described in Proposition 4. Conditional

on an obfuscating opponent, it has to be more profitable for a firm to obfuscate than to deviate by choosing a transparent pricing scheme. The existence depends on the prior beliefs as well as on λ .

First, we consider symmetric prior beliefs. Figure 4 presents a firm's profit for different obfuscation decisions and different values of λ . For small values of λ , obfuscating the price is a dominated strategy. Independent of the opponent's decision, it is optimal for a firm not to obfuscate. The condition in Proposition 4 does not hold because conditional on the opponent choosing to obfuscate, the best reply is to be transparent. The unique subgame perfect equilibrium is that both firms abstain from obfuscation in the first stage and set prices equal to zero in the second stage. The first-stage game reduces to a prisoner's dilemma as both firms could increase their profits if both obfuscate. However, only transparency is an equilibrium.

For large enough λ , the game may result in different equilibrium. First, as before, it is optimal to be transparent conditional on the opponent's decision not to obfuscate. Therefore it is a subgame perfect equilibrium that both firms do not obfuscate in the first stage and then set prices of zero in the second stage. However, conditional on an obfuscating opponent, it is now optimal to obfuscate as well. In Figure 4 we observe that for high enough values of λ , the profit of mutual obfuscation exceeds the profit of a separating equilibrium for the transparent firm. Thus, for symmetric priors and high enough values of λ , there exists a subgame perfect equilibrium where both firms obfuscate and set positive prices described in Proposition 4. As there exist two equilibria, the game is characterized by a coordination problem. Both firms have an incentive to coordinate to attain the high-payoff equilibrium with positive profits.

Next, we turn to analyze equilibria under the assumption of asymmetric prior beliefs of consumers. Consumers prefer one firm before processing information. Now, the degree of asymmetry in prior beliefs and the size of information costs determine if the game has one or two subgame perfect equilibria. Figure 6 shows the relationship between consumers' prior beliefs, information costs, and the existence of a obfuscation equilibrium. In detail, we show the area in which the condition of Proposition 4 holds. We observe that a higher asymmetry of prior beliefs requires a higher λ such that mutual obfuscation in the first stage is an equilibrium, everything else being equal. The intuition can be derived from Figure??. In this example and before processing information, consumers prefer firm 2 to firm 1. We show both firms' profit functions for increasing values of information costs λ , differentiating between the case in which both firms obfuscate and the case in which only one firm obfuscates. Proposition 4 requires that the profit from mutual obfuscation exceeds the profit in the separating case for each firm. Comparing the profit functions for asymmetric priors to those with symmetric priors in Figure 4, we see that the ex ante less preferred firm requires a higher value of λ , conditional on the the other firm chooses to obfuscate, to prefer obfuscation compared to a transparent pricing scheme. Intuitively, while the benefit of decreased competition persists for both firms, the market share and relative profits for the ex ante less preferred firm in the mutual obfuscation case will be lower. This in turn increases the relative attractiveness of choosing a transparent pricing scheme. Accordingly, for some values of λ where obfuscation would have been a best reply in the symmetric case, the ex ante less preferred firm now chooses a transparent pricing scheme such that mutual obfuscation is not an equilibrium anymore.

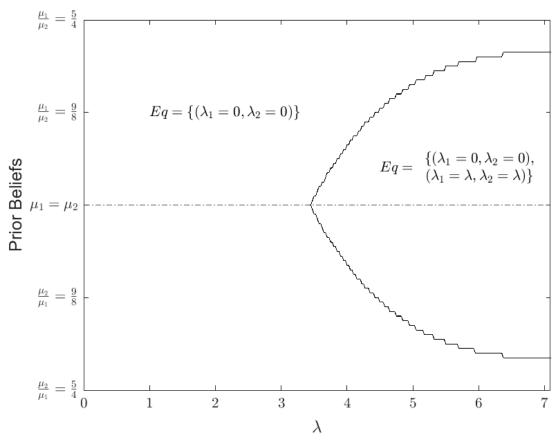


Figure 6: Equilibria in First Stage

Note: The graph describes equilibria regions for the first stage of the game given different prior beliefs and increasing values of the obfuscation parameter λ . The solid line represents the border between the equilibria regions.

5.3 New Competitor

We now consider the entry of a new competitor. The market entry of a new competitor may influence the obfuscation behavior of firms in equilibrium. We differentiate between two cases. First, the prior belief may be such that the consumer will not process any information about the new product and thus will not consider buying the product. The optimal reply of the new competitor in this case is either not to enter the market or to set a transparent price, which in turn will lead to zero equilibrium profits.²⁴ Thus, consumers' prior belief creates an additional source of a market entry barrier in a market with rational inattentive consumers. Besides a high expected price, this scenario may also arise if the consumer thinks the new product duplicates an existing one, i.e., yielding the same payoff, in contrast to the standard

²⁴The necessary and sufficient condition provided by Caplin et al. (2019) can be used as a simple test to determine whether the consumer would consider the new product or not.

logit model (Matejka and McKay, 2014; Caplin et al., 2019). In our analysis, we assume a second case in which the consumer's prior belief is such that the consumer finds it optimal to process some information about all products if all firms obfuscate, $\eta_i^0 > 0$, $\forall i$.

If two or more firms choose a transparent pricing scheme, a Bertrand type of situation occurs. In the second stage, transparent firms split the entire market, resulting in zero equilibrium profits. As a result, obfuscating firm(s) would incur a loss of $\pi_i = -\xi$. This implies that $\lambda_i = 0$ is always the best reply for firm i conditional on $\exists \lambda_{-i} = 0$ where $-i \in N \setminus i$. As in the duopoly case (see Proposition 3), there always exists a subgame perfect equilibrium in which all firms do not obfuscate and choose to set a price of zero, $s_i = (\lambda_i = 0, p_i = 0) \quad \forall i \in N$.

For the existence of a pure strategy equilibrium, in which all firms choose to obfuscate, the profits in the joint obfuscation case $(\lambda_i = \lambda, \forall i \in N)$ have to be higher than in the separating cases $(\lambda_i = 0, \lambda_{-i} = \lambda, \quad i = 1, 2, ..., N)$ for all firms. Otherwise, at least one of the firms would have an incentive to deviate, implying that no obfuscation equilibrium exists (see Proposition 4 for the equivalent condition in the duopoly case). Therefore, to determine whether we would expect a new market entry to increase or decrease the occurrence of obfuscation in equilibrium, the effect on firms' profits in the second-stage subgame perfect equilibrium is crucial. The derivation of the subgame perfect equilibria follows the same logic as in the duopoly case. In a market in which all firms obfuscate, the optimal pricing of the firm's strategy results in the following equilibrium profit for firm i (see Appendix C):

$$\pi_i^* = \frac{\lambda}{(1 - \eta_i^*(\mathbf{k}))} \eta_i^*(\mathbf{k}), \quad \forall i \in N$$

A new competitor affects the optimal behavior of firms through two channels. First, as $\eta_i(\mathbf{k}) > 0 \,\forall i \in N$, a new competitor decreases aggregated demand. This decreases the prices and profits of existing firms through increased market competition. In turn, obfuscation becomes less attractive and the obfuscation equilibrium less likely, compared to the duopoly case.

Second, a new market entry also changes the consumer's prior belief and thus alters the optimal information strategy of the consumer. As a result, the choice among existing alternatives may change, and a single existing firm may benefit from a new market entry under rational inattention, in contrast to any random utility model (e.g. Matejka and McKay, 2014). This may also shift the equilibrium in the first stage, potentially in both directions, depending on whether the new information strategy favors the less prominent or more prominent firm.

The outcome depends on the exact shape of the consumer's prior belief, particularly on the correlation of the new competitor's price with the price of existing alternatives. Deriving the exact cutoff levels for different beliefs goes beyond this paper's scope, but consider the following stylized example, in which the likelihood of an obfuscation equilibrium may increase. There are two firms in the market with completely independent prices according to the consumer's prior belief. One option has small variance, and the

other option has thicker tails, implying a high probability that the firm's price is either very low or very high, a risky option. The option with small variance seems more attractive to the consumer, such that obfuscation equilibria only exist for large values of λ . Now, there is a new competitor, which also appears risky to the consumer. Prices of the new competitor are perceived to be negatively correlated with prices of the less prominent firm but independent from the more prominent firm's prices. This negative perceived correlation increases the incentive for the consumer to investigate whether one of the risky options is very cheap, similar to the red bus—blue bus—train example from Matejka and McKay (2014). If, in addition, unconditional choice probabilities and resulting profits are more balanced after market entry, the likelihood of an obfuscation equilibrium may increase, even if aggregated demand in the obfuscation case decreases for existing firms.

In addition, a new competitor may increase the incentive to investigate prices at all. Similar to the duopoly case, equilibrium profits are increasing in η_i^0 , implying that, in the case of only one transparent firm, the transparent firm will again choose the maximum price that will claim the entire market \bar{p} (see Appendix C).²⁵ The higher the attractiveness for the consumer to investigate the obfuscated alternatives, the lower \bar{p} is, and thus the incentive for one particular firm to deviate from the obfuscation equilibrium is lower. As a consequence, the likelihood of an obfuscation equilibrium may increase.

6 Conclusion

This paper examines firms' equilibrium behavior in a duopoly with rational inattentive consumers. Before firms compete on the price of a homogeneous product, firms decide whether to obfuscate the prices or disclose all relevant information. In our motivating example of the mobile subscription market, mobile operators may obfuscate prices by using add-on pricing schemes with variable fees for the service's components, such as diverging fees for different operators and call time, non-linear prices for data usage, or varying roaming fees. Compared to a fixed-fee pricing scheme, assessment of the final prices is more complicated. For Sweden, we document a shift from variable add-on pricing schemes to fixed-fee contracts and decreased overall prices. Following our theoretical result, such a development may be explained by a digitalization-induced reduction of consumers' information cost, limiting the impact of firms' obfuscation behavior.

The presence of rational inattentive consumers may make it rational for firms to obfuscate prices in equilibrium. However, mutual obfuscation equilibria with high prices and profits only exist if consumers' information costs of learning about obfuscated prices are high enough. If information costs are low, the rent from obfuscation is small, and firms have an incentive to deviate from obfuscation in equilibrium.

²⁵This finding only holds true under certain conditions that are always fulfilled if we assume that, conditional on not choosing the transparent option, the other products are a priori homogeneous, or, less restrictively, if increasing the "prominence," i.e., unconditional choice probabilities, of firm 1 decreases the prominence of both other firms.

Obfuscation is not a mutually best reply anymore, and the unique equilibrium will be one with completely transparent prices and equilibrium profits of zero. Furthermore, we show that firms' mutual obfuscation in equilibrium is less likely if the consumer perceives an offer by one firm to be superior to other offers according to the consumer's prior belief.

Despite attentional costs for consumers, obfuscation has negative welfare implications, as it implies decreased competition in prices among firms. For these reasons, policymakers may want to limit obfuscation. According to our findings, this can be achieved by decreasing consumers' information costs. Potential policy measures include fostering the creation of neutral product comparison portals, ensuring consumers' access to digital services, and increasing comparability between products, such as forbidding some price dimensions that are difficult to access or standardizing product descriptions. Furthermore, new competitors may decrease obfuscation behavior in equilibrium, not only because of decreased market power of existing firms but also because differences in prominence between existing firms and new competitors alter consumers' information strategy and, thus, the optimal obfuscation behavior by firms. Therefore, lowering market entry barriers may have two positive effects on social welfare.

Our study has several limitations and implications that open several avenues for future research. First, firms' obfuscation decision in the first stage is binary, and consumers' information costs to learn about obfuscated prices are exogenously given, as well as homogeneous across firms and consumers. While this allows tractable results, it may be worthwhile to allow for more flexibility in these respects to assess questions about the fragmentation of markets and different scopes of obfuscation. Huettner et al. (2019) provide a suitable framework for the consumer side that can be applied.

Second, we consider a static model. Extending our framework to a dynamic setup would allow us to study the effects of memory and learning under rational inattention on market equilibria and optimal obfuscation behavior over time. Steiner et al. (2017) and Maćkowiak et al. (2018) study rational inattention in a dynamic framework. While dynamic applications of rational inattention in industrial organization are rare, this approach seems particularly promising in markets characterized by a dynamic environment.

Third, our empirical analysis is stylized. The market for mobile subscriptions is heavily affected by new technologies in different dimensions, on both the demand and supply sides. A thorough examination of the market is needed to precisely estimate the effects of information costs and digitalization on market structure and prices. Generally, the effect of digitalization on economic activity is an empirical question that has received a lot of attention from scholars in recent years (e.g., Goldfarb and Tucker, 2019). Combining the insights from the rational inattention literature with empirical work on these issues can help to better identify the decisive role of information in the far-reaching digital transformation process that reshapes many disciplines, including economics.

References

- Jason Abaluck and Jonathan Gruber. Evolving choice inconsistencies in choice of prescription drug insurance. The American Economic Review, 106(8):2145–84, 2016.
- Simon P Anderson, Andre De Palma, and Jacques François Thisse. Discrete choice theory of product differentiation. MIT press, 1992.
- Mark Armstrong. Ordered consumer search. Journal of the European Economic Association, 15(5): 989–1024, 2017.
- Mark Armstrong and Jidong Zhou. Paying for prominence. *The Economic Journal*, 121(556):F368–F395, 2011.
- Mark Armstrong, John Vickers, and Jidong Zhou. Prominence and consumer search. *The RAND Journal of Economics*, 40(2):209–233, 2009.
- Joseph Bertrand. Théorie mathématique de la richesse sociale. *Journal des Savants*, 67(1883):499–508, 1883.
- Tamer Boyacı and Yalçın Akçay. Pricing when customers have limited attention. *Management Science*, 64(7):2995–3014, 2017.
- Jeffrey R Brown and Austan Goolsbee. Does the internet make markets more competitive? Evidence from the life insurance industry. *Journal of Political Economy*, 110(3):481–507, 2002.
- Jennifer Brown, Tanjim Hossain, and John Morgan. Shrouded attributes and information suppression: Evidence from the field. *The Quarterly Journal of Economics*, 125(2):859–876, 05 2010.
- Erik Brynjolfsson and Michael D Smith. Frictionless commerce? A comparison of internet and conventional retailers. *Management Science*, 46(4):563–585, 2000.
- Andrew Caplin, Mark Dean, and John Leahy. Rational inattention, optimal consideration sets, and stochastic choice. The Review of Economic Studies, 86(3):1061–1094, 2019.
- Bruce I Carlin. Strategic price complexity in retail financial markets. *Journal of Financial Economics*, 91(3):278–287, 2009.
- Ioana Chioveanu. Prominence, complexity, and pricing. *International Journal of Industrial Organization*, 63:551–582, 2019.
- Ioana Chioveanu and Jidong Zhou. Price competition with consumer confusion. *Management Science*, 59(11):2450–2469, 2013.

- European Commission. Electronic communications market indicators collected by commission services, through national regulatory authorities, for the communications committee (cocom) january and july reports. 2019.
- Glenn Ellison. A model of add-on pricing. The Quarterly Journal of Economics, 120(2):585-637, 2005.
- Glenn Ellison and Sara Fisher Ellison. Search, obfuscation, and price elasticities on the internet. *Econometrica*, 77(2):427–452, 2009.
- Glenn Ellison and Alexander Wolitzky. A search cost model of obfuscation. The RAND Journal of Economics, 43(3):417–441, 2012.
- Xavier Gabaix and David Laibson. Shrouded attributes, consumer myopia, and information suppression in competitive markets. *The Quarterly Journal of Economics*, 121(2):505–540, 05 2006.
- Avi Goldfarb and Catherine Tucker. Digital economics. *Journal of Economic Literature*, 57(1):3–43, 2019.
- Michael D Grubb. Failing to choose the best price: Theory, evidence, and policy. *Review of Industrial Organization*, 47(3):303–340, 2015.
- Benjamin R Handel. Adverse selection and inertia in health insurance markets: When nudging hurts. The American Economic Review, 103(7):2643–82, 2013.
- Benjamin R Handel and Jonathan T Kolstad. Health insurance for "humans": Information frictions, plan choice, and consumer welfare. *The American Economic Review*, 105(8):2449–2500, 2015.
- Paul Heidhues, Botond Kőszegi, and Takeshi Murooka. Inferior products and profitable deception. *The Review of Economic Studies*, 84(1):323–356, 08 2016.
- Paul Heidhues, Johannes Johnen, and Botond Koszegi. Browsing versus studying: A pro-market case for regulation. *The Review of Economic Studies*, forthcoming.
- Tanjim Hossain and John Morgan. ... plus shipping and handling: Revenue (non) equivalence in field experiments on ebay. Advances in Economic Analysis & Policy, 5(2), 2006.
- Frank Huettner, Tamer Boyacı, and Yalçın Akçay. Consumer choice under limited attention when alternatives have different information costs. *Operations Research*, 2019.
- Bartosz Maćkowiak, Filip Matějka, and Mirko Wiederholt. Dynamic rational inattention: Analytical results. *Journal of Economic Theory*, 176:650–692, 2018.
- M Susan Marquis and Martin R Holmer. Alternative models of choice under uncertainty and demand for health insurance. The Review of Economics and Statistics, pages 421–427, 1996.

- Daniel Martin. Strategic pricing with rational inattention to quality. Games and Economic Behavior, 104:131–145, 2017.
- Keith M Marzilli Ericson. Consumer inertia and firm pricing in the medicare part d prescription drug insurance exchange. American Economic Journal: Economic Policy, 6(1):38–64, 2014.
- Filip Matějka and Alisdair McKay. Simple market equilibria with rationally inattentive consumers. *The American Economic Review*, 102(3):24–29, 2012.
- Filip Matejka and Alisdair McKay. Rational inattention to discrete choices: A new foundation for the multinomial logit model. *The American Economic Review*, 105(1):272–298, 2014.
- Filip Matějka. Rationally inattentive seller: Sales and discrete pricing. *The Review of Economic Studies*, 83(3):1125–1155, 2015.
- OECD. Working party on communication infrastructures and services policy, wireless market structures and network sharing. Organisation for Economic Co-operation and Development, Directorate for Science, Technoology, and Innovation. Committee on Digital Economy Policy., 2015.
- Eugene Orlov. How does the internet influence price dispersion? Evidence from the airline industry. *The Journal of Industrial Economics*, 59(1):21–37, 2011.
- Michele Piccione and Ran Spiegler. Price competition under limited comparability. *The Quarterly Journal of Economics*, 127(1):97–135, 2012.
- Bjørn-Atle Reme, Helene Lie Røhr, and Morten Sæthre. The poking effect: Price changes, information, and inertia in the market for mobile subscriptions. 2018.
- Andrew Rhodes. Can prominence matter even in an almost frictionless market? *The Economic Journal*, 121(556):297–308, 2011.
- Steven Salop and Joseph Stiglitz. Bargains and ripoffs: A model of monopolistically competitive price dispersion. *The Review of Economic Studies*, 44(3):493–510, 1977.
- Carl Shapiro. Consumer information, product quality, and seller reputation. The Bell Journal of Economics, pages 20–35, 1982.
- Christopher A Sims. Implications of rational inattention. *Journal of Monetary Economics*, 50(3):665–690, 2003.
- Dennis E Smallwood and John Conlisk. Product quality in markets where consumers are imperfectly informed. The Quarterly Journal of Economics, 93(1):1–23, 1979.
- Ran Spiegler. Choice complexity and market competition. Annual Review of Economics, 8:1–25, 2016.

- Dale O Stahl. Oligopolistic pricing with sequential consumer search. *The American Economic Review*, pages 700–712, 1989.
- Jakub Steiner, Colin Stewart, and Filip Matejka. Rational inattention dynamics: Inertia and delay in decision-making. *Econometrica*, 85(2):521–553, 2017.
- Joseph E Stiglitz. Imperfect information in the product market. *Handbook of Industrial Organization*, 1: 769–847, 1989.
- Greg Taylor. Raising search costs to deter window shopping can increase profits and welfare. *The RAND Journal of Economics*, 48(2):387–408, 2017.
- Peter W Turnbull, Sheena Leek, and Grace Ying. Customer confusion: The mobile phone market. *Journal of Marketing Management*, 16(1-3):143–163, 2000.
- Hal R Varian. A model of sales. The American Economic Review, 70(4):651–659, 1980.
- Chris M Wilson. Ordered search and equilibrium obfuscation. *International Journal of Industrial Organization*, 28(5):496–506, 2010.
- Susan E Woodward and Robert E Hall. Consumer confusion in the mortgage market: Evidence of less than a perfectly transparent and competitive market. *The American Economic Review*, 100(2):511–15, 2010.

Xearly Producer Price Index

90

2014

2016

2018

2020

Figure A.1: Producer Price Index

Note: The figure presents the yearly producer price index for telecommunication services in Sweden between 2014 and 2020. The baseline of the index is in the year 2015.

Appendix

A Producer Price Index and Average Revenues

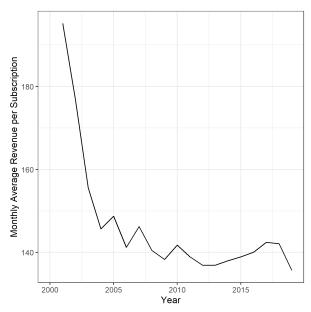
Our main empirical analysis argues that the decrease in the consumer price index in telecommunication services and equipment is a clear indicator of decreasing prices. Within this section, we show that the result is similar when evaluating the producer price index or average revenues. Figure A.1 shows the yearly producer price index for telecommunication services in Sweden between 2014 and 2020. Figure A.2 presents monthly average revenues in the mobile subscription market between 2000 and 2018. Similar to the results for the consumer price index, the producer price index and the average revenues decrease.

B Solving the Consumer Maximization Problem

Solving the consumer maximization problem, we mainly follow Matejka and McKay (2014). Formally, in the non-trivial cases the consumer chooses an information strategy to solve the following maximization problem:

$$\max_{F \in \Delta(\mathbb{R}^{2N})} \int_{\mathbf{k}} \int_{\mathbf{s}} E(k_{a(F(\mathbf{k}|\mathbf{s}))}) F(d\mathbf{s}|\mathbf{k}) \Omega_0(d\mathbf{k}) - \hat{c}(F)$$
s.t.
$$\int_{\mathbf{s}} F(d\mathbf{s}, \mathbf{k}) = \Omega_0(\mathbf{k})$$

Figure A.2: Monthly Average Revenue



Note: The figure presents the monthly average revenue per subscriber in the Swedish mobile subscription market between 2000 and 2015. Average revenues are adjusted for inflation and in prices of 2000.

Let S_i further be the set of signals **s** that result in action strategy a:

$$S_i = \{ \mathbf{s} \in \mathbb{R}^N : a(F(\mathbf{k}|\mathbf{s})) = i \},$$

where N is the number of options available to the consumer. Building upon this, we define the conditional probability of selecting action i depending on state \mathbf{k} :

$$\eta_i(\mathbf{k}) \equiv \int_{s \in S_i} F(d\mathbf{s}|\mathbf{k})$$

The unconditional choice probabilities before observing the costly signal but after processing all costless information is given by the following:²⁶

$$\eta_i^0 \equiv \int_{\mathbf{k}} \eta_i(\mathbf{k}) \Omega_0(d\mathbf{k}) \tag{4}$$

It can be shown that, due to convex information costs, one action strategy is selected in at most one posterior (Matejka and McKay, 2014). This implies that each action strategy is associated with a particular signal, which in turn allows us to rewrite the maximization problem of the consumer in terms of choice probabilities, which facilitates the derivation of the respective demand. Without loss of generality we restrict the derivation of the conditional demand to two options, i.e., two products the consumer can

²⁶Again, what is relevant to our problem is not the unconditional choice probability before any information $G(\cdot)$ is processed, but the unconditional choice probability after all costless information is processed.

choose from. Using the definition of Shannon entropy, our maximization problem then reads:

$$\max_{\eta \in \{\eta_i(\mathbf{k})\}_{i=1}^2} \sum_{i=1}^2 \int_{\mathbf{k}} k_i \eta_i(\mathbf{k}) \Omega_0(d\mathbf{k}) - \lambda(-\sum_{i=1}^2 \eta_i^0 log \eta_i^0 + \int_{\mathbf{k}} \sum_{i=1}^2 \eta_i(\mathbf{k}) log \eta_i(\mathbf{k})) \Omega_0(d\mathbf{k})$$
 (5)

such that

$$\eta_1(\mathbf{k}) + \eta_2(\mathbf{k}) = 1 \tag{6}$$

$$\forall i: \quad \eta_i(\mathbf{k}) \ge 0, \tag{7}$$

where η is the collection of conditional probabilities $\{\eta_i(\mathbf{k})\}_{i=1}^2$. The maximization problem is applicable to all cases in which at least one firm chooses to obfuscate its prices. However, the cases differ with respect to the relevant (conditional) prior beliefs $\Omega_0(\mathbf{k})$.

Both Firms Obfuscate

In the case in which both firms obfuscate ($\lambda_1 = \lambda_2 = \lambda$), the consumer chooses an information strategy based on $G_0(k_2|k_1 = \Omega_0(k_2))$ and homogeneous information costs across both products. Choosing an informative signal, she will thus consider the joint distribution about both prices.²⁷

Solving the corresponding Lagrangian, where $\zeta(\mathbf{k})$ signifies the Lagrange multipliers on (4) and $\tau_i(\mathbf{k})$ are Lagrange multipliers on (5), gives us the following first-order conditions (assuming an interior solution $\eta_i^0 > 0$):

$$k_i - \lambda \left(-log(\eta_i^0) - 1 + log(\eta_i(\mathbf{k}) + 1) + \tau_i(\mathbf{k}) - \zeta(\mathbf{k}) = 0 \right)$$

Matejka and McKay (2014) show that if $\eta_i^0 > 0$ and $k_i > -\infty$, it has to hold that $\eta_i(\mathbf{k}) > 0$, which in turn implies that the Lagrange multiplier on (6) is zero, $\tau_i(\mathbf{k}) = 0$. Taking the exponential of both sides and rearranging the first-order condition results in:

$$\eta_i(\mathbf{k}) = \eta_i^0 e^{(k_i - \zeta(\mathbf{k}))/\lambda} \tag{8}$$

Plugging (7) into (5) gives us:

$$e^{\zeta(\mathbf{k})/\lambda} = \sum_{i=1}^{2} \eta_i^0 e^{k_i/\lambda},$$

which in turn can be plugged back into (7) to arrive at our demand function (1) for $0 < \eta_i^0 < 1$. Note that η_i^0 is not only determined by the prior beliefs but is rather a result of the maximization problem itself. We plug the conditional choice probabilities (1) into the definition of the unconditional choice probabilities

²⁷This makes our problem directly applicable to the framework studied by Matejka and McKay (2014).

(3) to arrive at the normalization conditions that allow us to numerically solve for the unconditional choice probabilities:

$$\int_{\mathbf{k}} \left(\frac{e^{k_i/\lambda}}{\sum_{j=0}^2 \eta_j^0 e^{k_j/\lambda}} \right) \Omega_0(d\mathbf{k}) = 1, \quad \forall i \eta_i^0 > 0$$

If $\exists \eta_i^0 = 0$, then $\eta_i(\mathbf{k}) = 0$. This implies that the consumer does not process any information about i and, in the duopoly case, always buys the product of the competitor. Caplin et al. (2019) provide the necessary and sufficient boundary conditions for this case that can be applied to our simple two-product setting.

Only One Firm Obfuscates

The logic of calculating the choice probabilities if only one firm obfuscates is similar. Note that we assume here without loss of generality that $\lambda_1 = 0$ and $\lambda_2 = \lambda$. However, when deciding how much information to process (about firm 2's price), the consumer knows the price of product 1. This has some implications for the relevant conditional prior belief that is given by $\Omega_0(k_2) = G_0(k_2|k_1)$, implying that the uncertainty is one-dimensional, considering only product 2's price.²⁸

Plugging constraint (5) into the maximization problem above, noting that k_1 is perfectly known by the consumer, and assuming $0 < \eta_2^0 < 1$, the Lagrangian for the maximization problem of the consumer in the case where only one firm obfuscates can be rewritten to:

$$\max_{\eta_2(\mathbf{k})} \int_{k_2} k_2 \eta_2(\mathbf{k}) \Omega_0(dk_2) + k_1 \int_{k_2} (1 - \eta_2(\mathbf{k})) \Omega_0(dk_2)$$

$$-\lambda \left(-\eta_2^0 log \eta_2^0 - (1 - \eta_2^0) log (1 - \eta_2^0) + \int_{k_2} (\eta_2(\mathbf{k}) log \eta_2(\mathbf{k}) + (1 - \eta_2(\mathbf{k})) log (1 - \eta_2(\mathbf{k}))) \Omega_0(dk_2) \right)$$

Differentiating with respect to $\eta_2(\mathbf{k})$ and noting that $\eta_2(\mathbf{k}) > 0$ almost surely if $\eta_2^0 > 0$ and $k_2 > -\infty$ (see Matejka and McKay, 2014) gives us the following first-order condition:²⁹

$$(k_2 - k_1) - \lambda \left(-log(\eta_2^0) - 1 + log(1 - \eta_2^0) + 1 + log(\eta_2(\mathbf{k})) + 1 - log(1 - \eta_2(\mathbf{k})) - 1 \right) = 0$$

Combining the terms and taking the exponential on both sides gives us:

$$e^{(k_2-k_1)/\lambda} = \frac{1-\eta_2^0}{\eta_2^0} \frac{\eta_2(\mathbf{k})}{1-\eta_2(\mathbf{k})}$$

 $^{^{28}}$ Note that if the price of product 1 is low enough compared to the expected price of firm 2, the consumer will not process any information about product 2 and will just buy product 1. Similarly, if the expected price of product 2 is lower than p₁, the consumer will pay no attention and will just buy (uncertain) product 2. This case's logic is similar to that of the case in which the consumer can decide to buy a product/enter the market or choose an outside option with a certain outcome. A similar setting is discussed in Matejka and McKay (2014) or Boyacı and Akçay (2017) for a binary state variable and corresponding prior belief that follows a Bernoulli distribution. ²⁹Note that $\eta_i^l \equiv \int_{\mathbf{k}} \eta_i(\mathbf{k}) G_l(d\mathbf{k})$.

Rearranging and adding 1 (respectively $\frac{(1-\eta_2^0)}{(1-\eta_2^0)}$ and $\frac{(1-\eta_2^0)e^{k_1/\lambda}}{(1-\eta_2^0)e^{k_1/\lambda}}$) on both sides gives us:

$$\frac{1}{1 - \eta_2(\mathbf{k})} = \frac{(1 - \eta_2^0)e^{k_1/\lambda} + \eta_2^0 e^{k_2/\lambda}}{(1 - \eta_2^0)e^{k_1/\lambda}}$$

Using the definition of k_i gives us our final demand:

$$\eta_2(\mathbf{k}) = \frac{\eta_2^0 e^{k_2/\lambda}}{(1 - \eta_2^1) e^{k_1/\lambda} + \eta_2^0 e^{k_2/\lambda}}$$

$$\eta_1(\mathbf{k}) = 1 - \eta_2(\mathbf{k})$$

The normalization condition gives us our unconditional choice probabilities:

$$\int_{k_2} \left(\frac{e^{k_2/\lambda}}{\eta_2^1 e^{k_2/\lambda} + (1 - \eta_2^1) e^{k_1/\lambda}} \right) \Omega_1(dk_2) = 1$$

.

C Derivation of Subgame Perfect Equilibria with N Firms

In a market in which all firms obfuscate, the optimal information strategy results in a generalized multinomial logit demand function for all three products: 30

$$\eta_i(\mathbf{k}) = \frac{\eta_i^0 e^{(q-p_i)/\lambda}}{\sum_{j=1}^3 \eta_j^0 e^{(q-p_j)/\lambda}}, \quad \forall i \in N$$

Following Anderson et al. (1992) and as in the duopoly, the subgame perfect equilibria are uniquely determined by following system of equations:

$$p_i^* = c + \frac{\lambda}{(1 - \eta_i^*(\mathbf{k}))}, \quad \forall i \in N$$

with equilibrium profits of

$$\pi_i^* = \frac{\lambda}{(1 - \eta_i^*(\mathbf{k}))} \eta_i^*(\mathbf{k}), \quad \forall i \in N$$

This proposition and proof are the generalization of Proposition 2 to a market with N competing firms.

Proposition 6. With N-1 obfuscating firms (-i) and one transparent firm (i), the optimal price of the transparent firm $(\lambda_i = 0)$ is equal to \bar{p}_i , where $\bar{p}_i = \max p_i$ such that $\eta_i^0 = \eta_i(\mathbf{k}) = 1$.

Proof. As in Proposition 2, note that the transparent firm will not set a price below \bar{p}_i , as $\frac{\partial \pi_i}{\partial p_i} = 1 > 0$ if $p_i < \bar{p}_i$, $\eta_i(\mathbf{k}) = 1$.

³⁰See Appendix A for the derivation.

All firms will face a multinomial logit demand if $p_i \geq \bar{p_i}$ (see above). The maximized profit function for a given unconditional choice probability is equal to $\pi_i^*(\eta_i^0) = \frac{\lambda}{(1-\eta_i^*(\mathbf{k}))} \eta_i^*(\mathbf{k})$. Without loss of generality, we assume N=3 and that firm 1 is using the transparent pricing scheme.

Plugging in the corresponding demand function, the profit function of firm 1 can be rewritten to:

$$\pi_1^*(\eta_i^0) = \lambda \frac{\eta_1^0 e^{(q-p_1^*)/\lambda}}{\eta_2^0 e^{(q-p_2^*)/\lambda} + \eta_3^0 e^{(q-p_3^*)/\lambda}} = \lambda \frac{\eta_1^0 e^{-p_1^*/\lambda}}{\eta_2^0 e^{-p_2^*/\lambda} + \eta_3^0 e^{-p_3^*/\lambda}}$$

If we assume that, conditional on not choosing the transparent option (option 1), the other products are a priori homogeneous, implying equal unconditional choice probabilities (η_i^0) and optimal prices (see, e.g., Matějka and McKay, 2012). Then, the unconditional choice probabilities can be rewritten as $\eta_2^0 = \eta_3^0 = \frac{(1-\eta_1^0)}{2}$. This simplifies the maximized profit function to:

$$\pi_1^*(\eta_i^0) = \lambda \frac{\eta_1^0 e^{(q-p_1^*)/\lambda}}{\frac{(1-\eta_1^0)}{2} \left(e^{(q-p_2^*)/\lambda} + e^{(q-p_3^*)/\lambda}\right)} = 2\lambda \frac{\eta_i^0}{(1-\eta_i^0)} \frac{e^{(-p_1^*)/\lambda}}{\left(e^{(-p_2^*)/\lambda} + e^{(-p_3^*)/\lambda}\right)}$$

Applying the envelope theorem, differentiating with respect to η_1^0 equals:

$$\frac{\partial \pi_i^*}{\partial \eta_i^0} = \frac{2\lambda}{(1-\eta_1^0)^2} \frac{e^{(-p_1^*)/\lambda}}{\left(e^{(-p_2^*)/\lambda} + e^{(-p_3^*)/\lambda}\right)} > 0,$$

which is strictly larger than zero if $\eta_1^0 < 1$.

If the options are not a priori homogeneous, the derivate of the maximized profit function with respect to η_1^0 is given by:

$$\frac{\partial \pi_1^*}{\partial \eta_1^0} = \lambda \frac{e^{(-p_1^*)/\lambda} \left((\eta_2^0 e^{(-p_2^*)/\lambda} + \eta_3^0 e^{(-p_3^*)/\lambda}) - \eta_1^0 \left(\frac{\partial \eta_2^0}{\partial \eta_1^0} e^{(-p_2^*)/\lambda} + \frac{\partial \eta_3^0}{\partial \eta_1^0} e^{(-p_3^*)/\lambda} \right) \right)}{(\eta_2^0 e^{(-p_2^*)/\lambda} + \eta_3^0 e^{(-p_3^*)/\lambda})^2}$$

This is always larger than zero if $\left(\frac{\partial \eta_2^0}{\partial \eta_1^0} e^{(-p_2^*)/\lambda} + \frac{\partial \eta_3^0}{\partial \eta_1^0} e^{(-p_3^*)/\lambda}\right) \le 0$, which always holds if we assume that increasing the "prominence," i.e., unconditional choice probabilities, of firm 1 decreases the prominence of both other firms.³¹

³¹This assumption can further be relaxed by considering the entire numerator.