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THE ECONOMICS OF LEARNING:
Price Formation when Acquisition of Information is Possible but Costly

by
Bo Axell

This is a preliminary paper. Comments are welcome.

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THE ECONOMICS OF LEARNING: PRICE FORMATION WHEN ACQUISITION OF INFORMATION IS POSSIBLE BUT COSTLY

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Abstract

This paper presents some analyses of what would be the outcome of a price formation process in the presence of information costs. First, we examine the equilibria which obtain in search markets where consumers are uninformed about prices but informed about the distribution of prices. Second, we examine the case when consumers are not informed about the distribution of prices, and conclude that the equilibrium might differ considerably. Third, we show that in the case when firms are not informed about their profit opportunities, and therefore have to undertake experiments to find these out, a price dispersion equilibrium is locally stable, given reasonably rational experimental procedures.

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* I have benefitted from constructive advises from Harald Lang, in particular concerning the proof of the stability theorem; Proposition 1.

1. INTRODUCTION

The starting point for this paper is the contributions from Margret Bray and others about the learning process of rational expectation equilibrium. I will discuss the questions starting from my knowledge of how Bray (and others) have approached the problem in earlier contributions. Thomas Lindh presents in the seminar an overview of some of these articles. I will make some remarks about what I think is a more general aproach to the problem.

The main question in MB's work is: If a market is in disequilibrium, under what conditions will a learning mechanism function such that the prices converge to a competetive rational expectational equilibrium?

The question is very much like those which ask if there is any kind of mechanism that leads to the Arrow–Debreau equilibrium, namely the question of stability of the Walrasian equilibrium. This question can be divided into two questions, 1. local stability and 2. global stability. By local stability we mean that the market will return to equilibrium after a minor disturbance. Global stability refers to a scenario in which the market will approach equilibrium from a situation far away from equilibrium.

If a market is out of equilibrium, then the sellers of the market have to guess what will be the price on the commodity in next period. Then, knowing their cost function, they decide how much to produce. They will then enter the market place with their supply where there are demanders (customers) which are demanding in accordance with a demand curve

$$D(p) = m_1 - m_2 \cdot p.$$

The market clearing mechanism will then determine the price that makes the demand equal to the supply. The (unexplained) market maker (auctioneer) will then also coordinate those who are willing to supply with those who are willing to demand (free of charge).

My starting point is different and more general. What are the equilibria that will be the end point of a process that relies on a reasonably rational information gathering process?

In an Arrow–Debreau type of an economy there is no room for any type of learning since all markets (including future markets) equilibrate simultaneously. Hence there is simultaneous information about all future markets.

In a Lucas–type market there <u>is</u> a need for expectations since markets clear only one at the time. There are no future markets. Therefor expectations about future prices are of fundamental importance.

However, if there are no markets at all and the contact between agents must be on a bilateral basis (ie sellers and buyers must contact each other themselves) then we must ask what would be the process of price and quantity outcome and will that process at all converge to an equilibrium?

The works in the field of equilibrium of a search product market includes Diamond (–71), Axell (–74, –76, –77), Hey (–74), Salop, Stiglitz (–77), Reinganum (–79), McMinn (–80), Burdett, Judd (–83), Rob (–85) and Caron–Salmona, Lesourne (–87).

The simultanous equilibrium in product and labor markets is analyzed in Albrecht, Axell (–84) and in Albrecht, Axell and Lang (–86).

2. CONVERGENCE TO EQUILIBRIUM WITHOUT AN AUCTIONEER.

Let us assume a market that has been exposed to a shock of one kind or another (ie technical change – shifting the supply curve –, or a change in taste – shifting the demand curve) making the situation far from the conventional. In such a situation suppliers have very little to go on when estimating the "market price" which will prevail. They form their own predictions of a reasonable price for their commodity. If they offer their product to consumers at this price, then we must ask if the process of price and quantity change will be such that it will approach equilibrium and in particular if this equilibrium is the competitive rational expectation equilibrium.

If the equilibrium is of some other kind, we must then ask what does that equilibrium looks like and what is involved in the convergence process.

The disequilibrium situation after a considerable shock in the market can be described as a situation in which many suppliers are offering different prices – a situation with price dispersion which can be characterized as a situation with a price distribution of the price offers.

Now we have to characterize the price offer constraints. We will assume that every meeting is constrained such that the seller is offering a price on a "take it or leave it" basis. This means that the seller decides a price that he will offer. The consumer regards this as a "take it or leave it" offer and decides whether or not to accept it. If he finds it acceptable he accepts to buy the desired amount. If he does not find it acceptable he rejects it and starts trying to find another seller supplying the commodity in question. There is assumed to be no further negotiation. Samuelson (–84) showed that this is an optimal bargaining procedure. There are (infinitely) many sellers in this same position. Hence a general description of such a situation is a continous distribution of price offers.

However, starting from a price dispersion situation, it would give rise to a search process of the following kind.

Sellers are giving price offers and these price offers differ. The price offer situation can be described with a price offer distribution f(p), see fig.1 below. Note in particular that we assume that the sellers (or the buyers) cannot take help in the price formation process from any kind of "auction market mechanism", they have to find trading partners by means of their own efforts.

Now look at the situation in the period just after the shocks to the market. Different firms will have different predictions about what would be the best price offer. An arbitrary description of the sitation in that (disequilibrium) situation in that market would be to say that different firms are offering different prices. Hence we can summarise the situation with help of a price offer distribution.

Let us name this f(p) (the density function. We name the cumulative distribution function F(p)).

Let us assume that this function in this first period disequilibrium situation looks like the one in fig. 1. below.

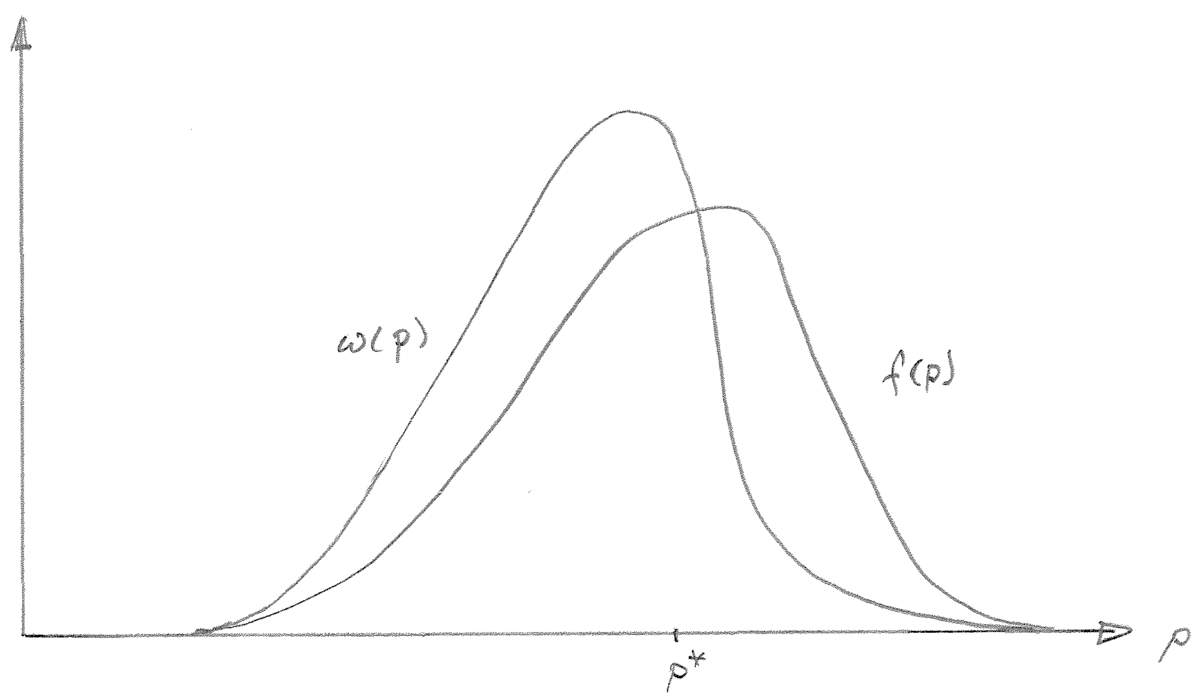

Fig. 1. The distribution of price offers.

It is easy to understand that if there are costs to collect information, then anyone who has low search costs would reject an offer like p^* in the distribution, while someone with very high search costs would accept.

If consumers face a situation in which different fims offer different prices, then the question is this; at what price(s) will consumers agree to buy? The answer is that it depends on a number of factors. What information do they have, eg about the distribution of price offers? What are their expectations about the change in this distribution in the future? What is the effort or cost to come into contact with an other firm? How does that cost differ between consumers? It is obvious that a price that a consumer would accept is a price low enough that the consumer does not find it worthwhile to try to find a lower price.

How do we specify such an acceptance rule? Of course we have to define a number of assumptions. One possible set of assumptions is this:

1. The consumers plan to buy exactly one unit of the commodity, regardless of how low the price offer may be.

2. The consumers know the actual distribution of price offers but do not know which firm is offering which price.

3. They expect the price offer distribution to remain the same for, at least, a reasonable duration so that considerations about its change will not influence their termination decision.

4. They have no costs of calculation of behavior, eg optimal termination.

Given these assumptions, the search process will be a reservation price search process.

If we describe the price offer distribution with F(p) for the distribution function (the cumulative distribution) and f(p) for the density function, the expected gain from further search for a consumer $\psi(r)$, given that he has found a firm with the offer r is;

$$\psi(r) = \int_0^r (r-p)\cdot f(p)dp.$$

If the marginal cost of search is c, then the optimal rule for the search behaviour is such that if $\psi(r) \geq c$ then continue search, while if $\psi(r) < c$ then accept r.

This means that the search behavior, under these assumptions, has the satisficing behaviour rule to calculate r in accordance with the solution to the equation:

$\psi(r) = c$, or hence;

$$\int_0^r (r-p)\cdot f(p)dp = c.$$

Solving r from this equation will yield the reservation price; any offer above such price should be rejected and the very first price offer found below that should be accepted.

This means of course that, given any reasonable search cost distribution, the distribution of actual purchases will be biased to the left in relation to the price offer distribution. Hence firms offering relatively low prices on average will get more accepting consumers than those firms offering relatively high prices.

Hence there will be a negatively sloping demand curve of the ordinary kind – firms charging low prices will on average get relatively more accepting customers than firms offering high prices.

The question is now, however, whether we can construct an equilibrium model that will maintain this as an equilibrium outcome, or if the market will collapse to a single price equilibrium.

Now, consider the firms situation. If a firm charges a high price, ie a price in the right end of the distribution, it will sell only to those who have very high search costs. If a firm, on the other hand, offers a low price, ie a price in the left end of the distribution, a majority of consumers contacting him will accept and buy. The explaination of this is that the presence of search costs make lower price offers more acceptable than high offers.

Given a known distribution of search costs $\gamma(c)$, it is possible to have a price dispersion in equilibrium. To show this we first note that demand, according to reasons explained above, is greater at low prices than at high prices. This gives rise to a negatively sloping demand curve. However the question is whether or not there could be an equilibrium with price dispersion on this foundation.

The result given in Axell (–77) is the following:

The conditions on $\gamma(c)$ for an equilibrium is:

1. γ must be negatively sloping and convex.

2. $\gamma(c) \to 0$ when $c \to \infty$
   $\gamma(c) \to \infty$ when $c \to 0$.

3. $\frac{\gamma(c)^{3/2}}{\gamma\ (c)}$ is decreasing.

4. $\lim_{c \to \infty} \frac{\gamma(c)^{3/2}}{\gamma\ (c)} = -\frac{\sqrt{B}}{2}$

5. $\lim_{c \to 0} \frac{\gamma(c)^{3/2}}{\gamma\ (c)} = 0.$

where B is a positive constant.

(The derivation of these results can be found in Axell(–76 and –77)

This means that the search cost density must approach both axes, must be negatively sloped and convex and that the degree of convexity must fullfill some conditions. This is in the case when there will be an equilibrium with all prices above marginal cost being existent (up to infinity). As Rob points out in Rob (–85) it is less demanding if we look for an equilibrium in a limited interval.

However, this means that there is a requirement that the distribution of search costs is not bounded away from zero, ie that at least some searchers have search costs infinitely close to zero.

It is easily shown that, if the distribution of search costs is bounded away from zero, the market will have only the monopoly price as the equilibrium price.

However, there are some important objections to this result. The most important pertain to our assumption about the searchers actual knowledge, in particular the knowledge about the distribution of prices.

A firm charging the <u>lowest</u> price in the market will sell to all the consumers attracted by it (ie those who have found it in their search process). However, if a consumer finds a seller with ϵ higher price and ϵ is greater than his marginal cost of search, then there is no benefit from further search. Hence a firm charging the lowest price in the market might equally well charge an ϵ higher price without losing any customers, though it might lose some sales to these customers because of a decrease in demand due to the ϵ–price increase. Therefore the equilibrium must be the monopoly price.

However, if the consumers do not know the distribution of prices, the equilibrium might very well differ a great deal.

The monopoly price outcome in a market characterized by search and costly information is astonishing. This outcome depends, however, on the special assumption that the price–distribution is known.

If we instead assume that the distribution of prices is not known we get a very different result.

To demonstrate this let us focus on the case when it is unknown what the lowest price offered in the market is.

Then any firm, charging the lowest price in the market, may face a number of consumers that believe that this price may not be the lowest. In addition, some consumers that face firms charging slightly higher prices are not aware of this fact and hence decide to continue search. Then the firms with the very lowest price in the market will have the opportunity to receive the consumers who have previously found sellers with prices almost equal to the lowest prices. If there are many of this type of consumers, the lowest price strategy might be well defended and hence profitable. Therefore a price–dispersion equilibrium could be defended with rational behaviour when there are information costs present.

3. THE LEARNING PROCESS – CONVERGENCE TO PRICE DISPERSION EQUILIBRIUM

The scenario presented above is an equilibrium analysis. We derived a configuration that fullfills the Nash conditions for an equilibrium, given that some information was aviable for free and some was aviable only at a cost.

The information that was avaible for free for the firms was in principle all information; the consumers search strategies, the search cost distribution, the other firms strategies (ie the price distribution) the cost function (for at least the own firm) and hence the profit function (as a function of price).

For consumers the actual distribution of offered prices was known at any given time. The incompleteness of information that was introduced in this analysis had to do with consumer knowledge as to which price was charged by which firm. Consumers must find this out by means of a search process, and there are costs connected to each search step.

We could show that there exists an equilibrium in such a market, given that the search cost distribution fullfills some requirements. However, this analysis did not say anything about the convergence to such an equilibrium configuration from a situation in disequilibrium.

Now, let us consider the question of convergence to equilibrium. We are then in a situation where the economy, and in particular the market, is outside its equilibrium configuration. What are then the forces on this market, and are they such that the market will approach its equilibrium configuration?

The information that agents have for free is still overwhelming in contrast to what is "realistic" in an economy. In our model, the consumers know at all times the true price distribution, and the firms know essentially everything.

We now will investigate whether or not the equilibrium configuration described above could be the endpoint of a learning process, starting in a disequilibrium situation. Then the knowledge for firms and consumers is obviously too much.

We continue this analysis of "convergence while learning" in two parts. The first introduces the assumption that consumers now do not know the distribution of prices and hence have to learn it during the search process, while still firms knows essentially everything. In the second, we assume that a firm does not know the demand (and hence not profit) at other prices than it itself actually charges during the period. To obtain information about demand and profits at other prices it has to undertake experiments with other prices. The consumer side is in this second case kept contant assuming as before that they know the true distribution at every instant.

4. LEARNING THE DISTRIBUTION OF PRICES

Let us first look at the consumer side. In the equilibrium model, we assumed that they knew at every instant of time the distribution of prices and also that the actual distribution would be persistent for ever. This is of course unrealistic. A consumer enters the market with no exact knowledge about what prices are charged in the market. However, he may may have some feelings about prices because of experiences in other similar markets and what he has heard from friends and has seen in the papers.

This "belief" will be called the consumers "prior distribution". We formulate this in terms of the parameters of the distribution. If it is known that the distribution is a Normal distribution, the prior is described by means of its mean and variance.

When the individual then searches offering firms by means of drawing from the urn of price (and/or wage) offers, this process will update his belief about the actual distribution in accordance to a Baysian updating rule.

5. PRICE FORMATION ON PRODUCT MARKETS WHEN INFORMATION IS IMPERFECT

The model presented below pertains to behavior on a homogenous product market where the consumer does not know the mean or the variance of the price distribution, although he does have some prior views of a form to be described. The individual must then make use of the information he receives in the course of searching to form his estimate of what the distribution looks like. When yet another price datum flows in on a certain date, the individual will revise his estimate of that distibution and form a new a posteriori distribution. He must then also consider whether it will be optimal to go on searching or to stop and accept the most favorable offer that is open to him. In other words the individual seeks out price data sequentially and estimates, after each new observation, the expected benefit from continued searching and compares that with the search cost. This gives us an adaptive search model.

The a posteriori Distribution

Let us assume that prices at which the firms in question offer homogeneous capital goods form a normal distribution F(p), whose mean is $\bar{p}_F$ and variance is σ_F^2.

The individual can, we assume, obtain an address list of the relevant firms free of charge (for instance, by consulting the yellow pages in the telephone directory). He cannot know the price any particular firm will ask without first seeking out the firm and negotiating with it. However, activity of this kind will involve a search cost, c.

Even before any firm is contacted, the individual is assumed to have some idea of the variance of the market. This conception may be based on experiences gained from similar markets or on earlier experiences of the same market. We call this initial subjective variance σ_1^2, to which the individual attaches a weight k, showing the degree of confidence he attaches to σ_1^2. The k may be regarded as a coefficient of inadaptability, and may be interpreted as the amount of imagined price data preceding the first actual datum that has provided information about σ_1^2.

After n price data, drawn at random from the address list, the individual forms an a posteriori distribution with the variance

$$\sigma_i^2 = \frac{k\sigma_i^2 + ns^2(p_1 \ldots p_n)}{k + n - 1} \tag{1}$$

where $s^2(p_1 \ldots p_n)$ is the sample variance of the n price observations. The mean in this distribution is formed by the sample mean

$$\bar{p}_{in} = \frac{\sum_{j=1}^{n} p_j}{n} \tag{2}$$

Here I refrain for the time being from introducing any subjective conception of the mean. It follows that the individual will behave as though no other information about the distribution mean were to be had apart from the offers received during the search process. We can interpret this to mean that a very low weight is assigned to the subjective conception of the mean compared with the weight that is assigned to the information provided by the random sample.

Rule for Optimal Stopping

We assume that the individual knows that the prices are normally distributed. At this point either of two basic alternatives present themselves. The first is a search process whereby every offer must either be accepted or rejected and whereby a rejected offer cannot be recalled. The other is a process whereby a rejected offer can be recalled if it later appears advantageous. For a capital goods market such as the market for cars, the latter alternative would appear to be the more realistic, so I elect to analyze it.

After, n drawings n price quotations have been obtained, whose sample variance is pooled with the subjective initial variance into σ_{in}^2 as indicated above. The mean on the market is expected to be identical with the sample mean p_{in}. We call this a posteriori distribution $f_{in}(p)$. One of the prices obtained is the lowest. Let us call this price p_m. The desicion rule assumed to guide the individual's behavior is: If the expected benefit of carrying the search one step further is greater than the search cost he will keep searching; otherwise he stops to accept the lowest existing offer, i.e. p_m.

If we assume a linear utility function in the interval, i.e. the absence of risk aversion, this decision rule may be formulated:

$E(\Delta p) > c \rightarrow$ keep searching.
$E(\Delta p) \leq c \rightarrow$ stop and accept p_m.

The search cost, c, is assumed to remain the same for all search steps.

$E(\Delta p)$ is then p_m minus the expected probability that a price less than p_m will be drawn next, times the expected price given that a price less than p_m will be found, plus expected probabilities that a price greater than p_m will be found times p_m(which from now on will of course remain the most favorable).

$$E(\Delta p) = p_m - \left[\int_{-\infty}^{p_m} f(p) \frac{\int_{-\infty}^{p_m} pf(p)dp}{\int_{-\infty}^{p_m} f(p)dp} + p_m\left[1 - \int_{-\infty}^{p_m} f(p)dp\right]\right]$$

$$E(\Delta p) = p_m - p_m + \int_{-\infty}^{p_m}\{f(p)p_m - f(p)p\}dp$$

$$E(\Delta p) = \int_{-\infty}^{p_m} f(p)(p_m - p)dp$$

Insertion of a normal distribution transposes the foregoing into:

$$E(\Delta p) = \frac{1}{\sigma_{in}2\pi} \int_{-\infty}^{p_m} \exp \frac{-(p-p_{in})^2}{2\sigma_{in}^2}(p_m - p)dp \qquad (3)$$

where accordingly:

$$\sigma_{in}^2 = \frac{k\sigma_i^2 + ns^2(p_1 \ldots p_n)}{k + n - 1}$$

and

$$p_{in} = \frac{\sum_{j=1}^{n} p_j}{n}$$

Number of Search Steps and the Acceptance-price Distribution

At this juncture two questions take on primary interest. First, how many steps will the search process be carried through before being stopped? Second, what price will be accepted at the cutoff point? Both these factors will be stochastically distributed because the optimal stopping date and hence the accepted price will depend on the sequence in which prices have been drawn.

We first investigate the condition for stopping after one drawing. The search process will be stopped if:

$$\frac{1}{\sigma_i 2\pi} \int_{-\infty}^{p_1} \exp \frac{-(p-p_1)^2}{2\sigma_i^2}(p_1-p)dp < c$$

where p_1 is the price obtained.

Solving for the integral results in:

$$\frac{1}{\sigma_i 2\pi} \left[\sigma_i^2 \exp \frac{-(p-p_1)^2}{2\sigma_i^2}\right]_{-\infty}^{p_1} < c$$

which gives us

$$\frac{\sigma_i}{2\pi} < c$$

as the condition for cutoff after one search step. In other words, stopping after one step will be independent of the price drawn. Only the initial conception of the variance and search cost will decide that.

Naturally, however, the accepted price is stochastic. The distribution of the acceptance price in this case is the same as the parent distribution.

If $\sigma_i > c \cdot 2\pi$, at least two search steps will be taken. The probability of stopping after two, three, four or more steps will depend on the sequence in which prices are drawn, which of course is a function of the parent distribution's mean and variance. The a posteriori distributions are dependent on σ_i and k. The stopping condition is obviously also dependent on c. As a result the probability density functions for N (number of steps) and p_{acc} (acceptance price) will be obtained which depend on σ_F, σ_i, k and c. In principle it should be theoretically feasible to find the distributions for N and p_{acc} and to determine how these depend on σ_F, σ_i, k and c, but this problem would seem to be unreasonably difficult to solve. Instead, I have opted for computer simulation to help me form an idea of the distributions which arise when the values for σ_i, c and k are ordered in sets. The results are presented in Axell(–74) and are not repeated here.

6. TESTING FOR DEMAND AND PROFITS

Limited information for the firms.

One of the assumptions in the model is that the firms have perfect information about the consumer behavior, from which they can calculate the expected profit function. Thus equilibrium will be such that the profit will be the same at all prices in the interval.

In a dynamic formulation of the model, we instead say that firms have limited knowledge about the demand (firms demand) at different prices.

Let us say that a firm will stick to a certain price for a period of three months. In this way it will learn the expected demand at that price perfectedly well. However, it is curious about the demand (and hence profit) at other prices.

Each firm faces a stochastic demand curve in each period. There are two reasons for this. In the first place, the stopping distribution $\omega(p)$ is the expected distribution. Normally ω at p will differ from its expected value, causing the demand at p to be stochastic. In the second place, even if the number of buying consumers in the interval $[p, p + \Delta p]$ is equal to the expected number, the consumers need not be uniformly distributed among the firms in this interval, because the number of consumers per firm need not be large.

7. FIRM BEHAVIOR

Given the assumed behavior of the consumers and the associated demand curve, each firm has to decide what price to charge in order to maximize profit. Parallel with the consumers' situation, firms lack perfect information both about exact consumer behavior and the resulting demand curve. However, each firm knows its own demand at the price it charges during a period. However, it may obtain information about the shape of the demand curve by experimenting with price changes. The parallel with the consumer's search activity is obvious: firms risk losing profits by 'searching out' the demand at other prices.

In this section we will derive an expression for the firms' price change based on assumptions of search behavior of the firms.

Each firm faces a stochastic demand curve in each period. There are two reasons for this. In the first place, the stopping distribution $\omega(p)$ is the expected distribution. Normally ω at p_i will differ from its expected value, causing the demand at p_i to be stochastic. In the second place, even if the number of buying consumers in the interval $(p_i, p_i + \Delta p)$ is equal to the expected number, the consumers need not be uniformly distributed among the firms in this interval, because the number of consumers per firm need not be large.

We can introduce this stochastic element into the firm's environment by adding a stochastic term to the demand function. We are then in position to derive the stochastic profit function. The stochastic term could in principle be derived from consumer search behavior. This, however, would be a very difficult task. For simplicity we assume instead that the stochastic environment of the firms can fairly well be described by adding a stochastic term u to the profit function. Then profit as a function of price is

$$\pi(p_i) = p_i\, q(p_i) - C(q(p_i)) + u, \qquad (4)$$

where the demand $q(p_i)$ is mathematically expected demand. $C(q(p_i))$ is the cost function, which is taken to be the same for all firms in the market. u is a stochastic term which is added to expected profit.

Let us now describe the firm's experimental behavior. We assume that:

1. All firms are risk neutral
2. A firm knows the expected demand (and thereby the expected profit) at the price it has charged itself during period t.
3. A firm does not know the demand at prices other than that which it has charged itself.

Let us regard a particular firm i charging price p_i in period t. The firm will, during this period, register the demand q_i. The firm realizes that it is facing a finitely elastic demand curve, but it does not know whether p_i is the very best price or whether it could raise its profit by increasing or decreasing the price. However, the fact that it has chosen p_i reveals that it has no reason to believe that a lower price is likely to be better than a higher price.

We now assume that if the firm undertakes an experiment with a price change, then it will be equally probable for it to raise as to lower its price. Further, we make the simplifying assumption that all firms are experimenting.

Consider a firm charging p_i. It receives a profit of $\pi(p_i) + u$, where u is a stochastic term. Let us assume that u shows the variability in profit during relatively short periods (days for instance). We also assume that the profit function is homoscedastic, i.e. u has the same density function at all prices. If the firm remains at p_i for a longer period, say a month or two, it will get a fairly good picture of the expected profit $\pi(p_i)$. If during a short subperiod the firm tries another price, for instance $p_i + \Delta p$, then it will get the profit $\pi(p_i + \Delta p) + u$ at that price.

Given the experimental price increase Δp, a fundamental question is this: What is the probability of an increase in profit? I.e. we ask what is the probability of the following relationship:

$$\mathrm{pr}(\pi(p_i + \Delta p) + u) \geq E(\pi(p_i)), \tag{5}$$

where $$E(\pi(p_i)) = \pi(p_i).$$

Let us call this probability ν_{i+1}. Making a Taylor expansion of $\pi(p_i)$ around p_i we get

$$\pi(p_i+\Delta p) = \pi(p_i) + \pi'(p_i)\, \Delta p + \frac{1}{2}\, \pi''(p_i)\, \Delta p^2 + \ldots\ldots + \ldots\ldots$$

Linearizing in the interval, i.e. dropping terms of second degree and higher, the probability (5) is

$$\nu_i^+ = \mathrm{pr}(u \geq \pi'(p_i)\, \Delta p). \tag{6}$$

We see that ν_i^+ is the probability that the stochastic term does not reduce the profit at $p_i + \Delta p$ from its expected value more than the actual difference in expected profit, expressed by means of the slope of π at p_i times Δp. This is in figure 2 the probability of falling within α during the experiment with $p_i + \Delta p$.

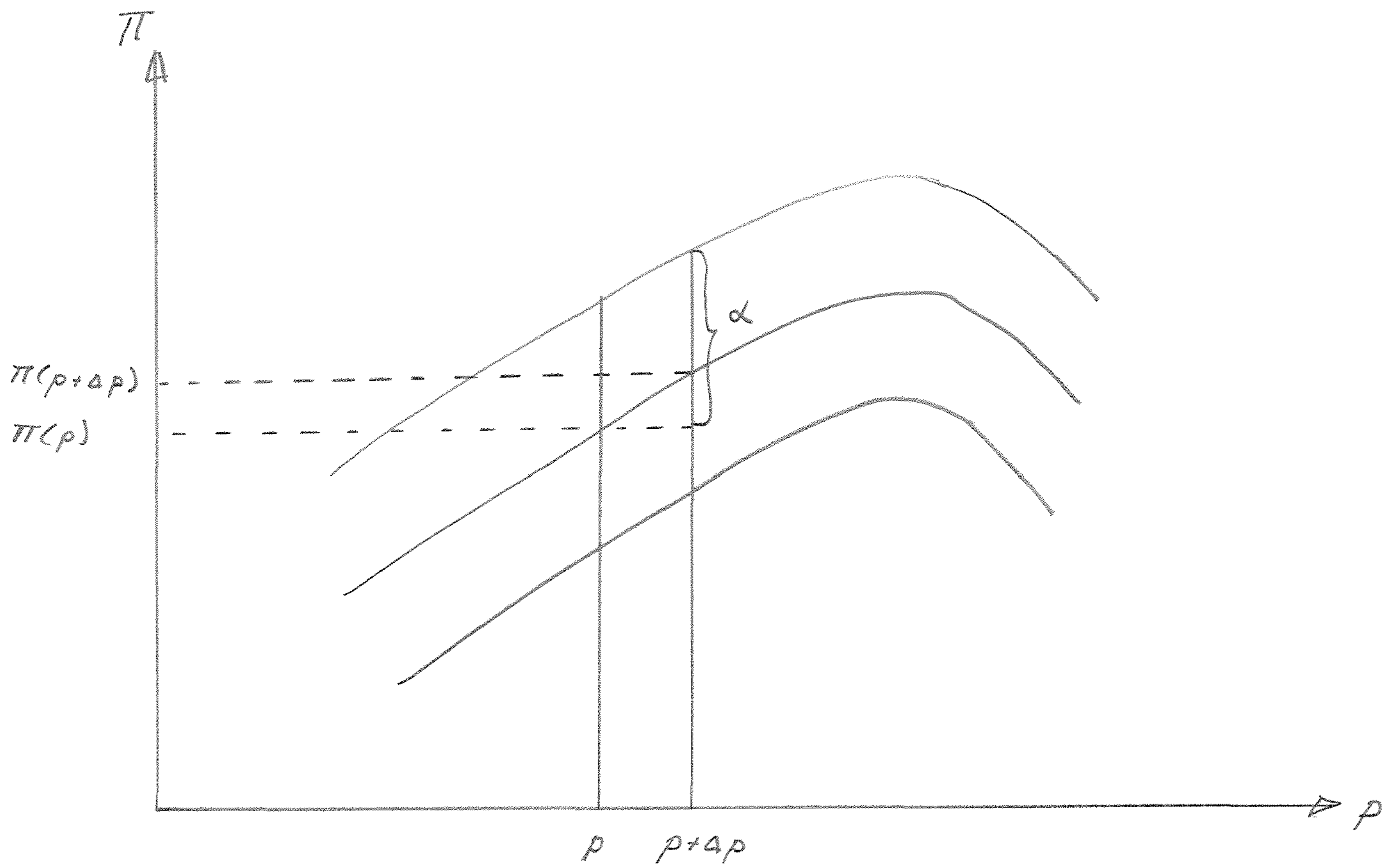

Figure 2

If the profit function is homoscedastic, i.e. the stochastic term u has the same probability density function at all prices, where this density function is $\zeta(u)$ with the probability distribution $Z(u)$, we get

$$\nu^{\dagger} = 1 - \int_{-\infty}^{-\pi'(p_i)\Delta p} \zeta(u)\, du = 1 - Z(-\pi'(p_i)\,\Delta p).$$

Since $\zeta(u)$ is not derived from consumer search, we have to assume a reasonable shape for it. The normal density function is perhaps a good choice, but in a complicated interdependent analysis it will cause great analytical problems. Simple expressions will appear if we assume instead that the stochastic profit terms are uniformly distributed. Since the important thing is to introduce a stochastic element into the firm's environment, it would seem that this distribution is no worse than any other. Let us thus assume that $\zeta(u)$ is a rectangular distribution with limits $-a$ and $+a$, i.e. $\zeta(u) = \frac{1}{2a}$. Then

$$Z(u) = \int_{-a}^{u} \frac{1}{2a}\, ds = \frac{u + a}{2a} \qquad -a \leq u \leq a.$$

The probability ν_i^+ is then

$$\nu_i^+ = \begin{cases} \frac{1}{2} + \frac{1}{2}\frac{\Delta p}{a}\, \pi'(p_i) & \text{if} \quad -a \leq \pi'(p_i)\, \Delta p \leq a\ , \\ 1 & \text{if} \quad \pi'(p_i)\, \Delta p > a\ , \\ 0 & \text{if} \quad \pi'(p_i)\, \Delta p < -a. \end{cases} \tag{7}$$

We see that this probability depends positively on the slope of the profit function and on the size of the price jump, but negatively on the variance of the stochastic term, as can be observed in figure 2.

If instead a firm tries the price $p_i - \Delta p$, then the probability of a profit increase is

$$\nu_i^- = \begin{cases} \frac{1}{2} - \frac{1}{2}\frac{\Delta p}{a}\, \pi'(p_i) & \text{if} \quad -a \leq \pi'(p_i)\, \Delta p \leq a\ , \\ 0 & \text{if} \quad \pi'(p_i)\, \Delta p > a\ , \\ 1 & \text{if} \quad \pi'(p_i)\, \Delta p < -a. \end{cases} \tag{8}$$

We see that $\nu_i^- = 1 - \nu_i^+$. Note that this follows from the approximation to a linear profit function in the interval $(p_i - \Delta p,\ p_i + \Delta p)$, evaluated at p_i.

Changes in the Distribution of Prices

In the previous section we derived the probability that a price change experiment will lead to increased profit at the experimental price. We now wish to study how firms actually change prices over time. Especially, we want to describe the aggregate effect of the behavior of the individual firms – how the price distribution, i.e. the distributions of firms over prices, will change over time.

We assume the following behavior of firms (in addition to the earlier assumptions): If a firm, charging the price p_i during a given period, experiments with the price $p_i + \Delta p$ during a subperiod and registers a higher profit at $p_i + \Delta p$, then it will charge the price $p_i + \Delta p$ during the next period; otherwise it will return to p_i. From this follows that the probability that a firm charging p_i will raise its price to $p_i + \Delta p$ is $\frac{1}{2}\nu_i^+$.[1]

[1] Remember that we have assumed that the probability it will experiment with a price increase is one half.

We have assumed the market to be an atomistic market, i.e. one in which the number of firms is very great. Then the probability of changing the price from, for instance, p_i to $p_i + \Delta p$ will show the proportion of firms at p_i changing price in that direction.

The frequency of firms charging p_i at time t is $f_t(p_i)$. The frequency of firms charging p_i at time t+1 is the share of those at $p_i - \Delta p$ at t which experimented with a price increase (i.e. one half) and obtained positive information (i.e. profit increase) plus the share of those at $p_i + \Delta p$ which experimented with a price decrease and obtained positive information, plus those at p_i that experimented with a price decrease or a price increase and obtained negative information. We thus get[2]

$$f_{t+1}(p_i) = \frac{1}{2} f_t(p_i - \Delta p)\, \nu_i + \frac{1}{2} f_t(p_i + \Delta p)(1-\nu_i) + \frac{1}{2} f_t(p_i)\, \nu_i + \frac{1}{2} f_t(p_i)(1-\nu_i),$$

which is

$$f_{t+1}(p_i) = \frac{1}{2} f_t(p_i - \Delta p)\, \nu_i + \frac{1}{2} f_t(p + \Delta p)(1-\nu_i) + \frac{1}{2} f_t(p_i) \tag{9}$$

Writing $f_t(p_i - \Delta p)$ with help of Taylor expansion we get

$$f_t(p_i - \Delta p) = f_t(p_i) - f_t'(p_i)\, p + \frac{1}{2} f_t''(p_i)\, p^2 - \dots + \dots .$$

In a corresponding way we have for $f_t(p_i + \Delta p)$:

$$f_t(p_i + \Delta p) = f_t(p_i) + f_t'(p_i)\, \Delta p + \frac{1}{2} f_t''(p_i)\, \Delta p^2 + \dots + \dots .$$

Disregarding terms of second degree and higher, we can write expression (9) as

$$f_{t+1}(p_i) - f_t(p_i) = \frac{1}{2} \left[f_t(p_i) - f_t'(p_i)\Delta p\right] \nu_i + \frac{1}{2} \left[f_t(p_i) + f_t'(p_i)\Delta p\right] (1-\nu_i) - \\ - \frac{1}{2} f_t(p_i),$$

which can be simplified to

$$f_{t+1}(p_i) - f_t(p_i) = f_t'(p_i)\, \Delta p(\tfrac{1}{2} - \nu_i).$$

[2] Note that we have changed notation of ν_i slightly. Here we think, for simplicity, that the linearization around the experimental price and the ordinary price does not differ too much.

Converting from discrete time intervals to continuous time, approximating the difference with the derivative, we have

$$\dot{f}(p_i, t) = f'(p_i)\,\Delta p(\tfrac{1}{2} - \nu_i).$$

If we now substitute for the expression for ν_i derived earlier we have

$$\dot{f}(p_i, t) = f'(p_i)\,\Delta p(\tfrac{1}{2} - \tfrac{1}{2} - \tfrac{1}{2}\tfrac{\Delta p}{a}\,\pi'(p_i)),$$

which is

$$\dot{f}(p_i, t) = -\tfrac{1}{2} f'(p_i)\,\pi'(p_i)\,\frac{\Delta p^2}{a}. \tag{10}$$

The expression for the profit function is

$$\pi(p_i) = p_i\,q_i(p_i) - C(q_i(p_i)),$$

where

$$q_i(p_i) = \frac{k}{m}\int_{p_i}^{\infty} \gamma(\tilde{F}(p))\,dp.$$

We get

$$\frac{d\pi}{dp_i} = \frac{k}{m}\left[\int_{p_i}^{\infty} \gamma(\tilde{F}_t(p))\;dp - p_i\;\gamma(\tilde{F}_t(p_i))\right] + \frac{dC}{dq_i}\frac{k}{m}\,\gamma(\tilde{F}_t(p_i)).$$

Rearranging terms we get

$$\frac{d\pi}{dp_i} = \frac{k}{m}\left[\int_{p_i}^{\infty} \gamma(\tilde{F}_t(p))\,dp + \gamma(\tilde{F}_t(p_i))\left[\frac{dC}{dq_i} - p_i\right]\right]. \tag{11}$$

The complete expression for the change of the price distribution will then be

$$\dot{f}(p_i, t) = -\frac{\Delta p^2}{2a} \cdot f_t'(p_i) \cdot \frac{k}{m} \cdot \left[\int_{p_i}^{\infty} \gamma(\tilde{F}_t(p))\, dp + \right.$$
$$\left. + \gamma(\tilde{F}_t(p_i))\left[\frac{dC}{dq_i} - p_i\right]\right]. \qquad (12)$$

This is a nonlinear differential equation of fourth degree. It will well suit the requirements for a Chaos situation, I hope.

7. THE CONVERGENCE TO EQUILIBRIUM – LOCAL STABILITY OF EQUILIBRIUM WITH EXPERIMENTAL LEARNING BY FIRMS

Now we want to analyse the question of the convergence of a market to equilibrium if it is close to the equilibrium configuration, and the firms are learning about demand and profits by means of the trial and error process described in the preceeding section.

The complete expression for the change of the price distribution, expressed in discrete time, will then be

$$f_{t+1}(p) - f_t(p) = -\frac{k \cdot \Delta p^2}{m \cdot 2a}\frac{df}{dp} \cdot \{\int_{p}^{\infty} \gamma[\tilde{F}(s)]ds +$$
$$+ \gamma[\tilde{F}(p)](mc-p)\}, \qquad (13)$$

where mc is a constant marginal cost. (13) is (12) expressed in discrete time.

There will be no change in the density $f_{t+1}(p)$ compared to $f_t(p)$ (i e equilibrium) if the RHS of (13) is zero. This will be the case if the expression inside the { } in (13) is zero.

This will be the case if $\gamma(\cdot)$ and $f(\cdot)$ are such that:

$$\int_p^\infty \gamma(\tilde{F}(s))ds = \frac{B}{p - mc} \qquad p > mc \tag{14}$$

where B is a positive constant.

The question is now: If f(p) and γ(c), both pdf, are constistent with the necessary and sufficient conditions for an equilibrium, ie fullfilling eq (14), is this equilibrium stable? In equilibrium $f_{t+1}(p) - f_t(p)$, i.e. (13), is zero. Hence, $f(\cdot)$ and $\gamma(\cdot)$ are such that the { } in (13) is zero. The question of stability is: If f(p) at p increases a bit (and decreases at some other price – f(p) is a pdf), what will be the sign of $f_{t+1}(p) - f_t(p)$, i.e., { } in (13)?

I want to show that if f(p) increases then the sign of { } is negative and if f(p) decreases it is positive (note that $\frac{df}{dp} < 0$ in equilibrium) and therefor resulting in a convergence to equilibrium, which then proves the stability of equilibrium.

First let us derive the general expression for stability. We want to make a small change in the equilibrium configuration. Let us introduce the function h(p) with the property $\int_0^\infty h(s)ds = 0$.

Substitute the function f(p) with $f(p) + \delta h(p)$ in (13).

The { } in (13) then becomes:

$$\int_p^\infty \gamma[\tilde{F}(s) + \delta\tilde{H}(s)]ds + \gamma[\tilde{F}(p) + \delta\tilde{H}(p)]\cdot(mc-p) \tag{15}$$

where $\tilde{H}$ is defined in the same way as F, i.e.:

$$H(p) = \int_0^p h(s)ds$$

$$\tilde{H}(p) = \int_0^p H(s)ds$$

Taking the derivative of (15) with respect to δ and then setting $\delta = 0$ gives:

$$\int_p^\infty \gamma'[\tilde{F}(s)]\tilde{H}(s)ds + \gamma'[\tilde{F}(p)]\tilde{H}(p)(mc-p). \tag{16}$$

PROPOSITION 1: The market is stable if expression (16) has the opposite sign to h(p). This will be the case in this case.

PROOF: Let h(p) have the particular form;

$$h(p) = \epsilon\delta_{p_1}(p) - \epsilon\delta_{p_2}(p),$$

which is the sum of two Dirac delta functions, where we assume that $p_1 < p_2$ and ϵ is positive <u>or negative</u>.

Since (16) is homogenous in h we can have $\epsilon = 1$. Then we have

$$\tilde{H}(p) = \begin{cases} 0 & \text{if } p \le p_1 \\ (p_2-p_1) > 0 & \text{if } p \ge p_2 \end{cases} \tag{17}$$

We want to show that (16) is < 0 at p_1 and > 0 at p_2. The first condition obviously holds, since the integral (the first term) is < 0, because $\gamma' < 0$ and the second term is $= 0$, because $\tilde{H}(p_1) = 0$.

Let us turn to the case when $p = p_2$.

At first, observe that lim.inf. of (16) when $p \to \infty$ is ≥ 0, because the integral $\to 0$ and the second term is always positive ($\gamma' < 0$, $(mc-p) < 0$, and $\tilde{H}(p) > 0$).
We want to show that (16) is > 0 for $p = p_2$.

Hence, it is sufficient to show that (16) is decreasing. Differentiating (16) gives:

$$- \gamma'[\tilde{F}(p)]\tilde{H}(p) + \gamma''[\tilde{F}(p)]F(p)\tilde{H}(p)(mc-p) - \gamma'[\tilde{F}(p)]\tilde{H}(p)$$

which is

$$-2\gamma'[\tilde{F}(p)]\tilde{H}(p) + \gamma''[\tilde{F}(p)]F(p)\tilde{H}(p)(mc-p) \tag{18}$$

For stability we want (18) to be < 0.

Because $\tilde{H}(p) = p_2 - p_1 > 0$ we want to show that:

$$2\gamma'[\tilde{F}(p)] - \gamma''[\tilde{F}(p)]F(p)(mc-p) > 0 \tag{19}$$

We can solve for $\gamma[\tilde{F}(p)]$ from equation (14).

We get

$$\gamma[\tilde{F}(p)] = \frac{B}{(p-mc)^2}$$

Differentiating we get:

$$\gamma'[\tilde{F}(p)]F(p) = \frac{-2B}{(p-mc)^3} \tag{20}$$

Differentiating again we get:

$$\gamma''[\tilde{F}(p)][F(p)]^2 + \gamma'[\tilde{F}(p)]f(p) = \frac{6B}{(p-mc)^4}$$

or, in other words:

$$\gamma''[\tilde{F}(p)][F(p)]^2 = \frac{6B}{(p-mc)^4} - \gamma'[\tilde{F}(p)]f(p) \tag{21}$$

Now, multiplying (19) with the positive F(p) and substituting for (20) and (21), we get:

$$\frac{-4B}{(p-mc)^3} - \{\frac{6B}{(p-mc)^4} - \gamma'[\tilde{F}(p)]f(p)\}(mc-p) =$$

$$= \frac{2B}{(p-mc)^3} + \gamma'[\tilde{F}(p)]f(p)(mc-p) \tag{22}$$

Stability requires that this expression is > 0. It obviously is. The first term is > 0 ($p > mc$). The second term is > 0, too, because $\gamma' < 0$, $f(p) > 0$ and $(mc-p) < 0$ ($p > mc$).

Q.E.D.

8. CONCLUSIONS

In this paper I have tried to project price formation in a market in a market economy considering the dynamics of both the learning processes and the mutual interdependence of, i/ the consumers and, ii/ the firms in the analysis.

While we could not get any conclusions about what whould happens if the consumers are uninformed about the distribution and hence have to learn it, we could conclude that if firms have to make experiments to find out the profit function, the market will approach the price dispersion configuration presented in Axell(–77), at least if the market is not too far away from that equilibrium configuration.

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