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**HOW CHANGES IN LABOR DEMAND  
AFFECT UNEMPLOYED WORKERS**

by

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## INTRODUCTION

In recent years a large amount of work has been accomplished on the empirical implementation of job search theory (see, for example, Barron (1975), and Kiefer and Neumann (1979)). Much of this work, however, has been hindered by the relatively well known fact that there are, in general, indeterminant consequences on the outcome of the job search process when there is a change in labor demand conditions.<sup>1</sup> The purpose of this short study is to specify a reasonably general restriction which implies that specific predictions can be made about the consequences of changes in the demand for labor conditions. Further, the restriction used, which is placed on the distribution of wage offers faced by the unemployed worker, adds some new insight into the nature of job search models.

A shift in the demand for labor conditions faced by an unemployed worker will involve at least one of the following types of changes.

Type 1: A change in the probability (per period) the worker finds a vacancy, given a particular search intensity.

Type 2: A change in the wage offer distribution.

The objective of the study is to investigate the consequences of both types of changes on (a) the expected duration of a completed spell of unemployment,  $D$ , and (b) the expected post unemployment wage of a worker,  $\bar{W}$ . For the majority of the study it will be assumed that an unemployed worker fully predicts the changes in labor market conditions. Nevertheless, the case where the worker is unaware of such changes will also be investigated. It has been noted that a type 1 change in labor market conditions may increase or decrease  $D$ . Further, it can be shown that a type 2 change may increase or decrease  $\bar{W}$ . Later in this study it will be demonstrated that if a particular restriction is placed on the cumulative distribution function

(cdf) describing the wage offers, then a fully predicted change of either type will lead to a reduction in  $D$  and an increase in  $\bar{w}$ .

Recently, several authors have studied the consequences of a type 1 change in labor market conditions (Barron (1975), Feinberg (1977), Axelsson and Löfgren (1977), and Björklund and Holmlund (1980)). Within the context of a job search model, Barron has shown that a type 1 improvement (when fully predicted) will increase an unemployed worker's reservation wage. Thus, a type 1 improvement implies the worker is more likely to receive offers (Barron's "Effect 1"), but less likely to accept an offer (Barron's "Effect 2"). Barron concludes that the net effect is, in general, indeterminate. He conjectures, however, that it would depend on such things as the cost of search and the unemployment compensation received by the worker. Feinberg subsequently demonstrated that if the cdf of wage offer is Uniform,  $D$  will always decline when there is a fully predicted type 1 improvement.<sup>2</sup> One objective of the present study is to generalize Feinberg's result.

To illustrate the restriction used to obtain our results let

$$(1) \quad V(z, G) = E\{w | w \geq z \text{ and } w \text{ is distributed according to cdf } G\}.$$

$V(\cdot, G)$  is termed the truncated mean function (tmf) of  $G$ . It is straightforward to show that the tmf of any particular cdf is a unique transformation of it. It will be shown that if the tmf has a slope less than 1, then  $D$  will always decline with a fully predicted type 1 improvement in labor market conditions. It should be noted that many well known types of distribution functions have a tmf which satisfy this restriction. Nevertheless, there are some exceptions.<sup>3</sup> When the restriction does not hold, it will be shown that a fully predicted type 1 improvement will increase  $D$  if the discount factor is low enough. The expected post-unemployment wage,  $\bar{w}$ , will always increase with a predicted type 1 improvement.

Both the Barron study and the Björklund and Holmlund study consider a type 2 change in the demand for labor conditions. However, both these

studies assume that such a change in demand for labor is not fully predicted by unemployed workers (Barron's "Effect 3"). Suppose for the moment that such a change is fully predicted. Within the context of a job search model, a type 2 change may be interpreted as a change in the cdf of wage offers. There are many ways in which a cdf of wage offers can be said to improve. For example, suppose the wage offer distribution is a Normal cdf. In this case a type 2 improvement may take the form of an increase in the mean and the variance of this distribution. In the present study such improvements will be ruled out by assumption. Specifically, it will be assumed that any change in the cdf of wage offers takes the form of a translation of location (in the case of a Normal cdf this would imply that the mean can change but not the variance).<sup>4</sup> In this case a fully predicted type 2 change in labor market conditions has an indeterminate effect on  $\bar{W}$ . Nevertheless, it will be shown that a predicted type 2 improvement will always increase  $\bar{W}$  if the tmf generated from the wage offer distribution has a slope less than 1. If this restriction is not satisfied, and if the discount factor is large enough, a type 2 improvement will lower  $\bar{W}$ . The expected duration of unemployment always decreases with a type 2 improvement when the discount factor is strictly positive.

Suppose now that a change in the demand for labor conditions is not predicted by the worker. In this case the worker will not alter his or her reservation wage when a change occurs. This implies a type 1 improvement will reduce the expected duration of unemployment and leave the expected post-unemployment wage unchanged. A type 2 improvement that is not predicted will reduce the expected duration of unemployment, but have, in general, an indeterminate effect on the expected post-unemployment wage. However, if the tmf of the distribution of wage offers has a slope less than 1, then such an improvement will increase  $\bar{W}$ . These results have important implications for the derivation of the short-run Phillips curve.

The restriction that the tmf of the wage offer distribution has a slope less than 1 has important implications for empirical work on job search as well as for the derivation of a short-run Phillips curve. Only if this restriction is satisfied will changes in the demand for labor conditions have predictable consequences on the outcome of a worker's job search process. Further, if this restriction is satisfied it is straightforward to construct downward sloping short-run Phillips curves, whereas the short-run Phillips curve may be upward sloping if the wage offer distribution does not satisfy this restriction. Before describing the model used some properties of truncated mean functions will be presented.

#### 1. TRUNCATED MEAN FUNCTIONS OF A CLASS OF DISTRIBUTIONS

Let  $\{F^\mu(\cdot)\}_{\mu \in R^1}$  denote a class of cdfs identical up to a translation of location, i.e., for given  $z$ ,  $F^\mu(z) = F^{\mu'}(z - (\mu - \mu'))$  for any  $\mu$  and  $\mu'$  in  $R^1$ . For notational convenience let  $H(z) = F^\mu(z)$  for any  $z$  when  $\mu = 0$ . In this case for any particular  $z$  we have

$$(2) \quad F^\mu(z) = H(z - \mu)$$

for any  $\mu \in R^1$ . For given  $\mu \in R^1$ , let  $V(\cdot, \mu)$  denote the tmf when the cdf is  $F^\mu(\cdot)$ . From (1) and (2) it follows that

$$\begin{aligned} (3) \quad V(z, \mu) &= \frac{\int_z^\infty w \, dF^\mu(w)}{(1 - F^\mu(z))} \\ &= \frac{\int_{z-\mu}^\infty w \, dH(w)}{(1 - H(z-\mu))} + \mu \\ &= \mu + V(z-\mu, 0). \end{aligned}$$

Clearly,  $V(\cdot, \mu)$  is a unique transformation of  $F^\mu(\cdot)$  for any given  $\mu \in R^1$ . This fact and (3) imply that the restrictions placed on  $H(\cdot)$  alone will determine

the properties of  $V(\cdot, \mu)$  for all  $\mu \in R^1$ . To simplify the exposition it will be assumed that  $H(\cdot)$  is strictly increasing and differentiable on the real line. Taking the partial derivative of (3) with respect to  $z$  yields

$$(4) \quad V_1(z, \mu) = V_1(z-\mu, 0) = \frac{h(z-\mu)}{(1-H(z-\mu))} [V(z-\mu, 0) - (z-\mu)]$$

where  $h(z-\mu) = H'(z-\mu)$ . Since  $H(\cdot)$  is strictly increasing,  $V(z-\mu, 0) > z-\mu$ , and thus  $V_1(z, \mu)$  for any  $z$  and  $\mu \in R^1$ , i.e., an increase in the truncation point,  $z$ , increases the conditional expectation. Further, from (2), (3), and (4) we obtain

$$(5) \quad V_2(z, \mu) = 1 - V(z, \mu) = 1 - V_1(z-\mu, 0),$$

where  $V_2(\cdot, \mu) = \partial V(z, \mu) / \partial \mu$ . Thus, a small shift to the right in the location of the cdf will increase the conditional expectation if and only if  $V_1(z, \mu) < 1$ . If  $H(\cdot)$  is assumed to be a particular type of cdf, then it is relatively easy to check if the slope of its tmf is less than 1 everywhere. For example, if  $H(\cdot)$  is assumed to be a Normal cdf or a Logistic cdf, then the implied  $V_1(x, 0) < 1$  for all  $x$ . However, if  $H(\cdot)$  is assumed to be a Student cdf, then  $V_1(x, 0) > 1$  for some  $x$ .

Recently, Goldberger (1980) has demonstrated the following claim.

CLAIM: If  $h(\cdot)$  is strictly logconcave, i.e.,  $\log H'(\cdot)$  is strictly concave, then the implied  $V_1(x, 0) < 1$  for all  $x \in R^1$ .

The above result provides a simple-to-check sufficient condition for the restriction used. If the probability density function of a particular cdf is not logconcave, the slope of the tmf has to be calculated to determine if it is less than 1 everywhere. The economic model will now be presented.

#### THE MODEL

In the first part of this section a simple infinite life job search model is briefly outlined. Consider an unemployed worker looking for a

job in a labor market. Each period the worker visits a firm and enquires if there a job. Let  $\lambda$  denote the probability any firm contacted has a vacancy. If the worker obtains a job offer, assume the wage rate offered is a random draw from cdf  $F(\cdot)$  for some fixed  $\mu \in R^1$ . Hence,  $\lambda(1-F^H(z))$  indicates the probability the worker obtains an offer with a wage rate at least as great as  $z$  in a period. The worker's expected discounted lifetime income given an offer with wage rate  $w'$  is accepted is assumed to be  $w' \sum_{i=1}^{\infty} (1+r)^{-i} = w'/r$ , where  $r$  denotes the discount factor.

Let  $\psi(z, \lambda, \mu)$  denote the unemployed worker's expected discounted lifetime income (net of job search costs) when the worker accepts the first job offer with a wage rate at least as great as  $z$ , i.e., if  $z$  is used as the reservation wage. It follows that

$$(6) \quad \psi(z, \lambda, \mu) = \frac{1}{(1+r)} \left[ u-c + \lambda \int_z^{\infty} (w/r) dF^H(w) + (1-\lambda(1-F^H(z)))\psi(z, \lambda, \mu) \right],$$

where  $u-c$  indicates the unemployment compensation per period minus the cost of visiting a firm. Given the worker desires to maximize expected discounted income it is well known that reservation wage  $R$  will be used where  $\psi(R, \lambda, \mu) = R/r$  (see Lippman and McCall (1976)). This fact, (2) and (6) imply

$$(7) \quad R = u-c + (\lambda/r) \int_{R-\mu}^{\infty} (w + \mu - R) dH(w).$$

$R$  will be termed the optimal reservation wage conditional on the parameters  $\lambda$  and  $\mu$ .

Within the context of the model developed, a type 1 change in the demand for labor conditions will be reflected in a change in  $\lambda$ , whereas a type 2 change will be reflected by a change in  $\mu$ . Using (4) and (7) we have

$$(8) \quad \begin{aligned} \frac{dR}{d\lambda} &= \frac{1}{[r+\lambda(1-H(R-\mu))]} \int_{R-\mu}^{\infty} (w + \mu - R) dH(w) \\ &= \frac{(1 - H(R-\mu))^2}{h(R-\mu)[r+\lambda(1-H(R-\mu))]} V_1(R-\mu, 0) > 0 \end{aligned}$$

and

$$(9) \quad \frac{dR}{d\mu} = \frac{\lambda(1 - H(R-\mu))}{[r + \lambda(1-H(R-\mu))]} > 0.$$

Thus, an improvement in the demand conditions of either type will increase the worker's optimal reservation wage when it is fully predicted. Note that (9) implies that  $dR/d\mu < 1$ , if the discount rate is strictly positive. This fact, in turn, implies that if all wage offers in the market are increased by 1, then the worker's optimal reservation wage will increase by less than 1 when  $r > 0$ . Although the above results are of theoretical interest they are not of much use in themselves for empirical work as workers' reservation wages are usually not directly observable. Nevertheless, researchers can often ascertain how long a worker was unemployed and what was the worker's post-unemployment wage. Consequently, the effect of a change in labor market conditions on  $D$  and  $\bar{W}$  will be analyzed.

To simplify the exposition the escape from unemployment probability,  $E$ , will be considered instead of the expected duration of unemployment,  $D$ . This is possible as  $D = 1/E$ , and hence

$$(10) \quad dD/dX \geq 0 \quad \text{as} \quad dE/dX \leq 0$$

for any variable  $X$ . The probability of finding an acceptable wage offer in a period can be written as

$$(11) \quad E = \lambda(1 - H(R-\mu))$$

where  $R$  satisfies (7). From (3) and (7) it follows that the expected post-unemployment wage can be written as

$$(12) \quad \bar{W} = \mu + V(R-\mu, 0)$$

where  $R$  satisfies (7). Using (11) and (12) yields

$$(13) \quad \frac{dE}{di} = \begin{cases} (1 - H(R-\mu)) - \lambda h(R-\mu) dR/d\lambda, & \text{if } i = \lambda \\ \lambda h(R-\mu) [1 - dR/d\mu], & \text{if } i = \mu. \end{cases}$$

$$(14) \quad \frac{d\bar{w}}{di} = \begin{cases} V_1(R-\mu, 0) dR/d\lambda, & \text{if } i = \lambda \\ 1 - V_1(R-\mu, 0) [1 - dR/d\mu], & \text{if } i = \mu. \end{cases}$$

If there is an unpredicted change in the demand for labor conditions the worker will not change his or her reservation wage. This fact, (10), (13), and (14) allow us to make the following claim.

- CLAIM 2: (i) An unpredicted type 1 improvement in labor market conditions implies (a) the expected duration of unemployment will decrease, and (b) the expected post-unemployment wage will remain unchanged.
- (ii) An unpredicted type 2 improvement in labor market conditions implies (a) the expected duration of unemployment will decrease, and (b) the expected post-unemployment wage will increase (decrease) if  $V_1(R-\mu, 0) < 1$  (if  $V_1(R-\mu, 0) > 1$ ).

As the above claims follow immediately from inspection of (10), (13), and (14) no proof will be presented.

In this final part of the study it will be assumed that any change in labor market conditions is fully predicted by unemployed workers. First, suppose there is a change in  $\lambda$  when all other parameters are held constant. From (8), (9), (13), and (14) we have<sup>5</sup>

$$(15) \quad \frac{dE}{d\lambda} = \frac{\lambda(1-H(R-\mu))^2}{[r + \lambda(1-H(R-\mu))]} \left[ \frac{r}{\lambda(1-H(R-\mu))} + [1 - V_1(R-\mu, 0)] \right]$$

and

$$(16) \quad \frac{d\bar{w}}{d\lambda} = V_1(R-\mu, 0) \frac{\lambda(1-H(R-\mu))}{[r + \lambda(1-H(R-\mu))]}.$$

Inspection of (10), (15), and (16) allows us to state the following claims without any formal proof.

CLAIM 3:

- (a)  $d\bar{w}/d\lambda > 0$
- (b)  $dD/d\lambda < 0$ , if  $V_1(R-\mu, 0) < 1$

- (c)  $dD/d\lambda > 0$  for some  $r > 0$ , if  $V_1(R-\mu, 0) < 1$

Claim 3 implies that the consequences of a type 1 improvement can be predicted fully only if the distribution of wage offers is such that the tmf has a slope less than 1. In this case a type 1 improvement which is fully predicted, will reduce the expected duration of unemployment and increase the expected post-unemployment wage. If the distribution of wage offers is such that the tmf has a slope greater than 1 in places, then a type 1 improvement will always increase the expected post-unemployment wage, but may increase the expected duration of unemployment if the discount factor is small enough.

Finally, suppose there is a shift in the cdf of wage offers that is fully anticipated by the unemployed workers, i.e., a predicted type 2 change labor market conditions. From (9), (13), and (14) it follows that

$$(17) \quad \frac{dE}{d\mu} = \lambda h(R-\mu) \frac{r}{[r + \lambda(1-H(R-\mu))]}$$

and

$$(18) \quad \frac{d\bar{W}}{d\mu} = 1 - V_1(R-\mu, 0) \left[ \frac{r}{[r + \lambda(1-H(R-\mu))]} \right].$$

Using (10), (17), and (18) the following claims can be stated without formal proof.

CLAIM 4:

- (a)  $dD/d\mu < 0$ .
- (b)  $d\bar{W}/d\mu > 0$ , if  $V_1(R-\mu, 0) < 1$
- (c)  $d\bar{W}/d\mu < 0$  for large enough  $r$ , if  $V(R-\mu, 0) > 1$

Claim 4 establishes that if the tmf of the distribution of wage offers has a slope less than one everywhere, then a type 2 improvement will reduce the expected duration of unemployment and increase the expected post-unemployment wage. Although the expected duration of unemployment will

will always fall with a fully anticipated type 2 improvement, the expected post-unemployment wage cannot be guaranteed to increase with such an improvement if  $V_1(R-\mu,0) < 1$ . If  $V_1(R-\mu,0) > 1$ , then a type 2 improvement can reduce the expected post-unemployment wage if the discount factor  $r$  is large enough.

From the above analysis it can be concluded that if the distribution of wage offers is such that its tmf has a slope less than 1 everywhere, then there are predictable consequences to a type 1 or type 2 change in labor market conditions independent of the discount factor used by the unemployed workers. When the distribution of wage offers is such that its tmf has a slope greater than 1 in places sign predictions cannot be made without knowledge of the discount rate.

#### FOOTNOTES

1. See Barron (1975) for details.
2. Feinberg (1977) conjectures, and goes far in establishing, that if the cdf of wage offers is Normal, then a type 1 improvement lowers the expected duration of unemployment.
3. Goldberger (1980) specifies the tmf of several well known types cdfs. He also shows which have a slope less than 1.
4. There is little evidence about actual distribution of wages in labor markets and how they change through time.
5. In Barron's model the discount factor is assumed to be zero. Thus, (15) reduces to  $dE/d\lambda = (1-H(R-\mu))(V_1(R-\mu,0)-1)$  when  $r = 0$ . In this special case  $dD/d\lambda < 0$  if and only if  $V_1(R-\mu,0) < 1$ .

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