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Production Efficiency of Nodal and Zonal Pricing in Imperfectly Competitive Electricity Markets

Mahir Sarfati, Mohammad Reza Hesamzadeh and Pär Holmberg

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Mahir Sarfati[a,b,], Mohammad Reza Hesamzadeh[a,d,], Pär Holmberg[b,c,d,]

[a]*Electricity Market Research Group (EMReG), KTH Royal Institute of Technology, Sweden*
[b]*Research Institute of Industrial Economics (IFN), Sweden*
[c]*Energy Policy Research Group (EPRG), University of Cambridge, UK*
[d]*Program on Energy and Sustainable Development (PESD), Stanford University, CA, USA*

Abstract

Electricity markets employ different congestion management methods to handle the limited transmission capacity of the power system. This paper compares production efficiency and other aspects of nodal and zonal pricing. We consider two types of zonal pricing: zonal pricing with Available Transmission Capacity (ATC) and zonal pricing with Flow-Based Market Coupling (FBMC). We develop a mathematical model to study the imperfect competition under zonal pricing with FBMC. Zonal pricing with FBMC is employed in two stages, a day-ahead market stage and a re-dispatch stage. We show that the optimality conditions and market clearing conditions can be reformulated as a mixed integer linear program (MILP), which is straightforward to implement. Zonal pricing with ATC and nodal pricing is used as our benchmarks. The imperfect competition under zonal pricing with ATC and nodal pricing are also formulated as MILP models. All MILP models are demonstrated on 6-node and the modified IEEE 24-node systems. Our numerical results show that the zonal pricing with ATC results in large production inefficiencies due to the inc-dec game. Improving the representation of the transmission network as in the zonal pricing with FBMC mitigates the inc-dec game.

Keywords: Congestion management, Zonal pricing, Flow-based market coupling
JEL Classification: C61, C72, D43, L13, L94

1. Introduction

Electricity markets around the world mainly use two congestion management methods: (a) nodal pricing and (b) zonal pricing. Electricity markets in the US and New Zealand employ nodal pricing. In contrast the European electricity markets favor zonal pricing.

Nodal pricing considers all the transmission constraints in the day-ahead market. Nodal prices could differ if congestion occurs in the transmission system. Each producer

Email addresses: `sarfati@kth.se` (Mahir Sarfati), `mrhesamzadeh@ee.kth.se` (Mohammad Reza Hesamzadeh), `par.holmberg@ifn.se` (Pär Holmberg)

is paid in accordance with the local price at the node where it is located. The nodal market is cleared in a single stage. Currently, nodal pricing is not an option for the integrated European electricity market [1]. One reason is that switching to the nodal pricing would require a fundamental change in the way European electricity markets are structured (i.e. current arrangements for cross-border trading would need to be redeveloped, implying significant IT and procedural changes) and the cost impact of this would, in the short-term, likely out weight the benefits [2].

Instead, the European Union (EU) favors zonal pricing. Zonal pricing aggregates nodes into zones with uniform prices. Ideally, zonal pricing should consider all transmission constraints and the nodal prices inside a zone should be the same [3], [4], [5] and [6]. In practice, a simplified transmission network is taken into account. According to the representation of the transmission constraints, there are two different versions of zonal pricing that are considered by EU: (i) Zonal pricing with Available Transmission Capacity (ATC) and (ii) Zonal pricing with Flow-Based Market Coupling (FBMC). Zonal pricing with ATC is for example employed in the Nordic electricity market. The previous design of the Central Western European (CWE) market also used zonal pricing with ATC. This method ignores the intra-zonal transmission constraints in the day-ahead market and the power flow between two zones is only limited by a pre-defined value (ATC). Since the transmission constraints are ignored for intra-zonal flows and approximated for the inter-zonal flows, the day-ahead dispatch may overload some transmission lines. These lines need to be relieved in the re-dispatch stage. In this second stage, all transmission constraints are taken into account and the overloaded lines are relieved. The re-dispatch generates net-expenses, which are paid by the system operator.

Some of the European markets are changing their day-ahead market to flow-based zonal markets, "Flow-based Market Coupling" (FBMC) [7]. FBMC considers constraints on a set of representative transmission lines, the Critical Branches (CBs) in the day-ahead market stage. Any transmission line (no matter if it is an inter-zonal or intra-zonal line) which has a strong impact on the cross-border trades can be selected as a CB. Hence, zonal pricing with FBMC is a better representation of transmission constraints than zonal pricing with ATC. However, overloadings may still occur in the transmission network since the CBs are decided before the clearing of the day-ahead market. Hence, a re-dispatch stage is still needed in zonal pricing with FBMC.

We consider a market-based re-dispatch[1] with countertrading as in UK and the Nordic countries. A general problem in this case is that different representations of the transmission constraints in the two stages result in predictable price difference, so that producers can make an arbitrage profit. In references [10], [11], [12] and [13] show that export constrained producers can make an arbitrage profit by overselling power in the day-ahead market and buying back power at a lower price in the re-dispatch stage. This type of bidding behavior is called the "increase-decrease game (inc-dec game)" [12],

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[1]Some electricity markets, i.e. Germany, uses a regulated re-dispatch mechanism where the producers are paid according to their true costs [8, 9].

[13] and [14] or "decrease game (dec game)" [15] and [16]. The inc-dec game was for example observed during the electricity crisis in California [16], [17] and [18] and in the British electricity market [19]. Several researchers analyze the inc-dec game in markets with zonal pricing and ATC. Authors in [12] consider a competitive market and analyze the imperfections caused by arbitrage opportunities. The study in [20] considers both imperfect competition and arbitrage opportunities but their analysis is limited to two-node networks. Authors in [13] model a two-stage price-bid game to evaluate designs of zonal power market with imperfect competition.

One of the purposes of employing zonal pricing with FBMC is to reduce the inc-dec game opportunities [14]. Before the CWE market employed zonal pricing with FBMC, this method and zonal pricing with ATC were run in parallel for 18 months. It was reported that FBMC lead to a significant increase in social welfare and improved price convergence [21]. Author in [22] simulated both zonal pricing methods, using 12-month data from the CWE market, and finds that FBMC improves social welfare by between €50M and €100M. The studies in [23] and [24] confirm these findings. However, these studies ignore distortions due to imperfect competition. In particular, the impact of changes in the market design and in the arbitrage opportunities on the bidding behavior of producers were not taken into account by studies in [21], [22], [23] and [24]. To the best of our knowledge, this paper is first to study the inc-dec game in markets with zonal pricing and FBMC. In addition we consider imperfect competition.

This paper develops a mathematical model to analyze imperfect competition and the inc-dec game in markets with zonal pricing with FBMC. We consider two stages, a day-ahead market stage and a re-dispatch stage. The imperfect competition of producers in these two-stages is modeled as a two-stage game. The two-stage game is formulated as a two-stage EPEC which is further reformulated as a two-stage MILP. Our market-clearing model in the day-ahead stage is related to [24], but they assume perfect competition.

Zonal pricing with ATC and nodal pricing are used as benchmarks in this study. For zonal pricing with ATC, we consider a similar two-stage zonal model as in [13] and [25]. But, in [13] and [25], the imbalance uncertainties are also taken into account. Hence, the second stage of the market is used both for balancing the imbalances and for relieving the overloaded lines (re-dispatching). In this study, we assume that there is no uncertainty in the market. Accordingly, the second stage in our zonal pricing with ATC model is only used for re-dispatching. We formulated the two-stage game under zonal pricing with ATC as a two-stage MILP model. For nodal pricing, a one-stage game is considered to model the imperfect competition. The one-stage game is formulated as a one-stage MILP model. Our one-stage MILP model is related to [26], [27], [28] and [29]. Based on our three MILP models, we analyze the imperfect competition and the inc-dec game for each congestion management method and we compare the production efficiencies and other aspects of nodal and zonal pricing. The study in [13] analyzes the two-stage zonal markets with ATC using a two-stage EPEC. However, it reformulates the two-stage EPEC as a Mixed Integer Bilinear Program.

We solve for Nash equilibria (NE) in the one-stage game. Strategies are assumed to be sequentially rational in the two-stage game, so in that game we solve for Subgame

Perfect Nash Equilibria (SPNE). There are multiple equilibria, and we solve for the worst equilibrium, which has the highest total dispatch cost. The developed MILP models are demonstrated in 6-node and the modified IEEE 24-node systems. Our numerical results show that the zonal pricing with ATC has 10.5-18.7% higher total production cost as compared to the nodal pricing. This is due to the inc-dec game. Changing the zonal design to the zonal pricing with FBMC does not avoid the inc-dec game but it improves market efficiency by 8.6-14.9% as compared to the zonal pricing with ATC. We investigate the impact of ramping costs on the market efficiency and we see that it has substantial impact on the market efficiency. In one of our examples, without ramping costs, zonal pricing with FBMC is as efficient as nodal pricing.

The main contribution of this paper is twofold: (i) We develop a MILP model which formulates the two-stage bidding game of multiple producers in zonal markets with FBMC. (ii) Using this MILP model, we compute production inefficiencies and other aspects of the design which we compare with the benchmarks, nodal pricing and zonal pricing with ATC.

The rest of the paper is organized as follows: Section 2 explains the three congestion management methods. Sections 3, 4 and 5 derive the MILP problems which model the imperfect competition for nodal pricing, zonal pricing with ATC and zonal pricing with FBMC, respectively. Section 6 presents the simulation results of the 6-node and the modified IEEE 24-node example systems. Section 7 closes the paper.

2. Congestion Management Methods

In this study, we develop mathematical models to analyze imperfect competition between producers for three congestion management methods: Nodal pricing, Zonal pricing with Available Transmission Capacity (ATC) and Zonal pricing with Flow-Based Market Coupling (FBMC). Table 1 illustrates the considered transmission constraints and the auction format of each congestion management method. We do not have any uncertainty in the model. Hence, all trade in the second-stage is countertrading, which the market operator uses to relieve congestion. As in practice, we assume that marginal pricing is used in the day-ahead market. Similar to the Nordic countries and UK, we assume re-dispatches are paid as bid.

Table 1: Considered transmission constraints and auction format of each congestion management method

		Considered transmission constraints	Auction format
Nodal pricing		All transmission constraints	Marginal pricing
Zonal pricing with ATC	stage 1	only inter-zonal constraints	Marginal pricing
	stage 2	All transmission constraints	Pay-as-bid
Zonal pricing with FBMC	stage 1	only critical branches	Marginal pricing
	stage 2	All transmission constraints	Pay-as-bid

We consider a transmission network where nodes $n = 1, ..., \mathcal{N}$ are connected by transmission lines $k = 1, ..., \mathcal{K}$ with transmission capacities F_k (MW). There is a fixed demand D_n (MWh) at each node. There are multiple producers $u = 1, ..., \mathcal{U}$ with production capacities G_u which are located in some of the nodes in the network.

We assume there is no uncertainty in our model. This deterministic setting is used in our paper in order to simplify our model. Since our paper is one of the first papers which studies the inc-dec game in zonal markets with FBMC, we focus on the model without making it cloudy with increased complexity of uncertainty modeling. This assumption is commonly used in the relevant literature as in [12], [26], [29] and [30]. However, the case of stochastic framework is a good extension of this work. We assume that each producer makes one price bid per plant, which corresponds to the design in Colombia. But most other markets would allow producers to make several bids for one plant, so that the bid price would increase with the output of the plant. In practice, producers choose bid prices from a discrete set. We also consider problems with a finite set of permissible prices, but we approximate reality by considering a coarse price grid with just a few allowed price levels. We assume that the market operator discloses all relevant parameters, i.e. transmission network parameters, nodal demands, before the day-ahead market stage. Moreover the marginal costs of producers are common knowledge among producers. This assumption is a standard assumption in the relevant literature as in [31], [32], [33], [34], [35] and [36].

The following three sections derive the mathematical models of imperfect competition for nodal pricing, zonal pricing with ATC and zonal pricing with FBMC. The detailed list of symbols used in the mathematical models can be found in Appendix B.

3. Imperfect Competition under Nodal Pricing

Under nodal pricing all transmission constraints are considered in the day-ahead market, and therefore there is no need for the re-dispatch stage. For this reason, a market under nodal pricing can be modeled by a one-stage game. We mathematically model this one-stage game using the same approach as in [26]. Fig. 1 illustrates the derivation of the one-stage model.

We assume each generator has a finite set of biding strategies $s = 1, ..., \mathcal{S}$. After all generators submit their strategic price bids, $\hat{c}_u$ (\$/MWh), the market operator collects the bids and decides the dispatch of each producer, g_u (MWh), and the local market price, λ_n (\$/MWh), in each node.

The Nash equilibrium of the one-stage game is an outcome where no producer can increase its profit, π_u (\$/h), by changing its bid unilaterally. This is modeled in inequality (1).

$$\pi_u(s_u^*, s_{-u}^*) \geq \pi_u(s_u, s_{-u}^*) \quad \forall u \tag{1}$$

Inequality (1) ensures that for each producer the profit in the Nash equilibrium strategy $\pi_u(s_u^*, s_{-u}^*)$ is greater or equal than the profit in all alternative strategies $\pi_u(s_u, s_{-u}^*)$ while holding its rivals' strategies fixed. We can formulate the profit of producer u as
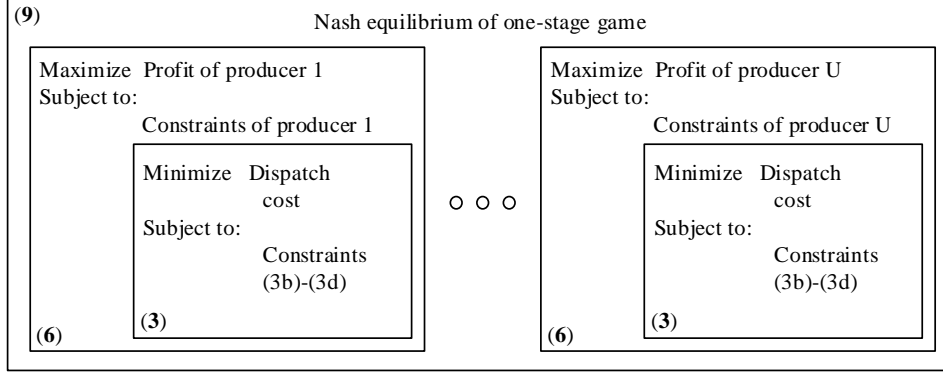
Figure 1: The derivation of one-stage game model under nodal pricing, mathematical formulations of each box is stated in bold fonts in paranthesis

in (2).

$$\pi_u = (\sum_{u:n} \lambda_n - C_u)g_u \tag{2}$$

In (2), π_u is the profit of producer u, $\sum_{u:n} \lambda_n$ is the nodal price faced by producer u which is located at node n and C_u is the marginal cost of producer u. The price and dispatch in the nodal market are decided by an economic dispatch problem. It is formulated in (3).

$$\underset{g_u}{\text{Minimize}} \sum_{u} \hat{c}_u g_u \tag{3a}$$

Subject to:

$$\sum_{u} g_u = \sum_{n} D_n \ : (\zeta) \tag{3b}$$

$$F_k - \sum_{n} H_{k,n}(\sum_{n:u} g_u - D_n) \geq 0 \ : (\tau_k) \ \forall k \tag{3c}$$

$$0 \leq g_u \leq G_u \ : (\rho_u, \theta_u) \ \forall u \tag{3d}$$

Problem (3) is a linear program (LP). Objective function (3a) expresses the dispatch cost. Constraint (3b) is the energy balance constraint. Constraints (3c) and (3d) represent both upper and lower limits of the transmission network and the generation limits, respectively. In (3c), $H_{k,n}$ is the nodal Power-Transfer-Distribution-Factors (PTDF). Since Problem (3) is a linear program (LP), its Karush-Kuhn-Tucker (KKT) conditions are both necessary and sufficient. The stationary, dual feasibility and strong duality conditions of (3) are derived in (4a), (4b) and (4c), respectively.

$$-\hat{c}_u + \zeta - \sum_{u:n}(\sum_{k} H_{k,n}\tau_k) + \rho_u - \theta_u = 0, \ \forall u \tag{4a}$$

$$\tau_k, \rho_u, \theta_u \geq 0 \tag{4b}$$

$$-\sum_{u} \hat{c}_u g_u - (-\zeta \sum_{n} D_n + \sum_{k} \tau_k(F_k + \sum_{n} H_{k,n}(\sum_{n} D_n)) + \sum_{u} \theta_u G_u) = 0 \tag{4c}$$

Nodal price is defined by the subtraction of the marginal cost of transmission congestions from the marginal cost at the reference node [37]. The marginal cost at the reference node is given by the Lagrange multiplier of energy balance constraint (3b), ζ, and the marginal cost of transmission congestion for line k is given by the Lagrange multiplier of transmission-network-limit constraint (3c), τ_k. Mathematically, the nodal price faced by producer u located at node n is expressed by $\lambda_u = (\zeta - \sum_{u:n} \sum_k H_{k,n} \tau_k)$. Using this expression and stationary condition (4a), it can be written as $\lambda_u = \hat{c}_u - \rho_u + \theta_u$. If we put this expression in (2), we have $\pi_u = (\hat{c}_u - C_u)g_u - \rho_u g_u + \theta_u g_u$. The complementary slackness conditions of (3d) are $\rho_u g_u = 0$ and $\theta_u(G_u - g_u) = 0$. Using these conditions, we can replace $\rho_u g_u$ by 0 and $\theta_u g_u$ by $\theta_u G_u$ in the profit expression. Finally the profit function (2) is reformulated as in (5).

$$\pi_u = (\hat{c}_u - C_u)g_u + \theta_u G_u, \ \forall u \tag{5}$$

Now we can formulate the strategic bidding model of a producer in the nodal power market as in (6).

$$\underset{g_u, \hat{c}_u, \zeta, \tau_k, \rho_u, \theta_u, \pi_u}{\text{Maximize}} \quad \pi_u = (\hat{c}_u - C_u)g_u + \theta_u G_u \tag{6a}$$

Subject to:

Constraints $(3b) - (3d), (4)$ $\tag{6b}$

$\hat{c}_u \in [\underline{C}_u, \overline{C}_u], \ \forall u$ $\tag{6c}$

Objective function (6a) maximizes the profit of producer u while satisfying the KKT conditions of (3) and the bid limits in (6c). In maximization problem (6), bilinear term $\hat{c}_u g_u$ appears in (4c) and (6a). We approximate the price bids $\hat{c}_u$ in interval $[\underline{C}_u, \overline{C}_u]$ by a set of discrete values as in [29]. This approximation is modeled using binary variable $x_{u,a}$ in (7). Parameter B_a is set in a way that the upper and lower limits and pre-defined number of values in between can be considered.

$$\hat{c}_u = \sum_a B_a x_{u,a} C_u \tag{7}$$

This approximation transforms the type of bilinearity in term $\hat{c}_u g_u$ from the product of two continuous variables to the product of a continuous and a binary variable. The latter type of bilinearity can be linearized by McCormick reformulation [38]. We replace $\hat{c}_u$ by its definition in (7) and linearize bilinear term $x_{u,a} g_u$. To do this, we first define a new variable $\vartheta_{u,a}$ and replace bilinear term $x_{u,a} g_u$ by $\vartheta_{u,a}$. Then we add the constraints in (8) to the model in (6).

$$g_u + G_u(x_{u,a} - 1) \leq \vartheta_{u,a} \leq g_u \tag{8a}$$

$$0 \leq \vartheta_{u,a} \leq G_u x_{u,a} \tag{8b}$$

After the linearization of $x_{u,a} g_u$, the resulting model is a MILP model which represents the strategic bidding problem of a producer. Since each producer chooses its bid from a discrete set, the set of its alternative strategies $\{\hat{c}_u(s_1, s^*_{-u}), \hat{c}_u(s_2, s^*_{-u}), \hat{c}_u(s_{\mathcal{U}}, s^*_{-u})\}$ can be formed by different combinations of binary variable $x_{u,a}$. We can

calculate each producer's profit in all alternative strategies while holding its rivals' strategies fixed. This enables us to replace the objective function of each producer's bidding problem by (1). This transforms the problem of each producer into a system of mixed-integer linear constraints (MILCs). Solving all producers' MILCs together gives us the Nash equilibrium of the one-stage game.

Our model may have multiple Nash equilibria, to tackle this, we employ the worst Nash equilibrium (WNE) concept where we search the Nash equilibrium which has the worst (highest) dispatch cost [26] and [39]. The WNE problem is formulated in (9).

$$\underset{\hat{\Psi}}{\text{Maximize}} \quad \sum_u \hat{c}_u g_u \tag{9a}$$

Subject to:

Nash equilibrium constraint (1) $\qquad\qquad$ (9b)

For Nash equilibrium strategy s_u^*, $\forall u$:

$$\{(3b) - (3d), (4), (5), (7), (8)\} \tag{9c}$$

For all alternative strategies (s_u, s_{-u}^*), $\forall u$:

$$\{(3b) - (3d), (4), (5), (7), (8)\} \tag{9d}$$

The set of decision variables in (9) is $\hat{\Psi} = \Psi \cup \Psi(s_u, s_{-u}^*)$ where $\Psi = \{g_u, \zeta, \tau_k, \rho_u, \theta_u, \pi_u, \hat{c}_u, x_{u,a}, \vartheta_{u,a}\}$. Constraints (9c) and (9d) are written for Nash equilibrium strategy s_u^*, and for all alternative strategies (s_u, s_{-u}^*), respectively.

4. Imperfect Competition under Zonal Pricing with ATC

Zonal pricing with ATC has two stages. The first stage is the day-ahead market stage and the second stage is the re-dispatch stage. All producers can trade power in both stages. The considered market has zones $z = 1, ..., \mathcal{Z}$ which aggregates specific nodes. The trade between neighboring zones $l = l, ..., \mathcal{L}$ is limited by the ATC value, $\bar{F}_l (MW)$. Timing of the information exchanges between the market operator and the producers under zonal pricing with ATC is illustrated in Fig. 2.

Before the first stage, the market operator decides the ATCs between zones and announces them. In the first stage of the two-stage game, each producer considers the given ATCs, the Nash equilibrium in the re-dispatch stage and its rivals' (predicted) day-ahead bids and decides its own day-ahead bid, $\hat{c}_u$ ($/MWh). The market operator receives the day-ahead bids from all producers and clears the day-ahead market. The day-ahead dispatch results, g_u (MWh), and the zonal prices, δ_z ($/MWh), are announced by the market operator. In the second stage, each producer considers the day-ahead dispatch and its rivals' (predicted) regulation bids and decides its up- and down-regulation bid, $\hat{c}_u^{up}$ ($/MWh) and $\hat{c}_u^{dn}$ ($/MWh). The market operator decides the dispatch of the up- and down-regulation, g_u^{up} (MWh) and g_u^{dn} (MWh), in order to relieve the overloading induced by the day-ahead dispatch.

The two-stage game under zonal pricing with ATC is modeled in our previous paper [25] where the imbalance uncertainties are modeled and studied. Accordingly, the second
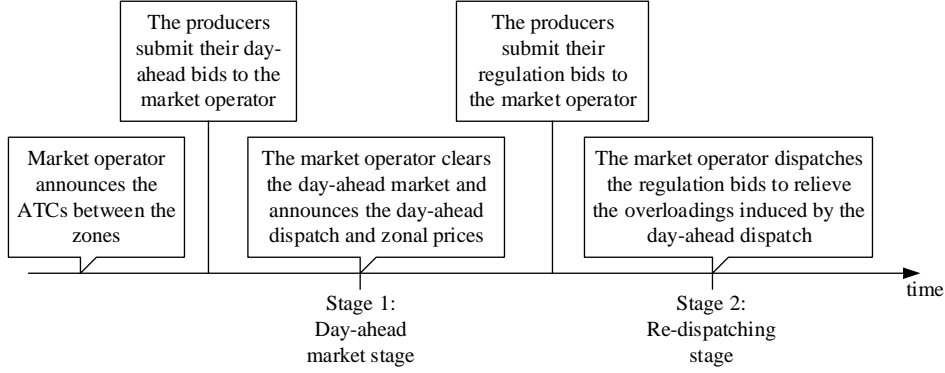
Figure 2: Timing of information exchanges under zonal pricing with ATC

stage is cleared for supplying the imbalances and for re-dispatching. In this paper, we assume that there is no uncertainty in the market. This simplifies our model since the second stage of the market is only used for re-dispatching. Fig. 3 illustrates the derivation of the two-stage game mathematical model. The SPNE in this two-stage game can be found by backward induction. Thus we start to derive our mathematical model with the last stage, the re-dispatch stage.
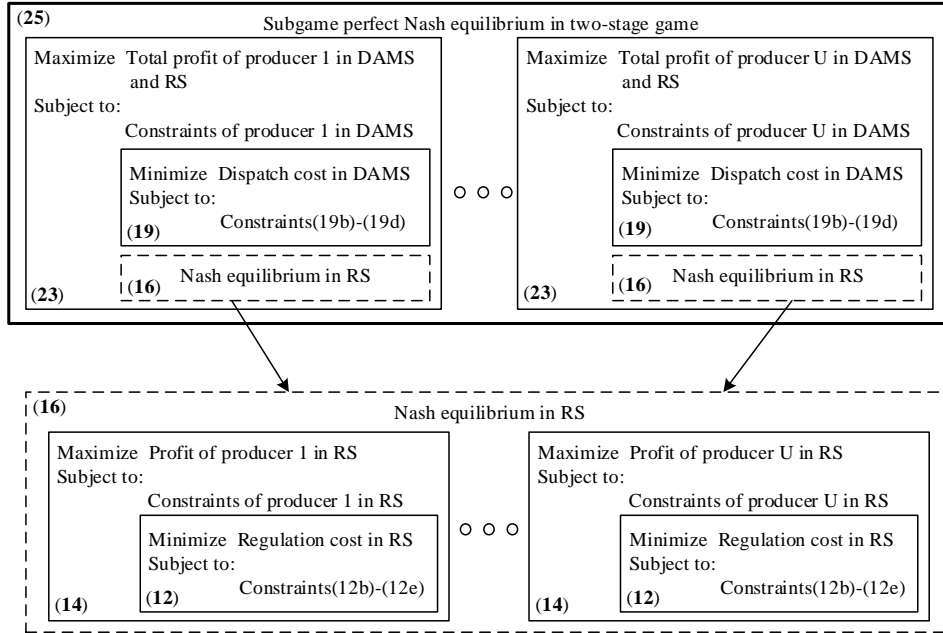

Figure 3: The derivation of two-stage game model under zonal pricing, mathematical formulations of each box is stated in bold fonts in paranthesis, DAMS: Day-ahead market stage, RS: Re-dispatch stage

4.1. Nash equilibrium in the re-dispatch stage

The Nash equilibrium in the re-dispatch stage is an outcome where no producer can increase its profit in the re-dispatch stage, ϕ_u (\$/h), by changing its bid unilaterally. This is modeled in inequality (10).

$$\phi_u(\omega_u^*, \omega_{-u}^*) \geq \phi_u(\omega_u, \omega_{-u}^*), \quad \forall u \tag{10}$$

In inequality (10), $\phi_u(\omega_u^*, \omega_{-u}^*)$ and $\phi_u(\omega_u, \omega_{-u}^*)$ represents the profit in the re-dispatch stage for Nash equilibrium strategy (ω_u^*) and for alternative strategies (ω_u) while holding its rivals' strategies fixed. In the re-dispatch stage, each producer is paid according to its own price-bid. The profit of producer u in the re-dispatch stage is formulated in (11).

$$\phi_u = (\hat{c}_u^{up} - C_u^{up})g_u^{up} + (C_u^{dn} - \hat{c}_u^{dn})g_u^{dn} \tag{11}$$

The re-dispatch stage is used for relieving the overloading in the transmission system induced by the day-ahead dispatch. The market operator dispatches the down-regulation bids to reduce the injections from the export-constrained nodes and it dispatches the up-regulation bids to increase the injections from the import-constrained nodes. The dispatch is decided by the solution of a bid-based economic dispatch problem. This bid-based economic dispatch problem is formulated in (12).

$$\underset{g_u^{up}, g_u^{dn}}{\text{Minimize}} \sum_u (\hat{c}_u^{up} g_u^{up} - \hat{c}_u^{dn} g_u^{dn}) \tag{12a}$$

$$\sum_u g_u + g_u^{up} - g_u^{dn} - \sum_n D_n = 0 \; : (\alpha) \tag{12b}$$

$$F_k - \sum_n H_{k,n}(\sum_{n:u} g_u + g_u^{up} - g_u^{dn} - D_n) \geq 0 \; : (\mu_k) \; \forall k \tag{12c}$$

$$0 \leq g_u^{up} \leq G_u - g_u \; : (\kappa_u, \beta_u) \; \forall u \tag{12d}$$

$$0 \leq g_u^{dn} \leq g_u \; : (\psi_u, \varphi_u) \; \forall u \tag{12e}$$

Objective function (12a) minimizes the total regulation net-expenses. The energy balance constraint is represented in (12b). Constraint (12c) models both upper and lower bounds of the transmission limits and $u : n$ represents the connection of producer u to node n. The regulation capacity constraints are represented in (12d) and (12e). g_u is the given dispatch level in the day-ahead market. The Lagrange multipliers corresponding to the constraints are presented in parentheses following the related constraint. Optimization model (12) is LP so its KKT conditions are both necessary and sufficient. The stationary, dual feasibility and strong duality conditions of (12) are derived in (13a)-(13b),(13c) and (13d), respectively.

$$-\hat{c}_u^{up} + \alpha - \sum_{u:n}(\sum_k H_{k,n}\mu_k) + \kappa_u - \beta_u = 0, \ \forall u \tag{13a}$$

$$\hat{c}_u^{dn} - \alpha + \sum_{u:n}(\sum_k H_{k,n}\mu_k) - \psi_u - \varphi_u = 0, \ \forall u \tag{13b}$$

$$\mu_k, \kappa_u, \beta_u, \psi_u, \varphi_u \geq 0 \tag{13c}$$

$$\sum_u(-\hat{c}_u^{up}g_u^{up} + \hat{c}_u^{dn}g_u^{dn}) - (\alpha(\sum_u g_u - \sum_n D_n) + \sum_u(\beta_u(G_u - g_u) + \varphi_u g_u) +$$
$$\sum_k \mu_k(F_k - \sum_n H_{k,n}(\sum_{n:u} g_u - D_n)) = 0 \tag{13d}$$

The strategic bidding model of a producer in the re-dispatch stage is formulated in (14).

$$\underset{\Phi}{\text{Maximize}} \quad \phi_u = (\hat{c}_u^{up} - C_u^{up})g_u^{up} + (C_u^{dn} - \hat{c}_u^{dn})g_u^{dn} \tag{14a}$$

Subject to:

Constraints $(12b) - (12e), (13)$ \hfill (14b)

$$\hat{c}_u^{up} \in [\underline{C}_u^{up}, \overline{C}_u^{up}], \ \forall u \tag{14c}$$

$$\hat{c}_u^{dn} \in [\underline{C}_u^{dn}, \overline{C}_u^{dn}], \ \forall u \tag{14d}$$

Objective function (11) maximizes the profit of producer u in the re-dispatch stage while satisfying the KKT conditions of (12) and the regulation bid limits in (14c) and (14d). The set of decision variables in (14) is $\Phi = \{g_u^{up}, g_u^{dn}, \alpha, \mu_k, \kappa_u, \beta_u, \psi_u, \varphi_u, \hat{c}_u^{up}, \hat{c}_u^{dn}\}$. Maximization problem in (14) has two bilinear terms. These are (a) $\hat{c}_u^{up}g_u^{up}$ in (14a) and (13d) and (b) $\hat{c}_u^{dn}g_u^{dn}$ in (14a) and (13d). We approximate the regulation bids $\hat{c}_u^{up}$ and $\hat{c}_u^{dn}$ in interval $[\underline{C}_u^{up}, \overline{C}_u^{up}]$ and $[\underline{C}_u^{dn}, \overline{C}_u^{dn}]$ by a set of discrete values. It is modeled by binary variables $x_{u,a}^{up}$ and $x_{u,a}^{dn}$ in (15).

$$\hat{c}_u^{up} = \sum_a B_a x_{u,a}^{up} C_u^{up}, \quad \hat{c}_u^{dn} = \sum_a B_a x_{u,a}^{dn} C_u^{dn} \tag{15}$$

After replacing $\hat{c}_u^{up}$ and $\hat{c}_u^{dn}$ by the expressions in (15), the type of the bilinearity in terms $\hat{c}_u^{up}g_u^{up}$ and $\hat{c}_u^{dn}g_u^{dn}$ is transformed from the product of two continuous variables to the product of a continuous variable and a binary variable. This type of bilinearity can be linearized by McCormick reformulation as explained before.

After the linearization of the bilinear terms, the resulting model becomes a MILP. Since each producer chooses its regulation bid from a discrete set, we can transform each producer's bidding problem into a MILC model as explained before. The solution of all producers' bidding problem simultaneously gives us the Nash equilibrium in the re-dispatch stage. This is formulated by a feasibility problem in (16).

11

Find $\hat{\Phi} \cup \hat{\Phi}(\omega_u, \omega^*_{-u})$ $\qquad$ (16a)

Such that

Nash equilibrium constraint (10) $\qquad$ (16b)

For Nash equilibrium strategy ω^*_u, $\forall u$:

$\Big\{$Constraints $(11), (12b) - (12e), (13), (15)$ and linearization of $x^{up}_{u,a} g^{up}_u$

and $x^{dn}_{u,a} g^{dn}_u$ as in $(8)\Big\}$ $\qquad$ (16c)

For all alternative strategies $(\omega_u, \omega^*_{-u})$ $\forall u$:

$\Big\{$Constraints $(11), (12b) - (12e), (13), (15)$ and linearization of $x^{up}_{u,a} g^{up}_u$

and $x^{dn}_{u,a} g^{dn}_u$ as in $(8)\Big\}$ $\qquad$ (16d)

Where in (16), $\hat{\Phi} = \Phi \cup \{x^{up}_{u,a}, x^{dn}_{u,a}\}$. Constraints (16c) and (16d) are written for Nash equilibrium strategy ω^*_u, and for all alternative strategies $(\omega_u, \omega^*_{-u})$, respectively.

4.2. The SPNE of the two-stage game

The SPNE of the two-stage game is an outcome where no producer can increase its total profit both in the day-ahead market stage and in the re-dispatch stage by changing its bidding strategy unilaterally. This is modeled in inequality (17).

$$\pi_u(s^*_u, s^*_{-u}) + \phi_u(s^*_u, \omega^*_u, s^*_{-u}, \omega^*_{-u}) \geq \pi_u(s_u, s^*_{-u}) + \phi_u(s_u, \omega_u, s^*_{-u}, \omega^*_{-u}), \ \forall u \qquad (17)$$

In equality (17), $\pi_u(s^*_u, s^*_{-u})$ and $\pi_u(s_u, s^*_{-u})$ represent the profit in the day-ahead market stage in SPNE strategy (s^*_u) and in the alternative strategy (s_u), respectively. The profits in the re-dispatch stage in SPNE strategy (s^*_u, ω^*_u) and in alternative strategy (s_u, ω_u) are represented in $\phi_u(s^*_u, \omega^*_u, s^*_{-u}, \omega^*_{-u})$ and $\phi_u(s_u, \omega_u, s^*_{-u}, \omega^*_{-u})$, respectively.

The profit of producer u in the day-ahead market stage is formulated in (18).

$$\pi_u = (\sum_{z:u} \delta_z - C_u) g_u \qquad (18)$$

Here δ_z is the zonal price in the day-ahead market stage. The price and the dispatch of the producers in the day-ahead market stage is determined by an economic dispatch model. It is formulated in (19).

$$\underset{g_u}{\text{Minimize}} \sum_u \hat{c}_u g_u \qquad (19a)$$

Subject to:

$$\sum_u g_u = \sum_n D_n \ : (\xi) \qquad (19b)$$

$$\bar{F}_l - \sum_z H'_{l,z}(\sum_{z,u}(g_u) - \sum_{z:n}(D_n)) \geq 0 \ : (\gamma_l) \ \forall l \qquad (19c)$$

$$0 \leq g_u \leq G_u \ : (\eta_u, \nu_u) \ \forall u \qquad (19d)$$

Objective function (19a) minimizes the dispatch cost in the day-ahead market stage while satisfying the energy balance constraint (19b), the available transmission capacity limits between zones (19c) and the generation limits (19d). In (19c), $H'_{l,z}$ is the zonal PTDF for zonal pricing with ATC. Problem (19) is a LP, so therefore its KKT conditions are necessary and sufficient. The stationary, the dual feasibility and the strong duality condition of (19) are set out in (20a),(20b) and (20c), respectively.

$$- \hat{c}_u + \xi - \sum_{u:z}(\sum_l H'_{l,z}\gamma_l) + \eta_u - \nu_u = 0, \ \forall u \tag{20a}$$

$$\gamma_l, \eta_u, \nu_u \geq 0 \tag{20b}$$

$$- \sum_u \hat{c}_u g_u - (-\xi \sum_n D_n + \sum_l \gamma_l(\bar{F}_l - \sum_z H'_{l,z}(\sum_{z:n} -D_n)))$$
$$+ \sum_u \nu_u G_u = 0 \tag{20c}$$

The zonal price in zone z in the day-ahead market stage is calculated by $\delta_z = \xi - \sum_l H'_{l,z}\gamma_l$. Using this expression, stationary condition (20a) and the complementary slackness conditions for (19d), the day-ahead profit in (21) is reformulated in (21).

$$\pi_u = (\hat{c}_u - C_u)g_u + \nu_u G_u, \ \forall u \tag{21}$$

Similar to the regulation bids in the re-dispatch stage, the day-ahead bids are approximated by a set of discrete values. This is modeled by binary variables $x_{u,a}$ in (22).

$$\hat{c}_u = \sum_a B_a x_{u,a} C_u \tag{22}$$

Now, we can formulate the strategic bidding problem of producer u in both day-ahead market and re-dispatch stages in (23).

$$\underset{\Theta}{\text{Maximize}} \ \pi_u + \phi_u \tag{23a}$$

Subject to:

Constraints $(16), (19b) - (19d), (20), (21), (22)$ \hfill (23b)

The set of decision variables in (23) is $\Theta = \{g_u, \xi, \gamma_l, \eta_u, \nu_u, \pi_u, x_{u,a}\}$. Bilinear term $x_{u,a}g_u$ appears in (20c) and (21). It is linearized by McCormick reformulation. Since day-ahead dispatch decision (g_u) is a variable in maximization problem (23), we have three bilinear terms. These are **(a)** $\mu_k \sum_n H_{k,n}(\sum_{n:u} g_u)$ in (13d), **(b)** $\beta_u g_u$ in (13d) and **(c)** $\varphi_u g_u$ in (13d). To linearize these bilinear terms, we use the lemma proposed in [25]. This lemma states that since day-ahead bids are selected from a finite set of discrete values, the corresponding dispatch, g_u, is also selected from a finite set of discrete values. Accordingly, g_u is a discrete variable which can be formulated by binary variables $y_{u,r}$ in (24).

$$g_u = \sum_r E_{u,r}y_{u,r}, \ \forall u \tag{24}$$

Here parameter $E_{u,r}$ is the day-ahead dispatch of producer u in bid combination r. If the number of the producers in the day-ahead market stage is $\mathcal{U}$ and each of them has $\mathcal{A}$ bid alternatives, then there can be $\mathcal{A}^{\mathcal{U}}$ different bid combinations. Here we define index r which is $r = 1, ..., \mathcal{A}^{\mathcal{U}}$. Parameter $E_{u,r}$ is calculated by solving problem (19) for all $\mathcal{A}^{\mathcal{U}}$ bid combinations. The problem (19) in each bid combination is independent from each other so the calculation of parameter $E_{u,r}$ can be done in parallel.

This approach transforms the bilinear terms in **(a)-(c)** to the product of a binary variable and a continuous variable. They are linearized by McCormick reformulation as explained before. The resulting model which represents the bidding problem of a producer in both stages is a MILP. Since each producer chooses its bid from a discrete set, we can transform each producer's bidding problem into a MILC model. The solution of all producers' two-stage bidding problem simultaneously gives us the SPNE of the two-game. Our model may have multiple SPNE. To tackle this situation, we employ the worst SPNE (WSPNE) concept where we find the SPNE which has the worst (highest) total dispatch cost in two stages. The WSPNE is formulated in (25).

$$\underset{\hat{\Theta}}{\text{Maximize}} \sum_u \hat{c}_u g_u + \hat{c}_u^{up} g_u^{up} - \hat{c}_u^{dn} g_u^{dn} \tag{25a}$$

Subject to:

SPNE Constraint (17) $\tag{25b}$

For SPNE strategy (s_u^*, ω_u^*), $\forall u$:

$$\left\{ \text{Constraints } (16), (19b) - (19d), (20), (21), (22), (24) \right\} \tag{25c}$$

For all alternative strategies $(s_u, s_{-u}^*, \omega_u, \omega_{-u}^*)$, $\forall u$:

$$\left\{ \text{Constraints } (16), (19b) - (19d), (20), (21), (22), (24) \right\} \tag{25d}$$

The set of decision variables in (25) is $\hat{\Theta} = \tilde{\Theta} \cup \tilde{\Theta}(s_u, s_{-u}^*, \omega_u, \omega_{-u}^*)$ where $\tilde{\Theta} = \Theta \cup \{y_{u,r}\}$. Constraints (25c) and (25d) are written for SPNE strategy (s_u^*, ω_u^*), and for all alternative strategies $(s_u, s_{-u}^*, \omega_u, \omega_{-u}^*)$, respectively.

5. Imperfect Competition under Zonal Pricing with FBMC

Similar to the zonal pricing with ATC, the zonal pricing with FBMC is also employed in two stages. The first stage is the day-ahead market stage and the second stage is the re-dispatch stage. In the day-ahead market stage, the market operator considers a better representation of the transmission system in the market clearing mechanism. This reduces the overloading in the intra-zonal transmission lines but it may not avoid it entirely. Hence, a re-dispatch may still be required after the day-ahead market stage is cleared.

Before the producers submit their bids to the day-ahead market, the market operator forecasts the cross-border trades and it announces the critical branches (CBs) and their corresponding Remaining Available Margins (RAMs). Accordingly, the producers play the two-stage game as for zonal markets with ATC. The CBs and their RAMs are

calculated using the market operator's historical data. In this study, we decide CBs and the related parameters in three steps.

Step 1: Estimation of Generation Shift Keys (GSKs)
GSKs are important parameters when calculating power transfer distribution factors (PTDFs) for the market. The GSKs indicate the contribution of 1 MWh injection from node n to the balance in zone z. In practice, the GSKs are calculated by considering a base case which is derived from a snapshot of the power system showing the injections, the power flows in the transmission network and the changes in the network topology [40]. This procedure is not documented for the public. In this study we calculate the GSKs, $R_{n,z}$, by nodal injections divided by zonal net injections as in [24] and [40]. This is formulated in (26).

$$R_{n,z} = \frac{\sum_{n:u} g_u^* - D_n}{\sum_{n:z}((\sum_{n:u} g_u) - D_n)} \tag{26}$$

Here $\sum_{n:u} g_u^*$ is the production of producer u which is located at node n. To find the production level of producer u, the nodal economic dispatch model in (3) is performed. Since our model has no uncertainty, we assume that the same outcome has been repeated for several years. Accordingly, the market operator has a historical data of all producers' day-ahead bids under zonal pricing with FBMC. The market operator considers these historical data of the bids while solving the nodal economic dispatch model in (3). The GSKs cannot be determined in case of balanced zones since denominator $\sum_{n:z}((\sum_{n:u} g_u) - D_n)$ becomes zero.

Step 2: Calculation of Zonal PTDF
After the estimation of GSKs, the zonal PTDF is calculated by means of GSK and the nodal PTDF. It is formulated in (27).

$$\hat{H}_{k,z} = \sum_n H_{k,n} R_{n,z} \tag{27}$$

The zonal PTDF matrix, $\hat{H}_{k,z}$, indicates the power flow in line k when 1 MWh is injected from zone z.

Step 3: Decision of CBs and RAMs
The CBs are the transmission lines which are impacted by the cross-border trades [41]. Therefore they are considered in the economic dispatch problem in the day-ahead market stage of the zonal pricing with FBMC. The lines are decided as CBs if their zone-to-zone PTDF is larger than a certain threshold [42]. The zone-to-zone PTDF, T_z, is calculated as in (28).

$$T_k = \sum_{z, z \neq z'} |\hat{H}_{k,z} - \hat{H}_{k,z'}| \tag{28}$$

The RAM is the transmission capacity of the CB which is used in the day-ahead

market stage of the zonal pricing with FBMC. In practice, the market operator does not assign the whole physical transmission capacity of a line for the day-ahead market stage. Instead, it saves some capacity to ensure the secure operation of the power system if an outage occurs. In this study, we do not model any outage in the power system. Accordingly, we assume that the market operator assigns all physical transmission capacity RAM of the corresponding transmission line. However, our assumption can be easily adjusted to model the practice.
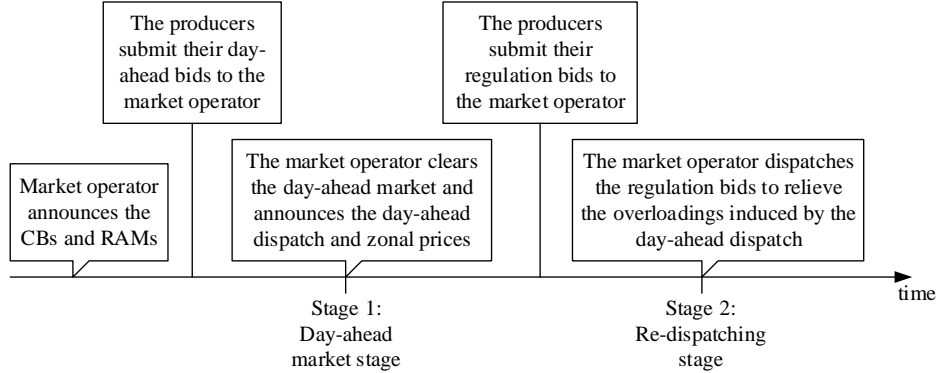

Figure 4: The timing of information exchanges under zonal pricing with FBMC

Before the first stage, the market operator calculates and announces the CBs and RAMs. The information exchange is illustrated in Fig. 4. In the first stage, the producers consider the announced information, its rivals' (predicted) day-ahead bid and the Nash equilibrium in the re-dispatch stage. The market operator receives the day-ahead bids from all producers and clears the day-ahead market. Then, the producers are informed by their dispatch level and the market price. In the second stage, each producer considers the day-ahead dispatch and its rivals' (predicted) regulation bids to decide its regulation bids. The market operator receives the regulation bids from all producers and it dispatches the regulation bids in order to relieve the overloading induced by the day-ahead dispatch.

The two-stage game and its mathematical model is rather similar to section 4. The model of the Nash equilibrium in the re-dispatch stage is exactly the same. The only difference is the model of the transmission constraints in the day-ahead market stage. In economic dispatch problem (19), we replace (19c) by (29). In (29), $\mathcal{I}$ is the set of selected CBs.

$$F_k - \sum_z \hat{H}_{k,z}(\sum_{z:n} g_u - \sum_{z:n} D_n) \geq 0 : \ (\hat{\gamma}_k) \ \forall k \in \mathcal{I} \tag{29}$$

The economic dispatch model in the day-ahead market stage of zonal pricing with FBMC becomes (19a),(19b),(29) and (19d). We write the KKT conditions of the resulting model and its stationary, the dual feasibility and the strong duality condition are set out in (30a),(30b) and (30c), respectively.

16

$$-\hat{c}_u + \xi - \sum_{u:z}(\sum_{k \in I} \hat{H}_{k,z}\hat{\gamma}_k) + \eta_u - \nu_u = 0, \ \forall u \tag{30a}$$

$$\hat{\gamma}_{k \in I}, \eta_u, \nu_u \geq 0 \tag{30b}$$

$$-\sum_u \hat{c}_u g_u - (\xi \sum_n D_n + \sum_{k \in I} \hat{\gamma}_k (F_k - \sum_z \hat{H}_{k,z}(\sum_{n:z} -D_n)))$$
$$+ \sum_u \nu_u G_u = 0 \tag{30c}$$

In (31), we formulate the WSPNE model of the two-stage game under zonal pricing with FBMC.

$$\underset{\hat{\Lambda}}{\text{Maximize}} \ \sum_u \hat{c}_u g_u + \hat{c}_u^{up} g_u^{up} - \hat{c}_u^{dn} g_u^{dn} \tag{31a}$$

Subject to:
   SPNE Constraint (17) $\tag{31b}$

   For SPNE strategy $(s_u^*, \omega_u^*), \ \forall u:$

$$\left\{\text{Constraints } (16), (19b), (19d), (21), (22), (24), (29), (30)\right\} \tag{31c}$$

   For all alternative strategies $(s_u, s_{-u}^*, \omega_u, \omega_{-u}^*), \ \forall u:$

$$\left\{\text{Constraints } (16), (19b), (19d), (21), (22), (24), (29), (30)\right\} \tag{31d}$$

The set of decision variables in (31) is $\hat{\Lambda} = \Lambda \cup \Lambda(s_u, s_{-u}^*, \omega_u, \omega_{-u}^*)$ where $\Lambda = \tilde{\Theta} \cup \{\hat{\gamma}_k\} \setminus \{\gamma_l\}$. Constraints (31c) and (31d) are written for SPNE strategy (s_u^*, ω_u^*), and for all alternative strategies $(s_u, s_{-u}^*, \omega_u, \omega_{-u}^*)$, respectively.

6. Numerical Results

This section demonstrates the bidding behaviors of producers for the market designs that we consider. To have a congested network and to observe an inc-dec game, we modified the 6-node system in [43] and IEEE 24-node system in [44]. We decide on these modifications by running several computational experiments on the example systems. We use the 6-node system to show how the parameters for the zonal pricing with FBMC are calculated and to show the inc-dec game and its impacts on an example system. The applicability of proposed models into a large power system is shown for the modified IEEE 24-node system. The MILP models (9), (25) and (31) are solved by the CPLEX solver of the GAMS platform. In order to reduce the solution time, the GUSS facility of GAMS is used for calculating $E_{u,r}$ before solving MILP models (25) and (31).

6.1. 6-node example system

The single line diagram of 6-node system is illustrated in Fig. 5.

The 6-node system has three competing generators which are located at nodes 1, 2 and 4. In order to observe inc-dec game, the cost data in [43] are modified. The data related to the generators are presented in Table 2. The system has two zones and the
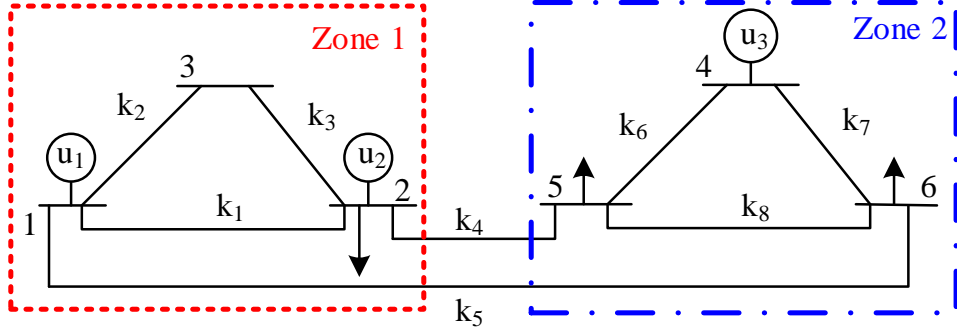
Figure 5: Single line diagram of 6-node example system

ATC between two zones is set to 90% of the total transmission capacity of line k_4 and k_5. To have a zonal system with different zonal prices, the transmission capacities of lines are set to $[F_{k_1}, F_{k_2}, F_{k_3}, F_{k_4}, F_{k_5}, F_{k_6}, F_{k_7}, F_{k_8}] = [70, 150, 100, 200, 250, 250, 180, 100]$ MW. We assume that each transmission line has an impedance of 1 per-unit. Each producer has three bidding actions for day-ahead price bids with 0%, 10% mark-up and 10% mark-down. The permissible up-regulation and down-regulation bids have 0%, 10%, 20% mark-up and 0%, 10%, 20% mark-down, respectively.

Table 2: Producer data in the 6-node system

u	C_u ($/MWh)	C_u^{up} ($/MWh)	C_u^{dn} ($/MWh)	G_u (MW)	Load	D_n (MWh)
u_1	16.5	20.5	12	500	n_2	300
u_2	14.9	19	11.5	400	n_5	300
u_3	16	19.5	12.5	400	n_6	300

The details of deciding the CBs and calculation of FBMC parameters for the 6-node system can be found in Appendix A. The lines k_4 and k_5 are found as the CBs for the zonal pricing with FBMC. We simulate all three congestion management methods. There are typically multiple equilibria. We report the worst-dispatch-cost equilibrium for each congestion management method, i.e. the WNE of the one-stage game related to nodal pricing and the WSPNE of the two-stage games related to zonal pricing with ATC and zonal pricing with FBMC. These equilibria are presented in Table 3.

Table 3: The worst-cost equilibrium in each congestion management method

	NP	ZP with ATC			ZP with FBMC		
	$\hat{c}_u$ ($/MWh)	$\hat{c}_u$ ($/MWh)	$\hat{c}_u^{up}$ ($/MWh)	$\hat{c}_u^{dn}$ ($/MWh)	$\hat{c}_u$ ($/MWh)	$\hat{c}_u^{up}$ ($/MWh)	$\hat{c}_u^{dn}$ ($/MWh)
u_1	18.15	14.85	24.6	9.6	18.15	24.6	9.6
u_2	16.39	16.39	22.8	9.2	13.41	22.8	9.2
u_3	17.6	17.6	23.4	10	14.4	23.4	10

18

For nodal pricing, all producers choose price bids which are higher than their marginal costs. In the zonal pricing with ATC, export-constrained producer, u_1, plays inc-dec game by choosing a day-ahead bid which is lower than its marginal cost. Producers u_2 and u_3 choose their day-ahead bids which are higher than their marginal costs, which is similar to the nodal market. The market operator dispatches producers u_1 at 500 MW, u_2 at 205 MW and u_3 at 195 MW. This day-ahead dispatch causes overloading in line k_1, between node 1 and 2, by 103.5 MW. The market operator dispatches the down-regulation bid of u_1 at 177.5 MW and up-regulation bid of u_2 at 177.5 MW to relieve this overloading.

In the zonal pricing with FBMC, producers u_2 and u_3 choose day-ahead bids which are lower than their marginal costs and producer u_1 chooses a price bid which is higher than its marginal cost. In this SPNE, producer u_3 decides its day-ahead bid in order to play inc-dec game. Producer u_2 bids a sufficiently low price to ensure that u_1 does not find it more profitable to deviate and undercut. Hence, setting a high price, 18.15 \$/MWh, is the most profitable strategy for producer u_1. This is related to the high-price equilibrium [45]. In the day-ahead market stage, producers u_1, u_2 and u_3 are dispatched 100 MWh, 400 MWh and 400 MWh, respectively. We observe that the day-ahead dispatch overloads the line between node 4 and node 6, line k_7, by 20 MW. Note that line k_7 is not a CB. The market operator dispatches down-regulation bid of u_3 and up-regulation bid of u_1 in order to relieve the overloading. We see that producer u_3 plays the inc-dec game and the choice of its day-ahead bid overloads line k_7. The critical branches, k_4 and k_5, are not congested and the zonal price in both zones are set by producer u_1's bid, 18.15 \$/MWh. The dispatch and the profits of each producer is summarized in Table 4.

Table 4: The dispatch and the profits of each producer in each congestion management method

	NP		ZP with ATC					ZP with FBMC				
	g_u	π_u	g_u	g_u^{up}	g_u^{dn}	π_u	ϕ_u	g_u	g_u^{up}	g_u^{dn}	π_u	ϕ_u
u_1	138.4	228.3	500	0	177.5	-55	425.9	100	38.4	0	165	157.3
u_2	400	1282.4	205	177.5	0	305.5	674.4	400	0	0	1300	0
u_3	361.6	578.6	195	0	0	674.4	0	400	0	38.4	860	95.9

Table 4 shows that under zonal pricing with ATC, producer u_1 makes a 55 \$/h loss in the day-ahead market stage and it makes a 425.9 \$/h profit in the re-dispatch stage.

Table 5 illustrates the volume of overloading, the total production cost in both stages, the total profit and the net-expenses of the market operator. The total production cost is calculated by summing the production cost in the day-ahead stage, $\sum_u C_u g_u$, and the production cost in the re-dispatch stage, $\sum_u C_u^{up} g_u^{up} - C_u^{dn} g_u^{dn}$. We observe that zonal pricing with ATC has the highest overloading due to the inc-dec game. Hence, it has the highest total production cost. Zonal pricing with FBMC reduces the volume of overloading by 81%. Accordingly, the inc-dec game is reduced as well. The total production cost in the FBMC approach is 8.6% lower than the one in the ATC approach. In this example system, nodal pricing becomes the most efficient congestion management

method since it has no overloading after the dispatch of the day-ahead market stage. Accordingly, it has the lowest total production cost. The total profit of the producers is the highest in the zonal pricing with FBMC. We observe that under zonal pricing with FBMC, no congestion occurs in the day-ahead market stage, accordingly all producers are paid by the same price which is set by the most expensive producer (producer u_1). In both zonal designs, the market operator has positive net-expenses (losses) due to the inc-dec game. However, under nodal pricing the market operator makes a profit of 190.1$/h.

Table 5: Production cost, total profit, and total payments in each congestion management methods, DAMS: Day-ahead market stage, RS: Re-dispatch stage

	NP	ZP with ATC	ZP with FBMC
Volume of overloading (MW)	0	103.5	20
Total production cost ($/h) ⓐ	14029.2	15666.8	14316.9
Total profit ($/h) ⓑ	2089.3	1662.8	2578.2
Total load payments ($/h) ⓒ	16308.6	15477	16335
Net-expenses of market operator ($/h) ⓐ+ⓑ-ⓒ	-190.1	1852.6	560.1

In Table 2, we assume that changing the production of the producers in the real-time operation is more expensive than their marginal cost i.e. last minute ramping is costly. Hence, the marginal cost of up-regulation of each producer, C_u^{up}, is set higher than its marginal cost, C_u and the marginal cost of down-regulation of each producer, C_u^{dn}, is set lower than its marginal cost. We observe that the ramping costs has an important influence on the market efficiency. To further investigate this issue, we set $C_u = C_u^{up} = C_u^{dn}$ and simulate the 6-node system. All other parameters are kept the same. Since nodal pricing does not have the re-dispatch stage, we only simulate zonal pricing with ATC and zonal pricing with FBMC. The WSPNE and the dispatch levels in two stages are the same as in Table 3 and 4 in both zonal designs. The total production cost is calculated by 14140.5$/h for zonal pricing with ATC and 14029.2$/h for zonal pricing with FBMC. We see that the ramping cost has a substantial impact on the total production cost. One other interesting observation is that without ramping costs, zonal pricing with FBMC is as efficient as nodal pricing.

6.2. Modified IEEE 24-node example system

The single line diagram of the IEEE 24-node system is illustrated in Fig 6.

To observe congestions in the network and an inc-dec game, we consider five producers with large installed capacities. The data related to the producers are presented in Table 6.

We split the IEEE 24-node system into 3 zones and the ATCs between zones are set to 90% of the total transmission capacity of the inter-zonal lines. To ensure that a
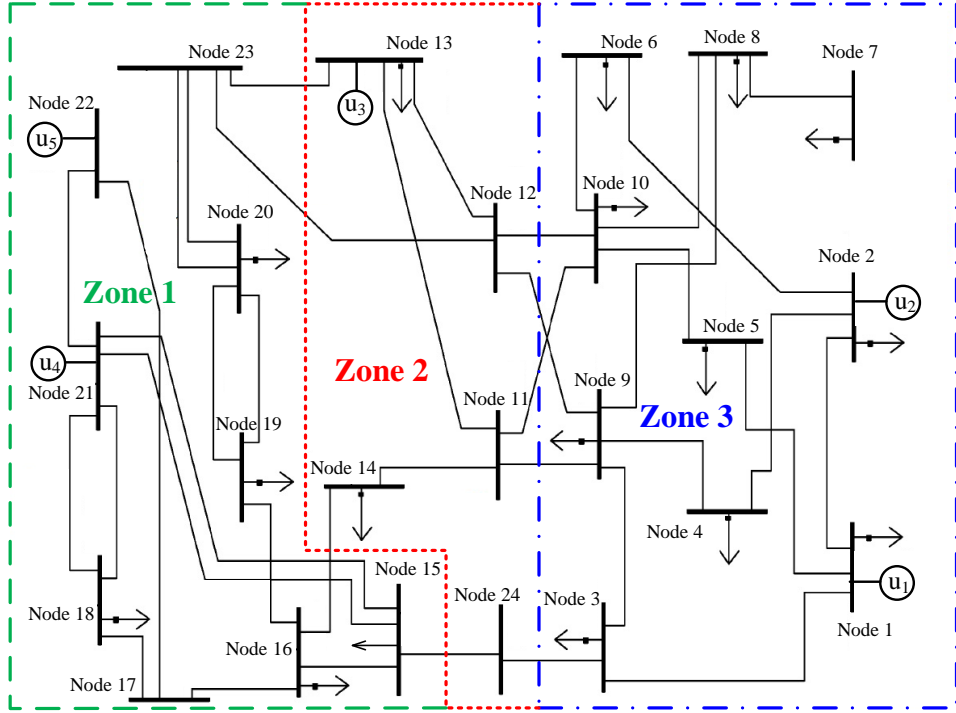
Figure 6: Single line diagram of the modified IEEE 24-node example system

Table 6: Producer data in the modified IEEE 24-node system

u	Location (Node)	C_u ($/MWh)	C_u^{up} ($/MWh)	C_u^{dn} ($/MWh)	G_u (MW)
u_1	1	17.5	25.5	14	1000
u_2	2	18	23.5	13.5	800
u_3	13	17	22.5	11.5	1000
u_4	21	16	20.5	10.5	1000
u_5	22	16.7	21.5	11	1700

feasible dispatch can be found for every bid combination for every congestion management method, the transmission capacities of lines between nodes 1-5, 16-17 and 16-19 are changed from the values in [44] to 350 MW, 800 MW and 800 MW, respectively. We assume that each producer has 3 bidding actions for day-ahead price bids: 0%, 10% mark-up and 10% mark-down. The permissible up-regulation and down-regulation bids have 0%, 10%, 20% mark-up and 0%, 10%, 20% mark-down, respectively.

We follow the steps explained in Section 5 and demonstrated on the modified IEEE 24-node network to find the CBs. We find 13 transmission lines as the CBs.

Table 7 illustrates the worst-cost equilibria (WNE in one-stage game and WSPNE in two-stage game) for each congestion management method. Under nodal pricing, all producers choose price bids higher than their marginal costs. Under zonal pricing with

Table 7: The worst-cost equilibrium in each congestion management method

	NP	ZP with ATC			ZP with FBMC		
	$\hat{c}_u$ ($/MWh)	$\hat{c}_u$ ($/MWh)	$\hat{c}_u^{up}$ ($/MWh)	$\hat{c}_u^{dn}$ ($/MWh)	$\hat{c}_u$ ($/MWh)	$\hat{c}_u^{up}$ ($/MWh)	$\hat{c}_u^{dn}$ ($/MWh)
u_1	19.25	15.75	30.6	11.2	19.25	30.6	11.2
u_2	19.8	19.8	28.2	10.8	19.8	28.2	10.8
u_3	18.7	18.7	27	9.2	15.3	27	9.2
u_4	17.6	17.6	24.6	8.4	14.4	24.6	8.4
u_5	18.37	15.03	25.8	8.8	18.37	25.8	8.8

ATC, producers u_1 and u_5 play the inc-dec game by choosing day-ahead bids which are lower than their marginal costs. The market operator dispatches u_1 at 1000 MW, u_4 at 145 and u_5 at 1700 MW. This day-ahead dispatch overloads the line between nodes 1 and 2 and the line between nodes 21 and 22 by 294.7 MW and 501.9 MW, respectively. The market operator dispatches the down-regulation bids of u_1 at 216.6 MW and u_5 at 790.9 MW and the up-regulation bids of u_2 at 387.3 MW and u_4 at 620.2 MW to relieve the overloadings. In the zonal pricing with FBMC, producers u_3 and u_4 choose day-ahead bids which are lower than their marginal costs and producers u_1, u_2 and u_5 choose day-ahead bids which are higher than their marginal cost. In the day-ahead market stage, producers u_1, u_3 and u_4 are dispatched 795 MWh, 1000 MWh and 1050 MWh, respectively. We observe that the day-ahead dispatch overloads the line between nodes 1 and 2 by 184.7 MW. The market operator dispatch down-regulation bid of u_1 at 195.8 MW and up-regulation bid of u_2 at 195.8 MW in order to relieve the overloading. The dispatch and the profits of each producer are summarized in Table 8.

Table 8: The dispatch and the profits of each producer for each congestion management method

	NP		ZP with ATC					ZP with FBMC				
	g_u	π_u	g_u	g_u^{up}	g_u^{dn}	π_u	ϕ_u	g_u	g_u^{up}	g_u^{dn}	π_u	ϕ_u
u_1	58	101.5	1000	0	216.6	100	606.4	795	0	195.8	1391.3	548.3
u_2	0	0	0	387.3	0	0	1820.2	0	195.8	0	0	920.3
u_3	807	1372.6	0	0	0	0	0	1000	0	0	2250	0
u_4	1050	2872.9	145	620.2	0	232	2542.7	1050	0	0	3412.5	0
u_5	930	1552.6	1700	0	790.9	1530	1739.9	0	0	0	0	0

Table 9 shows that the zonal pricing with FBMC, which uses a better representation of the transmission system in the day-ahead market stage, reduces the total volume of overloading by 76.8% as compared to the zonal pricing with ATC. We observe that the zonal pricing with ATC has the highest total cost due to the inc-dec game. Zonal pricing with FBMC reduces the inc-dec game, accordingly the total production cost reduces by 14.9% as compared to zonal pricing with ATC. Table 9 shows that the nodal pricing is the most efficient congestion management method also for this example system. It has a total production cost which is 23.9% and 5.3% lower than the one in ATC and

FBMC approaches, respectively. The producers' total profit is the highest in the zonal pricing with ATC due to the inc-dec game. The market operator makes losses in both zonal pricing designs due to the inc-dec game. However under nodal pricing, it makes an profit of 906.1 $/h.

Table 9: Production cost, total profit, and total payments in each congestion management methods, DAMS: Day-ahead market stage, RS: Re-dispatch stage

	NP	ZP with ATC	ZP with FBMC
Volume of overloading (MW)	0	796.6	184.7
Total production cost ($/h) ⓐ	47065	58292.7	49572.6
Total profit ($/h) ⓑ	5899.4	8571.1	8522.3
Total load payments ($/h) ⓒ	53871	50072	54766.3
Net-expenses of market operator ($/h) ⓐ+ⓑ-ⓒ	-906.1	16791.8	3328.6

Our numerical results show that under zonal pricing with ATC, producers which are located at the export-constrained nodes, u_1 in 6-node system and u_1 and u_5 in the modified IEEE 24-node system, plays the inc-dec game. This causes production inefficiencies. Zonal pricing with ATC has 10.5-18.7% higher total production cost the one under nodal pricing. Zonal pricing with FBMC represents the transmission network better than zonal pricing with ATC and this causes a reduction in the inc-dec game and an increase in the production efficiency by 8.6-14.9%. Since zonal pricing with FBMC cannot avoid the inc-dec game, it is 2-5.3% less efficient than the nodal pricing benchmark.

As in the 6-node system, we set $C_u = C_u^{up} = C_u^{dn}$ and simulate the modified IEEE 24-node system. All other parameters are kept the same. For the found equilibria, we calculate the total production cost for zonal pricing with ATC and zonal pricing with FBMC by 48191.8$/h and by 47810.4$/h, respectively. Similar to the results in the 6-node system, the ramping costs has a large impact on the total production in the modified IEEE 24-node system.

7. Conclusion

This paper analyzes imperfect competition in electricity markets under nodal and zonal pricing. We consider two types of zonal pricing: zonal pricing with ATC and zonal pricing with FBMC. The imperfect competition under nodal pricing is modeled as a one-stage game. The zonal pricing consists of two stages: the day-ahead market stage and the re-dispatch stage. We consider a two-stage game to model imperfect competition in these two stages. Mathematically, we formulate the two-stage game related to each type of zonal pricing as a two-stage EPEC and then it is reformulated as a two-stage MILP model. To tackle multiple equilibria, we use the concept of the worst-dispatch-cost equilibrium and consider the equilibrium which has the highest total dispatch cost.

The developed MILP models are demonstrated using the 6-node and the modified IEEE 24-node example systems.

Our numerical results show that the inc-dec game in zonal pricing with ATC can have large production inefficiencies and large profits for producers especially if ramping costs are high. The zonal pricing with FBMC has a better representation of the transmission system in the day-ahead market stage and it reduces inc-dec game but it cannot avoid it. Accordingly, it is less efficient than nodal pricing.

Acknowledgments

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Appendix A. Demonstration of the calculation of the FBMC parameters in the 6-node system

We first illustrate how the CBs are selected by the market operator. The nodal PTDF matrix is calculated as in [37] and it is illustrated in Table A.10.

Table A.10: Nodal PTDF matrix

	n_1	n_2	n_3	n_4	n_5	n_6
k_1	0.25	-0.333	0.042	-0.042	-0.083	0
k_2	0.125	-0.167	-0.521	-0.021	-0.042	0
k_3	-0.125	0.167	-0.479	0.021	0.042	0
k_4	0.375	0.5	0.438	-0.063	-0.125	0
k_5	0.625	0.5	0.563	0.063	0.125	0
k_6	-0.125	-0.167	-0.146	0.354	-0.292	0
k_7	0.125	0.167	0.146	0.646	0.292	0
k_8	0.25	0.333	0.292	0.292	0.583	0

The market operator considers the day-ahead bids as $\hat{c}_{u1}$=18.15 \$/MWh, $\hat{c}_{u2}$=13.41 \$/MWh, and $\hat{c}_{u3}$=14.4 \$/MWh. Given these bids, the nodal net injection at each node is calculated by solving optimization problem (3). Accordingly, GSKs are calculated by expression (26) and they are illustrated in Table A.11.

Table A.11: Generation shift keys

	n_1	n_2	n_4	n_5	n_6
z_1	0.779	0.221			
z_2			-0.395	-0.698	-0.698

Using GSKs and the nodal PTDF matrix, the zonal PTDF matrix is calculated by expression (27) and it is illustrated in Table A.12.

Table A.12: Zonal PTDF matrix

	k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8
z_1	0.121	0.061	-0.061	0.403	0.597	-0.134	0.134	0.268
z_2	-0.042	-0.021	0.021	-0.062	0.062	-0.344	-0.052	0.292

The zone-to-zone PTDF matrix is calculated by expression (28) and it is illustrated in Table A.13.

Table A.13: Zone-to-zone PTDF matrix

	k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8
T_k	0.163	0.082	0.082	0.465	0.535	0.21	0.186	0.024

We set the threshold to 0.4 when selecting the CBs in the zone-to-zone PTDF matrix. Accordingly, lines k_4 and k_5 are selected as CBs.

Appendix B. Nomenclature

The list of the notation is presented below.

Indices

u	Producer, $u = 1, ..., \mathcal{U}$
n	Power system node, $n = 1, ..., \mathcal{N}$
z, z'	Zone, $z = 1, ..., \mathcal{Z}$
k	Transmission line, $k = 1, ..., \mathcal{K}$
l	Inter-zonal line, $l = 1, ..., \mathcal{L}$
s	Bidding strategy in the day-ahead stage, $s = 1, ..., \mathcal{S}$
ω	Bidding strategy in the re-dispatch stage, $\omega = 1, ..., \Omega$
a	Bidding action of generator, $a = 0, ..., \mathcal{A}$
r	Day-ahead bid combination, $r = 1, ..., \mathcal{A}^{\mathcal{U}}$

Set

J	Set of transmission lines
$\mathcal{I}$	Set of critical branches, $\mathcal{I} \subset J$

Parameters (upper-case letters)

G_u	Installed capacity of producer u,
C_u	Marginal cost of producer u,
C_u^{up}	Marginal up-regulation cost of producer u,
C_u^{dn}	Marginal down-regulation cost of producer u,
$\overline{C}_u, \underline{C}_u,$	Upper (lower) limit of day-ahead bid of producer u,
$\overline{C}_u^{up}, \underline{C}_u^{up},$	Upper (lower) limit of up-regulation bid of producer u,
$\overline{C}_u^{dn}, \underline{C}_u^{dn},$	Upper (lower) limit of down-regulation bid of producer u,
$H_{k,n}$	Nodal PTDF matrix,
$H'_{l,z}$	Zonal PTDF matrix for zonal pricing with ATC,
$\hat{H}_{k,z}$	Zonal PTDF matrix for zonal pricing with FBMC,
F_k	Capacity of transmission line k,
$\bar{F}_l$	Available transmission capacity of inter-zonal line l,
$E_{u,r}$	Day-ahead dispatch of producer u in day-ahead bid combination r,
D_n	Demand at node n,
B_a	Step size of bidding action a,
$R_{n,z}$	Generation shift keys matrix,
T_k	Zone-to-zone PTDF matrix,

Variables (lower-case letters)

g_u	Day-ahead dispatch of producer u,
$g_u^{up}, (g_u^{dn})$	Up (down) regulation provided by producer u
$y_{u,r}$	Binary variable for modeling day-ahead dispatch of producer u in bid combination r,
x_u	Binary variable of day-ahead bidding decision of producer u,
$x_u^{up}, (x_u^{dn})$	Binary variable for up-regulation (down-regulation) bidding decision of producer u,

$\hat{c}_u$	Price bid of unit u,
$\hat{c}_u^{up}, (\hat{c}_u^{dn})$	Up-regulation (down-regulation) price bid of unit u
λ_n	Day-ahead market price at node n,
δ_z	Day-ahead zonal price at zone z,
ϕ_u	Profit of producer u in re-dispatch stage,
π_u	Day-ahead profit of producer u.