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Determinants of Economies of Scope in Retail*

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Abstract

This paper studies the determinants of economies of scope and quantifies their impact on the extensive and intensive product margins in retail. We use a framework based on a multiproduct technology to model stores' incentives to expand product variety. Using novel Swedish data on product categories and stores, we find that high-productivity stores offer more product categories and sell more products of all categories. Stores with high-demand shocks specialize in fewer product categories and sell more products of top-selling categories. Policy simulations of regional programs that target the determinants of economies of scope show that investment subsidies and mentoring support for low-productivity stores increase the number of product categories and sales per product category, especially benefiting stores in rural markets.

Keywords: economies of scope; productivity; retail; product variety; technology; competition

JEL Classification: L11, L13, L25, L81, M21

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1 Introduction

Services and retail businesses account for a rapidly growing share of economic activity. In recent years, ample investments have been made in new technologies such as mobile payment systems, and a drastic increase in warehouse clubs and a shift in consumer preferences from products to services have occured (Hortacsu and Syverson, 2015; Goolsbee, 2020). These structural changes require retailers to improve their businesses of buying multiple products from wholesalers and efficiently delivering such products to consumers while assuring quality. Buildings, equipment and supply chain facilities yield economies of scope that make it cheaper to sell many products together than to sell them separately (Panzar and Willig, 1981). Despite massive changes in the retail landscape, we lack knowledge of the determinants of economies of scope and their impact on the number of product categories (the extensive margin) and sales per product category (the intensive margin).[1]

This paper studies the determinants of economies of scale and scope in retail using a framework that models stores' multiproduct sales technology and the local market environment. We explore the trade-off between productivity and demand shocks for variety of products offered. While technology helps stores improve productivity and handle greater product variety, they can choose to offer less variety and allocate resources to provide convenience to customers, e.g., by improving service and shopping experience. We estimate the model using novel and detailed data on product categories and stores in Swedish retail between 2003 and 2009. Then, we evaluate how subsidies for investments in technology and mentoring support to foster productivity drive the number of product categories, sales per product category, store-level sales and market shares. The analysis explores differences between rural and urban markets that are of interest to policymakers in light of regional development programs including investment subsidies and mentoring support (Nordregio, 2011).

Descriptive patterns in the data motivate our research framework. Stylized facts show that stores frequently adjust their product categories. We measure product variety by the number of product categories a store offers for sale.[2] Stores with high market shares have high labor

[1] See Ellickson (2007), Basker et al. (2012), Hortacsu and Syverson (2015), and Hsieh and Rossi-Hansberg (2019).

[2] Product variety has been introduced by the entry literature (i.e., pay a fixed cost to increase variety), but we still need explanations for why firms offer multiple products in service industries. In general, economies of scope can arise from two sources: local complementarities and fixed costs (Gorman, 1985; Ellickson, 2007). Local

productivity, offer many product categories, and sell more per product category. Our data also suggest that it is important to explore heterogeneity across local markets and dynamic patterns over time, as indicated by the increase in the median market share, the four-firm concentration ratio, and the Herfindahl index (HHI). This descriptive evidence is consistent with the idea that stores utilize economies of scale and scope and productivity improvements to offer a wider variety of products. Accordingly, our framework explicitly models complementarities between economies of scale and scope in a local market setting.

We provide a novel approach based on a multiproduct technology to understand economies of scope and the role of adjustment of store inputs in altering product variety.[3] Our framework allows quantifying the increase in store-level sales resulting from offering product categories using the same resources and how the sales of a product category are affected by increasing sales of other product categories in the store. The gains from selling a larger variety of products arise from lower average costs or from increasing sales in new related product markets. Adjustments in product categories occur because retailers change their inputs or target a better match with local market demand. How many product categories to offer and how much to sell of each category are open empirical questions that depend on store resources and local demand conditions. Our framework is appealing for evaluating regional policy programs related to economies of scope and for exploring differences between rural and urban markets.

Our model highlights mechanisms through which productivity and demand shocks drive intensive and extensive product-category margins. We use the implications of the equilibrium behavior of the store's dynamic optimization problem to recover two sources of store-level heterogeneity, i.e., revenue productivity and demand shocks, which are both observed by stores but not by the researcher.[4] Revenue productivity follows an endogenous stochastic process, whereas demand shocks follow an exogenous process. Our measure of demand shocks includes features related to product quality, location, checkout speed, the courteousness of store employees, parking, bagging services, and cleanliness. To recover revenue productivity and demand shocks, we rely on two output equations – involving product-category sales and market share in-

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complementarities imply that a higher level of output of one product reduces the marginal cost of other outputs. Fixed costs can ensure economies of scope in the absence of local complementarities.

[3]See Mundlak (1964), Fuss and McFadden (1978), and Maican and Orth (2020).

[4]Unlike in manufacturing, it is difficult to define and measure technical productivity in services due to complexity of product aggregation and economies of scale and scope (Oi, 1992).

dex functions – and store's demand functions for labor and inventory, accounting for investment in technology, product variety and the local environment in which a store operates (Doraszelski and Jaumandreu, 2013; Kumar and Zhang, 2018; Maican and Orth, 2020).[5] Market shares contain information about demand shocks, and rich sales data for the universe of stores allow us to use local market shares together with demand for inventory to recover store-specific demand shocks. It is important for identification that the sales equation depends on both productivity and demand shocks, whereas the market share index function depends on only demand shocks (Ackerberg et al., 2007). We allow stores to learn from demand, i.e., demand shocks provide information used by stores to improve their future productivity.[6] This mechanism of learning about demand has not yet received much attention in the structural productivity literature, and can be used to evaluate mentoring support policies provided by regional development programs in retail.

This paper contributes to the recent literature on development in services and retail industries. The analysis focuses on the supply side to investigate the determinants of economies of scope and evaluate policy programs in rural and urban markets. We model the role of technology, inputs and the dynamic nature of product variety and store primitives.[7] The proposed framework provides a tractable way of analyzing economies of scope at the firm/establishment level using census data combined with data on product categories and sales per category. Our framework is applicable to any industry where many firms operate and offer a wide range of products for which data on price and detailed product characteristics are unavailable. In a rare case that data on product-level prices are available and can be linked to census data on services firms, our framework can be integrated with a more general demand framework that allows

[5]The carrying cost of inventory represents approximately 25 percent of the value of inventory and includes the capital cost, the storage space cost, the inventory service cost, and the inventory risk cost. To avoid running out of stock ("stock-outs"), retailers spend more on financing inventory than on advertising. Kumar and Zhang (2018) use the cost of goods to recover the distribution of demand shocks in manufacturing but do not model product variety.

[6]External factors such as trade liberalization and entry regulations have been found to be important determinants of this heterogeneity (De Loecker, 2011; Maican and Orth, 2015; Maican and Orth, 2017) in addition to firm-specific factors such as R&D investments (Doraszelski and Jaumandreu, 2013) or management (Syverson, 2011). Braguinsky et al. (2015) highlight the link between inventory, productivity and profitability.

[7]Hsieh and Rossi-Hansberg (2019) argue that consolidation in services is tied to investments in ICT, i.e., technologies that enable stores to produce at scale and to increase specialization among the top firms. See also Gorman (1985), Ellickson (2007), Basker et al. (2012), Bronnenberg and Ellickson (2015), Hortacsu and Syverson (2015), Berry et al. (2019), Ellickson et al. (2019).

for rich substitution patterns between products.[8] While we do not consider a dynamic game, a store's market share is affected not only by its own product variety choice but also by the product variety choices of other stores in the local market.

This paper also contributes to the literature that emphasizes the role of technology and demand in understanding firm performance, which mainly focuses on manufacturing (e.g., Olley and Pakes, 1996; Foster et al., 2008; Collard-Wexler, 2013; Asker et al., 2014; Collard-Wexler and De Loecker, 2015).[9] We highlight the trade-off between productivity and demand shocks for key performance indicators such as sales per product category, store-level sales and market shares in rural and urban markets. In particular, we contribute to the literature that uses the implications of equilibrium behavior for firms' decisions on inputs to estimate productivity (Olley and Pakes, 1996).[10] Most of the literature on productivity estimation considers single-output technology, which renders inference for multiproduct technology questionable (Bailey and Friedlaend, 1982). We explicitly model a multiproduct technology function with known theoretical micro foundations for multiproduct production and profit maximization (e.g., Mundlak, 1964; Fuss and McFadden, 1978). Because our multiproduct approach uses inputs at the firm/establishment level, identification and estimation are based on the well-established two-step methods in the production function literature (see the survey Ackerberg et al., 2007). The analysis, applying our approach to data on product categories and stores, is linked to a recent strand of research on understanding the productivity of multiproduct firms in manufacturing (e.g., De Loecker et al., 2016; Dhyne et al., 2017) and a companion paper on entry regulations in retail (Maican and Orth, 2020).[11]

[8] An example is a constant expenditure specification in an aggregate nested logit model. That retailers commonly offer thousands of separate products makes it difficult to handle individual product data together with census data. In fact, some aggregation is needed to make the analysis manageable. Most of the demand literature on product variety that allows for rich substitution patterns across products does not model the role of supply-side technology, inputs (labor, capital and inventory) or the dynamic nature of product variety and store primitives. This paper complements the literature on product variety using discrete choice demand models with product data (e.g., Berry and Waldfogel, 2001; Draganska and Jain, 2005; Sweeting, 2010; Sweeting, 2013; Eizenberg, 2014; Berry et al., 2016; Quan and Williams, 2018; Adams and Williams, 2019; Fan and Yang, 2019).

[9] By modeling the relationship between multiproduct technology and productivity, this paper adds to the literature that explores heterogeneity in performance in services, e.g., Basker (2007), Basker (2015), Maican and Orth (2015), Grieco and McDevitt (2017), Maican and Orth (2017), and Decker et al. (2018).

[10] See also Levinsohn and Petrin (2003), Doraszelski and Jaumandreu (2013), Ackerberg et al. (2015), and Gandhi et al. (2018).

[11] With the exception of Dhyne et al. (2017), this literature estimates input shares, which is difficult to use in retail. The nature of retail businesses suggests that in most cases it does not make sense to allocate employees to specific product categories. In addition, splitting capital is even more difficult in services. De Loecker et al.

The results show clear evidence that productivity improvements expand the intensive and extensive product-category margins. Stores sell more product categories and increase their sales, especially among bottom-selling product categories. Higher demand shocks, on the other hand, shrink the extensive product-category margin and encourage specialization. Stores with high-demand shocks thus focus on fewer product categories and sell more of their top-selling categories. Together, higher productivity and demand shocks increase store-level sales and market shares. We use the estimated model to quantify gains from improving economies of scope, which affect store sales. As a result, we observe that the increase in store median sales is two percentage points higher in rural than in urban markets if economies of scope improve by fifteen percent for all stores.

Counterfactual experiments examine regional policy programs in Sweden (Nordregio, 2011; SCB, 2015). Focusing on differences between rural and urban markets, we evaluate the impact of an investment subsidy through a thirty percent upward stock of technology and mentoring support that improves learning from demand to foster productivity. Implementing the policy on low-productivity stores shows that the number of product categories (the extensive margin) increases more in rural than in urban markets. Sales per category (the intensive margin) and store-level sales also increase, with magnitudes being larger for stores in rural markets. Store-level sales increase six percentage points more in rural than in urban markets. The corresponding difference for sales per category is four percentage points. The larger effects in rural markets are driven by more pronounced productivity improvements among stores there than in urban markets. Our results suggest that a policy that targets low-productivity stores will reduce the gap between rural and urban markets.

The next section introduces the Swedish retail industry and presents the data. Section 3 describes the model and discusses the identification and estimation. Section 4 presents the empirical results (Section 4.1) and shows the findings of policy experiments using the estimated model (Section 4.2). Section 5 describes robustness checks, and Section 6 summarizes the paper and draws conclusions.

(2016) and Dhyne et al. (2017) estimate productivity in manufacturing, accounting for multiproduct technology and using physical quantities, i.e., eliminating the average price from the productivity measure (also, see Valmari, 2016; Orr, 2018). Analyzing the impact of entry regulation on product variety, Maican and Orth (2020) present a general result for the identification of the transcendental multi-output service technology and discuss the restrictions on the parameters that need to be satisfied for profit maximization.

2 Swedish retail trade and data

Retail trade accounts for a substantial share of all workplaces in Sweden, and the sector employs more than 150,000 individuals (SCB, 2015). A drastic change in the retail landscape has occurred during recent decades. The rural areas of Sweden have experienced depopulation, lack of jobs and declining service provision. People have moved to cities, leaving the country-side areas behind. The demographic changes across Sweden have occurred along with a considerable structural change in retail trade. Most of retailers are situated in localities where the majority of the population lives. Stores have become larger and to a larger extent concentrate in cities and metropolitan areas. Sweden is divided into 290 municipalities, where 47 of them (16 percent) do not have at least five retail trade firms or have at least four retail firms that together employ at least 100 employees. As a result, policymakers have devoted ample time and interest to policy discussions about the development of retail services in rural markets. Several regional development programs have been implemented to support improvements in rural areas. The overall and common goals of the programs are to maintain commercial service in all parts of Sweden and to provide subsidies to firm investments.

Examples of initiatives date back to the beginning of the 1990s, when an organization called *All Sweden shall live!* was started with the aim of stimulating and supporting local development and improving rural policies in Sweden. A new regional development policy was announced by the Swedish parliament in 2001 after passing the bill 2001/02:4 titled *A policy for growth and viability for the country as a whole*, which specifically focused on maintaining a sustainable service level in all parts of Sweden. As part of the support, the Swedish Consumer Agency was tasked with finding new solutions for improvements of commercial services. For instance, a project called *Stores in the countryside* was one of the projects supported by the Swedish Consumer Agency and implemented by The Rural Service Association. The project aimed to improve stores in rural areas, e.g., by assigning mentors to improve communication between store managers and local authorities, and carry out store performance-related actions, such as store refitting and making changes in the distribution of products, improving the technical equipment, and modernizing inventory (Nordregio, 2011). After 2010, several of the projects meant to improve retailers' situation in rural areas have been running under the *Rural Development Program*, which receives support from the EU with the main aim of fostering competitiveness to achieve a balanced territorial development of rural economies and communities. Subsidies and investment support for technical equipment are examples of policy tools implemented by the program.

While we do not observe whether the stores in our sample participate in different development programs, we use the suggested policy tools in these programs to run various policy experiments and quantify programs' effectiveness for the development of Swedish retail. We particularly focus on these programs' common policies, such as providing subsidies for firm investments in technology and mentoring support, aiming to maintain retail services in all geographic areas.

Data, product variety, and local markets. This paper focuses on *Retail sale of new goods in specialized stores* (Swedish National Industry (SNI) three-digit code 524). This retail sector includes the following sub-sectors at the five-digit SNI level: clothing, footwear and leather goods, furniture and lighting equipment, electrical household appliances and radio and television goods, hardware, paints and glass, books, newspapers and stationery, and specialized stores.

We use two data-sets provided by Statistics Sweden. The first provides detailed annual information about all retail firms in Sweden (a census) from 2000 to 2009. The data contain financial statistics of input and output measures, i.e., sales, value added, the number of employees, capital stock, inventories, cost of products, and investment. Inventories capture the value of products held in stock by the end of each year and are taken from book values (accounting data). Sales are measured at output prices, whereas the costs of products and inventories are measured at input prices (what stores pay to wholesalers). Because of difficulties in measuring quantity units in retailing (and services) arising from the nature and complexity of product assortments, quantity measures of output and inventories are unavailable in many data-sets such as census data. In retail, we often refer to firms as stores. In our data, a unit of observation is an organization number.[12] We observe the municipality in which each organization number

[12]In a few cases in our data, an organization number can consist of more than one physical store (a multi-store) in the same municipality, for which we observe total, rather than average, inputs and outputs. Multi-store reporting constitutes less than five percent in our sample (Maican and Orth, 2015).

is physically located. Following previous studies of Swedish retail, we regard municipalities as *local markets* (Maican and Orth, 2015; Maican and Orth, 2018a). Therefore, an advantage of our data is that we can exploit local variations and study the impact of competition.

Our second data-set provides store-level information on the number of product categories and the values of these categories sold each year across Sweden. To the best of our knowledge, such detailed data on the number of product categories across stores and local markets in services industries have not previously been used in the literature. The data cover all product categories that a store sells on a yearly basis. Unique identification codes allow us to match products perfectly to stores.[13] To reduce the dimensionality of the product space in the empirical application, we use well-defined product categories to specify store products, e.g., shoes for women, shoes for men, and shoes for children. The number of product categories captures the extensive margin of product variety in a store. Thus, we define product variety as the number of product categories. Data on sales per product category capture the intensive margin of product variety (i.e., the intensive margin of product lines (product range) in a category). Most importantly, the combination of the two data-sets allows us to compute product market shares in a store and a store's market share in a geographic market, which provides rich information related to competition. Store market shares are computed using sales of stores that belong to the same five-digit sector (e.g., apparel) in a local market. Thus, an apparel retailer and a furniture retailer do not face the same competition. The mix of product- and store-level data is novel and, to the best of our knowledge, has not been used in service industries before.

Descriptive statistics and stylized facts. Table 1 shows the median and the interquartile range for the key variables in our data. The median store in our data has approximately 11 million SEK in sales, seven employees, and approximately four product categories. The number of product categories varies between one and 17 in our sample. The five-digit sector median market share is approximately 34-38 percent in a local market, and it is increasing over time. There has been an increase in the local concentration over time in our sample; e.g., the median C4 computed at the five-digit sector has increased from 91 to 94 percent.

Our data show variation in the number of product categories in stores over time. Unlike manufacturing, retailers frequently adjust product categories because retailers do not need to

[13] The product data set follows a classification system similar to that used for the sample data collected on prices and quantities in manufacturing (PRODCOM).

change technology. In retail, category repositioning is often necessary to improve the store's competitiveness and adjust to changes in consumer preferences. Stores adjust the number of product categories by considering the trade-off between costs of adjusting variety and future benefits from repositioning. We study store variation in the number of product categories in our data by using changes in the store's number of product categories between $t-1$ and t, i.e., yearly adjustments. The interquartile range of changes in the store's number of product categories is 2. The 10th percentile of the yearly adjustment in the number of product categories is -2, and the 90th percentile is 2. We observe adjustments in product categories in 52 percent of store-year observations in our sample. This result is confirmed by an analysis of category adjustments of individual stores in the sample: the median number of years a store adjusts product categories is approximately half of the total number of years in the sample. The mean of cumulated yearly adjustments of the number of product categories is positive (i.e., there has been an increase in product variety over time). There is also substantial variation in the yearly changes in the number of product categories across five-digit sub-sectors. For example, the median of the five-digit interquartile range is 1, and the maximum is 3.

The next step is to analyze how store performance and local environment affect product variety using simple correlations and reduced-form regressions. To proxy store performance and local concentration, we use covariates that are informative for policymakers even if such covariates are endogenous, i.e., affected by our recovered measures of store productivity and demand shocks, as discussed in the next sections. For a better understanding of the relationship between store performance and product variety (extensive and intensive product-category margins), we investigate the evolution of correlations over time in Table 2. Even if we expect the sign of correlations to be driven by store size, the changes in magnitude over time are due to productivity, demand shocks, and the local environment that affect the store performance measures. The number of product categories (the extensive margin) is negatively correlated with sales per cost of goods, which suggests that stores with fewer product categories sell more per unit cost. In addition, the number of product categories is positively correlated with capital stock per employee and the local market share (the benefits of economies of scope). These findings suggest that the trade-off between productivity and demand might play a key role in product selection. Capital per employee is positively correlated with the cost of goods per product category (not reported), implying that stores with high technology sell a wider range of products in a product category or sell high-quality products.

As to the intensive margins related to product variety, we focus on the average sales per product category and the entropy of product sales. Entropy measures store diversification in sales and is computed for each store j based on the market share of each product category i in the store, i.e., $E_{jt} = \sum_i ms_{ijt} ln(ms_{ijt})$ (Bernard et al., 2011). A large measure of entropy suggests that the store focuses on top sales categories. The average of sales per product category is positively correlated with measures that affect productivity and shopping quality, such as capital stock per employee and the average wage at the store. Stores with sales driven by top products (i.e., a large entropy) have large inventory per product category to ensure high shopping quality. Stores with high local market shares have low entropy, a large end-of-year inventory, and high labor productivity and capital stock per employee.

Using reduced-form regressions, we investigate the role of market shares, margins and local market concentration for product variety. Table 3 shows evidence of the relationships between a store's product variety and market share, margin and local market concentration using a fixed-effect estimator that controls for store heterogeneity.[14] The findings demonstrate that an increase in local market concentration is associated with fewer product categories, i.e., stores specialize. In addition, stores with large margins offer a smaller number of product categories.

Because of the increasing concentration in retail over time, we examine whether stores with a high market share have high productivity. Table 4 presents reduced-form evidence of the relationship between sales per employee (labor productivity) and stores' market share using an AR(1) specification. We observe a positive association between market share and labor productivity. This suggests that stores use information about demand to improve productivity. The persistence in labor productivity is approximately 86 percent. While all reduced-form results might be biased because of the endogeneity of market shares, margins and concentration measures, they help provide an understanding of the variation in the data. They also show evidence of the existence and determinants of superstar firms discussed in Autor et al. (2018).

[14]Store margins are proxied using the ratio (net sales - cost of goods)/net sales.

3 Empirical framework

This paper uses a framework that incorporates a multiproduct sales technology and local market information to study the determinants of economies of scale and scope in retail. The proposed model endogenizes the retailer's choices and emphasizes the factors behind the recent trends in retail toward larger stores that offer more product variety using economies of scale and scope. We consider a retail sector where all stores focus on a well-defined service activity, such as selling shoes or selling furniture. To generate sales, stores decide on the variety of products to offer for sale by choosing the products, inventory adjustments, labor, and investments in technology based on the observed information in the beginning of period t. While the framework allows modeling product variety through the choice of the number of products, we access only information on product categories in our data. Therefore, our measure of product variety in the model is the number of product categories.[15]

The proposed multiproduct sales technology models economies of scope to explain why stores offer a specific number of product categories given their resources. The framework is used to form a system of product sales equations for each store. This makes it possible to evaluate how much store-level sales increase if more product categories are added while resources remain unchanged, and how sales of a product category are affected by increasing sales of other product categories. We use the multiproduct technology together with the implications of equilibrium behavior from stores' decisions and local market information to recover store-specific revenue productivity and demand shocks.[16] Then, we evaluate how regional policy programs, targeting economies of scope through subsidies to investment in technology and mentoring support, affect stores' product variety (i.e., the extensive product-category margin), sales per product category (i.e., the intensive product-category margin), sales, market share and local market concentration.

Multiproduct service generating function. Stores use the same service technology to sell their products, and this technology does not depend on product category. Stores compete in

[15] In general, we refer to the number of products produced or sold by a firm as variety. Variety is measured by the number of product categories if there is no data on all products in a category (i.e., the product range in a category). If the researcher has information on product categories and the number of products in each category, the modeling choice is the number of products and not that of categories.

[16] Prices might be difficult to access due to differences in, e.g., pack sizes or units of measure for retail census data-sets. We do not observe prices in our data. However, a construction of a price index at the product category level will suffer from measurement errors due to different compositions of products inside a category across stores.

the product market and collect their payoffs. In the beginning of each time period, stores decide whether to exit the local market or to continue operating in it. If a store continues to operate, it chooses optimal levels of the number of product categories, products bought from the wholesaler and the adjustments in inventory before sales, investment in capital/technology, and labor (the number of employees).[17]

In the case of multiple products, the productivity of one input for a product is not independent of the other products provided by the store, which adds complexity to the store's profit maximization problem (Hicks, 1946; Mundlak, 1964). This complexity arises due to difficulties in aggregating the output, i.e., the composite output depends on other factors, including prices. We presume that the multiproduct service generating function for a store can be written as an implicit function, which assumes separability between inputs and outputs, $F(\mathbf{Q}, \mathbf{V}) = G(\mathbf{Q}) - H(\mathbf{V}) = 0$, where $\mathbf{Q}$ is the vector of outputs, and $\mathbf{V}$ is the vector of inputs. The implicit transformation function $F(\mathbf{Q}_j, \mathbf{V}_j) = 0$ for store j can be described by a transcendental function (a generalization of Cobb-Douglas) (Mundlak, 1964; Fuss and McFadden, 1978)[18]

$$Q_{1j}^{\tilde{\alpha}_1} \times \cdots \times Q_{np_j j}^{\tilde{\alpha}_{np_j}} exp(\tilde{\gamma}_1 Q_{1j} + \cdots + \tilde{\gamma}_{np_j} Q_{np_j j}) = L_j^{\tilde{\beta}_l} K_j^{\tilde{\beta}_k} A_j^{\tilde{\beta}_a} exp(\tilde{\omega}_j), \tag{1}$$

where np_j is the number of product categories of store j, Q_{ij} is the quantity of product category i sold by store j ($i = 1, 2, \cdots, np_j$), L_j is the number of employees, K_j is the capital stock, A_j is the inventory before sales, $\tilde{\omega}_j$ is the quantity-based total factor productivity (the technical productivity), $\tilde{\alpha}_1, \cdots, \tilde{\alpha}_{np_j}$ and $\tilde{\gamma}_1, \cdots, \tilde{\gamma}_{np_j}$ are parameters that define the production frontier and affect product-product and product-input substitutions playing a key role in profit maximization, and $\tilde{\beta}_l$, $\tilde{\beta}_k$ and $\tilde{\beta}_a$ are parameters that define product-input and input-input substitutions. To reduce the number of parameters to be estimated in empirical applications, Mundlak (1964) suggests using aggregation weights $\tilde{\gamma}_i = \tilde{\alpha}_y P_i$, where P_i is the price of product category i and $\tilde{\alpha}_y$ is a parameter to be estimated.[19]

Taking the logarithm of the multiproduct function (1) and indexing by time, we obtain the

[17] We treat each store as a decision-maker. The majority of stores in our sample are single establishments in a local market. We focus on investments in machinery and equipment and refer to them as investments in capital and technology. In retail, technology is embedded in machinery and equipment (hardware) that are used to generate sales.

[18] We follow the common notation, using uppercase letters for levels and lowercase letters for logarithms.

[19] The price of a product category is that of a representative basket of the category.

following service generating function:[20]

$$\sum_{i=1}^{np_{jt}} \tilde{\alpha}_i q_{ijt} + \tilde{\alpha}_y Y_{jt} = \tilde{\beta}_l l_{jt} + \tilde{\beta}_k k_{jt} + \tilde{\beta}_a a_{jt} + \tilde{\omega}_{jt} + \tilde{u}^p_{jt}, \tag{2}$$

where q_{ijt} is the logarithm of the quantity of product category i sold by store j in period t, Y_{jt} denotes total sales of store j in period t, l_{jt} is the logarithm of the number employees, k_{jt} is the logarithm of capital stock, a_{jt} is the logarithm of the sum of the inventory level in the beginning of period t (n_{jt}) and the products bought during period t, and $\tilde{u}^p_{jt}$ are i.i.d. remaining service output shocks. The service technology (2) is consistent with the theoretical micro foundations of the multiproduct technology frontier and profit maximization. It implies separability in inputs and outputs, and the productivity of resources in one product output is not independent of the level of output in other products. The term $\tilde{\alpha}_y Y_{jt}$ (output aggregation using sales) together with product output parameters $\tilde{\alpha}_i$ plays a key role in profit maximization in the multiproduct case. For example, if $\tilde{\alpha}_y = 0$, i.e., Cobb-Douglas specification in both inputs and outputs, then profit maximization does not hold when producing/selling multiple product categories.[21]

In our retail setting, inventories before sales a_{jt} appear as an input of the service generating function since the core activity of retail stores is to buy finished products from wholesalers and resell such products to consumers (Bils and Kahn, 2000).[22] A store's optimal inventory level balances two counteracting forces: Inventory reduces the risk of stock-outs and increases store attractiveness but is costly to adjust and hold in stock. Inventory provides a convenience yield to consumers because it reflects a reduction in shopping cost, i.e., less frequent stock-outs,

[20] In a companion paper, Maican and Orth (2020) present a general result on the identification of multi-output service generating functions, following Mundlak (1964), and discuss the restrictions on parameters that must be satisfied for profit maximization. We assume that all stores use the same service technology to sell their products and that such technology does not depend on the identity of the product category. As discussed in Maican and Orth (2020), this assumption helps reduce the number of parameters to be estimated. However, it can be relaxed to allow a separate technology for each product category if there are sufficient data for all product categories across markets over a long period of time.

[21] Mundlak (1964) and Maican and Orth (2020) discuss the importance of the form of the multiproduct function for profit maximization. To allow for a production technology where the marginal impact of inventory on sales depends on the store's inputs (e.g., with a translog form in inputs), we need to derive the restrictions on the parameters of the production technology such that profit maximization holds for any number of products and inputs.

[22] See also Humphreys et al. (2001), Iacoviello et al. (2011), and Wen (2011) for an extensive discussion of the differences and the role of input and output inventories. We model inventory as a type of capital that evolves endogenously based on products bought from the wholesaler and adjustments in inventory, and it is characterized by adjustment and holding costs.

provision of variety, and other benefits associated with the underlying retail services (Maican and Orth, 2018b). While we do not observe product inventory and stock-outs in the data, we use the information on the store's inventory demand to recover store-specific information on demand that is not observed in the data, i.e., demand shocks (discussed in detail below).[23]

The multiproduct setting requires aggregation over the different products to understand sales technology possibilities. To use the product sales to aggregate over products, we need product prices. Because product prices are commonly not observed for all products in many data-sets, we use the equilibrium price from a demand equation to model sales. In our model, a product category consists of physical products and store-specific services associated with each product, i.e., two stores do not sell exactly the same product even if product categories have the same label (e.g., shoes for kids).[24] The choice set of a consumer consists of the total number of product categories across stores in a local market. For simplicity of exposition, we assume that consumers have CES preferences over differentiated product categories. We then exploit the link between a CES demand system and a discrete choice demand system, which allows us to write the consumer choice probability equation consistent with CES preferences.

In the data, we observe product information only for a sample of stores and total sales for all stores in local markets. Therefore, we model the consumer's outside option as the set of product categories from stores with the same service activity in a local market for which we do not have product information. The consumer's decision is how much to buy of each product category from stores with product information available and how much to buy from the outside option in a local market. Using the CES setting, the consumer choice probability can be written as[25]

$$q_{ijt} - q_{ot} = -\sigma p_{ijt} + \mathbf{x}'_{ijt}\tilde{\boldsymbol{\beta}}_x + \sigma_a a_{jt} + \tilde{\mu}_{ijt}, \tag{3}$$

where p_{ijt} is the logarithm of the price of product category i, $\mathbf{x}_{ijt}$ are the observed determinants of the extensive and intensive margins of the utility function when consumers decide whether to buy and how much to buy of product category i, i.e., it includes product, store and local market characteristics such as population, population density and income, σ is the elasticity of

[23]Having annual data, we do not model stock-outs.

[24]In any data-set, it is unlikely that two stores sell the same product brands in a category and offer the same purchase service to consumers for each product.

[25]See, e.g., Anderson et al. (1987), Anderson and De Palma (2006), and Dube et al. (2020).

substitution, $\tilde{\mu}_{ijt}$ are demand shocks unobserved by the econometrician, e.g., the unobserved quality of product i in store j in period t, and q_{ot} is the outside option quantity.[26] The presence of a_{jt} in a demand equation captures the fact that consumers prefer in-stock products to minimize the search cost. To simplify the notation, we omit the local market index m if the store index j is present, and we refer to store j in market m (in our data, each store is unique).

We use the service production (2) and the price equation (the inverse demand from equation (3)) to obtain the sales-generating function at the store level, $y_{ijt} = q_{ijt} + p_{ijt}$ (Maican and Orth, 2020):

$$y_{ijt} = -\alpha_y y_{-ijt} + \beta_l l_{jt} + \beta_k k_{jt} + \beta_a a_{jt} + \beta_q y_{ot} + \mathbf{x}'_{jt}\boldsymbol{\beta}_x + \omega_{jt} + \mu_{jt} + u^p_{ijt}, \quad (4)$$

where y_{ijt} is the logarithm of sales of product category i in store j in market m in period t, y_{-ijt} is the logarithm of sales of product categories other than i in store j, y_{ot} measures the sales of the outside option captured by the sales of product categories from stores that belong to the same five-digit subsector (i.e., engaging in the same service activity) for which we do not have product information in local market m,[27] and u^p_{ijt} are i.i.d. remaining shocks to sales that are mean-independent of all control variables and store inputs. The new vector $\mathbf{x}_{jt}$ sums all observed characteristics at the store and market levels. In the empirical implementation, we use only local market variables in $\mathbf{x}_{jt}$, and therefore use the notation $\mathbf{x}_{mt}$ instead of $\mathbf{x}_{jt}$ in what follows. Online Appendix A shows the derivation of equation (4).[28]

The coefficient α_y provides information on economies of scope and plays a key role in both the level and persistence of productivity. By using sales, we can reduce the number of parameters to be estimated; specifically we estimate only the coefficient of sales of product categories other than product category i in store j, i.e., α_y, and not all coefficients α_i, $i = \{1, \cdots, np_{jt}\}$. The input coefficients in the multiproduct sales-generating function (4), i.e., β_l, β_k, β_a, β_q, are functions of the elasticity of substitution σ and are similar to the aggregate sales-generating function at the store (firm) level, which allows us to compare them with the estimates for a

[26] The demand system is similar to the logit discrete choice system based on unit demand, but the logarithm of price is used.

[27] Online Appendix A shows how to obtain y_{ot} using the price equation and multiproduct technology. If the outside option is "do not buy," y_{ot} represents total sales in market m (aggregate sales).

[28] To obtain equation (4), we rewrite the linear sum of product category sales $\sum_{i=1}^{np_{jt}} \left[\tilde{\alpha}_i y_{ijt} + \left(1 - \frac{1}{\sigma}\right)\tilde{\alpha}_y Y_{ijt}\right] \equiv \alpha_i y_{ijt} + \alpha_y y_{-ijt}$ and normalize $\alpha_i = 1$.

single-output technology.[29]

The observed and unobserved product characteristics are aggregated at the store level using $\tilde{\alpha}_i$ as weights. For example, μ_{jt} sums all remaining unobserved product category-specific demand shocks μ_{ijt} at the store level.[30] We refer to μ_{jt} as store j's specific demand shocks in period t. Demand shocks μ_{jt} measure factors related to product quality, location, checkout speed, the courteousness of store employees, parking, bagging services, and cleanliness. Although we can refer to such shocks as a measure of customer satisfaction and the quality of shopping in store j in period t, to avoid overinterpretation we simply refer to them as demand shocks.

In contrast to the case of manufacturing, it is difficult to define technical productivity in service industries (Oi, 1992). We cannot obtain a clean measure of technical productivity when estimating only one coefficient for the other product categories (i.e., α_y) and controlling for unobserved prices. The reason is that the coefficients of labor, capital and inventories include demand residuals even if we control for the elasticity of substitution. Therefore, the variable $\omega_{jt} \equiv (1 - 1/\sigma)\tilde{\omega}_{jt}$ measures revenue (sales) productivity, and we simply refer to it as store productivity in what follows. The productivity measure ω_{jt} might include sales shocks due to approximations in (4), but all these sales shocks are different from demand shocks μ_{jt} that affect consumer preferences for product categories in a store. In other words, we are able to separate productivity shocks ω_{jt} from store's demand shocks μ_{jt}, which are part of the demand and affect store market share. Both productivity ω_{jt} and demand shocks μ_{jt} are unobserved by the researcher, but are known to stores when decisions are made.

Choice of product variety. Stores know their productivity ω_{jt} and demand shocks μ_{jt} when they make their product category adjustments and input decisions based on the dynamic optimization problem given by the following Bellman equation (Maican and Orth, 2020):

$$V(\mathbf{s}_{jt}) = \max_{np_{jt},a_{jt},l_{jt},i_{jt}} \Big[\pi(\mathbf{s}_{jt}; np_{jt}, a_{jt}, l_{jt}, i_{jt}) - c_l(l_{jt}) - c_n(np_{jt}, a_{jt}) - c_i(i_{jt}, k_{jt}) + \beta E[V(\mathbf{s}_{jt+1})|\mathcal{F}_{jt}]\Big], \quad (5)$$

where $\mathbf{s}_{jt} = (\omega_{jt}, \mu_{jt}, k_{jt}, n_{jt}, np_{jt-1}, w_{jt}, y_{ot}, \mathbf{x}_{mt})$ is the state variable, w_{jt} is the logarithm of the average wage at store j, $\pi(\mathbf{s}_{jt})$ is the profit function and is a function of the logarithm of the

[29] The coefficients of the multiproduct sales technology are functions of σ, i.e., $\beta_q = 1/\sigma$, $\beta_l = \tilde{\beta}_l(1 - 1/\sigma)$, $\beta_k = \tilde{\beta}_k(1-1/\sigma)$ and $\beta_a = \overline{\beta}_a(1-1/\sigma)$. Parameters σ_a and β_a are included in $\overline{\beta}_a$, and they cannot be separately identified (see the identification section and online Appendix A).

[30] In fact, μ_{jt} is a weighted sum of all unobserved product category-specific demand shocks at the store level, $\mu_{jt} \equiv (1/\sigma)\sum_{i=1}^{np_{jt}} \tilde{\alpha}_i \mu_{ijt}$.

store's total sales y_{jt}, $c_l(l_{jt})$ is the labor cost, $c_n(np_{jt}, a_{jt})$ is the adjustment cost for product categories, which is increasing in inventory in the beginning of period n_{jt},[31] $c_i(i_{jt}, k_{jt})$ is the investment cost of new capital (equipment), which is increasing in investment choice i_{jt} and decreasing in current capital stock k_{jt} for each fixed i_{jt},[32] β is a store's discount factor, and $\mathcal{F}_{jt}$ represents the information available at time t.

The dynamic equation (5) is a complex optimization problem: to solve it, we need to fully model the cost structure at the store level. As discussed in Maican and Orth (2020), the existence of adjustment costs for product categories explains why stores might not increase the number of product categories. In this paper, we follow Olley and Pakes (1996) and Bajari et al. (2007), who instead of directly solving the optimization problem (5) use the nonparametric policy functions for identification and estimation.[33] The policy functions in (5) are functions of the store's state variables and capture complex decisions by stores, where current choices affect the future development of the store. The store's optimal number of product categories is $np_{jt} = np_t(\mathbf{s}_{jt})$, inventory demand is $a_{jt} = f_t(\mathbf{s}_{jt})$, labor demand is $l_{jt} = l_t(\mathbf{s}_{jt})$, and investment is $i_{jt} = i_t(\mathbf{s}_{jt})$. We use information on how stores choose the number of product categories (the extensive margin) and inputs from the estimated policy functions to solve the multiproduct technology and obtain sales per product category (the intensive margin) and store performance measures in the counterfactual policy experiments (Section 4.2).

Learning from demand. Store productivity ω_{jt} and demand shocks μ_{jt} are correlated over time, and are not observed by the researcher. We assume that demand shocks μ_{jt} follow a nonlinear AR(1) process

$$\mu_{jt} = \gamma_0^\mu + \gamma_1^\mu \mu_{jt-1} + \gamma_2^\mu (\mu_{jt-1})^2 + \gamma_3^\mu (\mu_{jt-1})^3 + \eta_{jt}. \tag{6}$$

[31] The modeling of evolution and adjustments in inventory follows the existing literature (e.g., Coen-Pirani, 2004). The inventory level in the beginning of period $t+1$ evolves according to $N_{jt+1} = \tilde{N}_t(A_{jt}, Y_{jt})$, where A_{jt} is the adjusted inventory before sales, i.e., the inventory in the beginning of the period N_{jt} adjusted with the products bought in period t, Y_{jt} is store-level sales, and $\tilde{N}_t$ is a function that is increasing in A_{jt} and decreasing in Y_{jt}. N_{jt+1} captures inventory in the beginning of period $t+1$ (or the end of period t) after sales in period t have been realized. For example, $N_{jt+1} = A_{jt} - Y_{jt}$, if A_{jt} and Y_{jt} are measured in physical units.

[32] Capital stock is a dynamic input that accumulates according to $K_{jt+1} = (1-\delta_k)K_{jt} + I_{jt}$, where δ_k is the depreciation rate. The investment I_{jt} in machinery and equipment is chosen in period t and affects the store in period $t+1$.

[33] Studying the impact of the entry regulation on product variety, Maican and Orth (2020) use value function approximation techniques to numerically solve the Bellman equation (also, see Ryan, 2012; Sweeting, 2013; Maican, 2019).

The model allows demand shocks that can be associated with the quality of the shopping experience to influence store productivity. In our setting, demand shocks can influence store productivity in at least two ways. The first is through productivity gains within stores that arise, for instance, because stores obtain opportunities to analyze information from consumers and use it to improve the shopping process and inventory management. For example, store employees are responsible for many small innovations (i.e., innovations on the floor) that improve the sales process inside the store. The second channel involves a selection effect from the exit of low-productivity stores.[34] Thus, productivity adjusts as a result of changes in the experienced demand shocks, although we also recognize that it is plausible that stores engage in other active efforts to increase their productivity. Our model quantifies the overall effect of demand shocks on productivity instead of modeling all possible sources of productivity improvement. Therefore, store productivity ω_{jt} follows an endogenous nonlinear AR(1) process where previous productivity ω_{jt-1} and demand shocks μ_{jt-1} affect the current productivity:

$$\begin{aligned}\omega_{jt} = \quad & \gamma_0^\omega + \gamma_1^\omega \omega_{jt-1} + \gamma_2^\omega (\omega_{jt-1})^2 + \gamma_3^\omega (\omega_{jt-1})^3 + \gamma_4^\omega \mu_{jt-1} \\ & + \gamma_5^\omega \omega_{jt-1} \times \mu_{jt-1} + \xi_{jt}.\end{aligned} \tag{7}$$

Above, η_{jt} and ξ_{jt} are shocks to demand and productivity, respectively, which are mean-independent of all information known at $t-1$.

Demand shocks and the market share index function. The store demand shocks μ_{jt} are defined as a weighted sum of product category-specific demand shocks of store j that arise from the demand system (3), i.e., μ_{ijt}, where the aggregation weights arise from the multiproduct service technology (1). Thus, demand shocks μ_{jt} include information that affects consumers' choices across stores and therefore affect the store's market share.

We recover demand shocks μ_{jt} using the recent developments from the production function literature, which suggests the use of an output index function and an input process to control for unobservables (Ackerberg et al., 2007). In our case, an informative output for demand shocks and product sales should be related to the store's market share. The input is inventory before sales, which incorporates information about μ_{jt}. Most importantly, recovering demand shocks μ_{jt} from a well-known aggregate demand system at the store level, where consumers obtain

[34] The selection effect is less important in our empirical setting even if we allow exit in the theoretical framework. The reason is that we observe few exits in our data sample.

utility from choosing a store, is complex because it requires data on prices and a definition of a product basket for which we need to construct a price index consistent with the multiproduct service technology.[35] We analyze all stores in well-defined five-digit subsectors, and, as in most service industries, price and quantity data are difficult to obtain in the absence of scanner data. Annual data on labor and capital also make it challenging to define a yearly product basket and price index even if price data exist.

We choose an index function that satisfies the following properties: (i) it aggregates stores' category sales from the multiproduct sales function in the output index r_{jt}; (ii) it is informative for store demand and consistent with the aggregate demand in a local market (e.g., it includes μ_{jt}); (iii) it allows μ_{jt} to appear additively to improve identification and (iv) it can be used together with multiproduct sales to compute total sales in a local market when there are changes in the local environment. We consider the output of an index function with store and market characteristics $\boldsymbol{\delta}_{jt}$ (that can include $\mathbf{x}_{mt}$) and μ_{jt} as arguments

$$r_{jt} = \boldsymbol{\delta}_{jt}\boldsymbol{\rho} + \mu_{jt} + \nu_{jt}, \tag{8}$$

where the output index $r_{jt} = ln(ms_{jt}) - ln(ms_{0t})$ is the ratio of the store market share and the market share of the outside option, ms_{jt} is the market share of store j in local market m in period t computed at the five-digit industry sector level using sales, ms_{0t} is the outside option, i.e., the market share of other stores in market m computed at the five-digit industry sector level (we have the same outside option as in equation (4), but here we use a share-based measure), and ν_{jt} is an error term that is mean-independent of all controls. In the empirical implementation, $\boldsymbol{\delta}_{jt}\boldsymbol{\rho} = \rho_{np}np_{jt} + \rho_{inc,1}inc_{mt} + \rho_{inc,2}inc^2_{mt}$, where np_{jt} is the number of product categories np_{jt}, and inc_{mt} is the logarithm of the average income in the local market. The main aim of the market share index function is to identify μ_{jt} separately from ω_{jt} and not to infer changes in price elasticities due to repositioning in product categories.

Next, we discuss why the market share index function is useful and how it relates to the multiproduct sales technology. First, sales are a commonly used output measure in services and

[35] For example, as in a nested-logit model, one can use the demand system and derive the probability of choosing store j as a function of p_{ijt} and μ_{ijt} using the conditional choice probability, which is not useful in the identification because p_{ijt} and μ_{ijt} are not observed. Even if we could write a logit demand model at the store level, the IIA problem would still exist because different retailers might sell very different product categories.

depend on both demand and supply factors. In our model, sales depend on both the store's demand shocks μ_{jt} and productivity ω_{jt}, whereas a store's market share depends only on μ_{jt}. In other words, the market share index function (8) and the sales-generating function (4) are linked through the demand shocks μ_{jt}, which ensure consistency and identification of the model. Because the sales-generating function (4) controls for capital stock k_{jt} and inventory a_{jt}, they are not part of μ_{jt}, and we do not need to control for them in the market share equation.[36] The number of product categories np_{jt} affects a_{jt}, which includes additional information such as the volume of each product, and products are aggregated based on monetary value.

Second, the market share index equation (8) is not a logit demand specification, but is informative for understanding store demand in a local market. It does not include the price, but includes product categories and residuals ν_{jt}. For identification, we cannot use the common nonparametric inversion strategy from the discrete choice literature to recover μ_{jt}. The reason is that μ_{jt} contains supply-side weights and the presence of remaining shocks ν_{jt}. The market share index function uses the same output as a logit demand consistent with CES assumptions because market shares include store demand information and the logarithm of store sales and the outside option yield a simple expression that can be integrated easily with sales per product category from the multiproduct sales equation. The ratio of market shares of two stores depends on the number of product categories they offer and demand shocks. Because store-specific demand shocks depend on the product-category mix, the market share ratio changes if one of the stores alters its product-category mix without changing the number of product categories. Nevertheless, one way to avoid the IIA problem specific to logit models in equation (8) is to group product categories by a store characteristic (e.g., store size) and rewrite equations (3) and (8) in a nested-logit form. However, this is beyond the aim of this paper.

Third, the multiproduct sales equation and the market share index function create a system of equations that can be used to compute total sales changes at the market level after exogenous policy changes once we have information on demand shocks (Maican and Orth, 2020). Specifically, equations (4) and (8) form two systems of equations: one at the store level (capturing sales per product) and one at the local market (capturing market shares), which are used to

[36] Even if we control for capital stock k_{jt} and inventory a_{jt} in the market share equation, we cannot separately identify their effects on demand and supply; i.e., we identify the net effect. See online Appendix A for a short discussion of identification of β_a.

predict changes in sales in policy experiments. This joint system can be solved by the nested fixed-point algorithm. We provide details of the numerical algorithm used to solve the joint system of equations at the end of this section. If the outside option sales are unaffected by changes in the local environment, the market share index function is not useful in counterfactuals. Thus, it is used only in identification to recover demand shocks μ_{jt}.

Economies of scope. Parameter α_y in the multiproduct sales function (4) provides information on the economies of scope. It measures percentage changes in sales of a product category if sales of other product categories in the store increase by one percent (i.e., sales cannibalization between product categories in the store).

We need to emphasize a few key aspects of the economies of scope parameter. First, multiproduct service technology (2) allows for rich information on service product substitution and models the relationship between sales per product category and total sales. Because stores do not sell the same product categories in our data, we cannot identify all parameters $\tilde{\alpha}_i$ and $\tilde{\alpha}_y$ without a selection over products or having panel data that covers a long time period. Therefore, we summarize all information on economies of scope into one parameter α_y. While this helps identification, the drawback is that we reduce the asymmetry of changes in sales. Thus, for all product categories, sales per product category decrease by the same amount if sales of other product categories increase. Second, we can allow for asymmetry effects by selecting stores that sell the same product categories, but this introduces a selection bias, and empirical findings might not be informative for what is happening in the local markets. Third and most importantly parameter α_y provides a critical understanding of stores' changes in total sales in the following cases: (i) changing the number of product categories while resources remain the same, and (ii) keeping the same product categories and changing the store's resources. To demonstrate the importance of the scope parameter, we provide an illustrative example of a store with three product categories and assume $\alpha_y = 0.85$ (Appendix B). Increasing the number of product categories from three to four while resources remain the same yields lower sales per product, whereas total store-level sales increase. This finding shows that the scope parameter drives store-level sales if more product categories are added. Reducing the scope parameter α_y from 0.85 to 0.80 while the number of product categories remains equal to three increases total sales by ten percent. This indicates that economies of scope are essential if competition is less intense between products in a store.

Identification and estimation. The multiproduct approach uses inputs at the firm (establishment) level, and therefore, the identification and estimation are based on the well-established two-step methods in the production function literature (Ackerberg et al., 2007). Our model consists of two equations (multiproduct sales and market share) and two unobservable processes (productivity and demand shocks), where one of the equations includes only one of them. The core of the identification of such a system of equations is discussed in detail by Ackerberg et al. (2007) (Section 2.4).[37] The outputs, the number of product categories, and the inputs are endogenous, i.e., are correlated with ω_{jt} and μ_{jt}. The identification and estimation follow Olley and Pakes (1996) and the subsequent literature and include the estimation of Markov processes for ω_{jt} and μ_{jt}. We estimate $\boldsymbol{\theta}$=(β_l, β_k, β_a, β_x, α_y, β_q, ρ_{np}, $\rho_{inc,1}$, $\rho_{inc,2}$) using a two-step estimator. In contrast to Olley and Pakes (1996), we have two unobservables to recover instead of one (also, see Maican and Orth, 2020). We use the store's labor demand function to recover productivity (Doraszelski and Jaumandreu, 2013; Maican and Orth, 2017).[38] We use the store's demand for inventory a_{jt} to recover the demand shocks μ_{jt}. The equations that are used in the estimation are the multiproduct sales function (4), the market share index equation (8) and the productivity and demand shock processes (7) and (6). The roadmap of the two-step estimator is as follows. In the first step, we recover ω_{jt} and μ_{jt} using a polynomial expansion in variables of the inverse labor and inventory demand functions in equations (4) and (8). In the second step, we use the productivity (7) and demand shock (6) processes to obtain the shocks $(\xi_{jt} + u^p_{ijt})$ and $(\eta_{jt} + \nu_{jt})$ as functions of parameters $\boldsymbol{\theta}$.

The general labor demand and inventory functions that arise from the stores' optimization problem are $\mathrm{l}_{jt} = \tilde{l}_t(\omega_{jt}, \mu_{jt}, k_{jt}, n_{jt}, w_{jt}, y_{ot}, \mathbf{x}_{mt})$ and $\mathrm{a}_{jt} = \tilde{a}_t(\omega_{jt}, \mu_{jt}, k_{jt}, n_{jt}, w_{jt}, y_{ot}, \mathbf{x}_{mt})$. To back out ω_{jt} and μ_{jt}, functions $\tilde{l}_t(\cdot)$ and $\tilde{a}_t(\cdot)$ must be strictly monotonic in ω_{jt} and μ_{jt}, which holds under mild regularity conditions specific to a general dynamic programming setting (Pakes, 1994). Maican and Orth (2020) discuss in detail all of such conditions required for invertibility.[39] By inverting these policy functions to solve for ω_{jt} and μ_{jt}, we obtain

[37] See also Matzkin (2008).

[38] Levinsohn and Petrin (2003) use intermediate inputs to recover productivity.

[39] Most importantly, the main requirement is that the Jacobian of the policy functions system be non-zero, which is satisfied in the empirical application because of the different nonlinear effects of state variables on the store's decisions.

$\omega_{jt} = f_t^1(l_{jt}, k_{jt}, n_{jt}, w_{jt}, a_{jt}, y_{ot}, \mathbf{x}_{mt})$ and $\mu_{jt} = f_t^2(l_{jt}, k_{jt}, n_{jt}, w_{jt}, a_{jt}, y_{ot}, \mathbf{x}_{mt})$, i.e., ω_{jt} and μ_{jt} are non-parametric functions of the observed variables in the state space and the controls.

We isolate demand shocks μ_{jt} stores receive using the market share index function and information on market shares in local markets. In particular, the market share index function helps recover demand shocks separately from productivity and ensures the identification of the model. By substituting μ_{jt} and ω_{jt} from the input demand function into (8), we can write the market share index function as $b_t(l_{jt}, k_{jt}, n_{jt}, w_{jt}, a_{jt}, y_{ot}, \mathbf{x}_{mt})$ and estimate it using an ordinary least squares (OLS) estimator based on a polynomial expansion of order two in its variables.[40] Thus, we obtain the predicted market share index $\hat{b}_t$. This allows us to write the demand shocks μ_{jt} as a parametric function, i.e., $\mu_{jt} = \hat{b}_{jt} - \rho_{np} np_{jt} - \rho_{inc,1} inc_{jt} - \rho_{inc,2} inc_{jt}^2$, which will be treated as an input in the multioutput sales-generating function. Then, if μ_{jt} and ω_{jt} from inverse input demand functions are substituted into (4), the sales-generating function becomes

$$y_{ijt} = -\alpha_y y_{-ijt} + \phi_t(l_{jt}, k_{jt}, n_{jt}, w_{jt}, a_{jt}, y_{ot}, \mathbf{x}_{mt}) + u_{ijt}^p, \tag{9}$$

where $\phi_t(\cdot) = \beta_l l_{jt} + \beta_k k_{jt} + \beta_a a_{jt} + \beta_q y_{ot} + \mathbf{x}'_{mt} \boldsymbol{\beta}_x + \omega_{jt} + \mu_{jt}$. An estimation of (9) using OLS and a polynomial expansion of order two yields an estimate of service output without service output shocks u_{ijt}^p, which gives us $\hat{\phi}_t$, used to obtain store productivity ω_{jt} as a function of parameters, $\omega_{jt} = \hat{\phi}_{jt} - \beta_l l_{jt} - \beta_k k_{jt} - \beta_a a_{jt} - \beta_q y_{ot} - \mathbf{x}'_{mt} \boldsymbol{\beta}_x - (\hat{b}_{jt} - \rho_{np} np_{jt} - \rho_{inc,1} inc_{jt} - \rho_{inc,2} inc_{jt}^2)$.

The next step is to use the information from the Markov processes for ω_{jt} and μ_{jt}, the service generating function (4) and the market share index function (8) to obtain the shocks $(\xi_{jt} + u_{ijt}^p)$ and $(\eta_{jt} + \nu_{jt})$ as functions of parameters of the model. We can identify coefficients $\boldsymbol{\theta}$ using moment conditions based on $(\xi_{jt} + u_{ijt}^p)$ and $(\eta_{jt} + \nu_{jt})$ and the generalized method of moments (GMM) estimator.[41] To identify $\boldsymbol{\theta}$, the following moment conditions are used: $E[\xi_{jt} + u_{ijt}^p |$ y_{-ijt-1}, l_{jt-1}, k_{jt-1}, a_{jt-1}, $\mathbf{x}_{mt-1}] = 0$ and $E[\eta_{jt} + \nu_{jt} | np_{jt-1}, inc_{mt-1}, inc_{jt-1}^2] = 0$.[42] In other words, we rely on the remaining shocks being uncorrelated with the previous variables to form

[40] We rely on the number of product categories being a store choice in each period and, therefore, a function of state variables. A polynomial expansion of order two is used because the state space is large in our model, and a polynomial expansion of order three shows no improvement in the estimation of the first stage.

[41] Our empirical results are robust to using moment conditions based on ξ_{jt} and η_{jt} to identify parameters β_l, β_k, β_a, β_x, β_q, ρ_{np}, $\rho_{inc,1}$, and $\rho_{inc,2}$ in the empirical application.

[42] Using Monte-Carlo simulations, Maican and Orth (2020) demonstrate identification of the multiproduct sales technology using labor demand as a proxy for productivity. The authors also show that the product sales system of (nonlinear) equations at the store level has a unique solution, which implies that we can compute product sales if we have information on inputs, productivity and demand shocks.

the moments.[43]

Parameters of inputs in the sales function (β_l, β_k, β_a) are identified using l_{jt-1}, k_{jt-1}, a_{jt-1} as instruments; i.e., we rely on the current remaining productivity and sales shocks being uncorrelated with previous inputs to form moment conditions. The economies of scope parameter α_y is identified using y_{-ijt-1} as an instrument; i.e., we rely on the previous output being uncorrelated with the current remaining sales and productivity shocks. Even though Monte-Carlo experiments show the robustness of the identification of the scope parameter using the previous output, below we also discuss an alternative estimator that is computationally more demanding. That previous local market characteristics $\mathbf{x}_{mt-1}$ and y_{ot-1} are uncorrelated with the current remaining sales and productivity shocks allows us to identify β_x and β_q (in general, $\mathbf{x}_{mt}$ can also be used as instruments because many market characteristics are exogenous). Finally, the coefficients of the market share index equation are identified by relying on the sum of the remaining demand shocks ($\eta_{jt} + \nu_{jt}$) being uncorrelated with the number of product categories, income, and income squared at $t-1$. It is important to note that the parameters of Markov processes (6) and (7) are estimated using the terms of these processes as instruments (they are functions of $\hat{\phi}_{jt-1}$ and $\hat{b}_{jt-1}$). Parameters $\boldsymbol{\theta}$ are estimated by minimizing the GMM objective function

$$\min_{\boldsymbol{\theta}} Q_N = \left[\frac{1}{N} W' v(\boldsymbol{\theta})\right]' A \left[\frac{1}{N} W' v(\boldsymbol{\theta})\right], \tag{10}$$

where $v_{jt} = (\xi_{jt} + u^p_{ijt}, \eta_{jt} + \nu_{jt})'$, W is the matrix of instruments, and A is the weighting matrix defined as $A = \left[\frac{1}{N} W' v(\boldsymbol{\theta}) v'(\boldsymbol{\theta}) W\right]^{-1}$.[44]

Alternative identification of the economies of scope parameters. We can use an alternative identification strategy for the economies of scope parameter. Instead of using the previous output of other products as an instrument, we can solve the system of output equations for each store, i.e., proceeding as in the counterfactual experiments. In other words, we fully endogenize product sales in the estimation. However, this estimator is computationally demanding because we have to solve the system of equations for each store-year observation and a new set of model parameters using the fixed-point iteration method. Monte-Carlo experiments demonstrate no tangible advantages of this alternative estimator over the above IV identification strategy if

[43] Ackerberg et al. (2007) and Wooldridge (2009) extensively discuss using previous variables as instruments in a two-step control function approach when estimating production functions.

[44] Standard errors are computed according to Ackerberg et al. (2012).

stores use the same sales technology for their products.

Alternative demand specifications. While our model is rich on the supply side, we acknowledge that the CES preferences are restrictive. However, the form of the multiproduct sales-generating function (4) is also consistent with a demand specification that allows for rich substitution patterns and avoids the IIA problem, e.g., a constant expenditure specification in an aggregate nested-logit model where price appears in the logarithmic form. The reason is that in a constant expenditure specification, we use the volume of sales for each product category, which allows us to aggregate products when integrating the demand system with the multiproduct function (2).[45] In a nested demand model, consumers choose stores and then products within a store implying that the output and input parameters will depend on the nest(s) parameter(s). In other words, the scope parameter α_y includes information about product correlation in the nested model at the store level. Because we do not focus on one specific product category (e.g., yogurt) in the empirical application and observe rich heterogeneity on the supply side in the data (i.e., in product categories), in what follows, we use a simple demand specification. Most importantly, our main empirical results are not driven by the demand assumption and are supported by various simple descriptive statistics and reduced-form specifications (see Section 2).

Numerical implementation of the model. Next, we describe how the estimated model can be used to compute changes in sales after policy changes. A numerical implementation of the model also helps improve the understanding of the integration of different parts of the model. Most of policy changes can be implemented by changes in the model's parameters and the evolution of productivity and demand shocks. Therefore, we need to solve numerically the system of equations formed by multiproduct sales and the market share index function.

For simplicity of exposition, let us assume that we have one store ($j = 1$) for which we observe the number of product categories and have recovered productivity and demand shocks by estimating the model. We want to use the estimated multiproduct sales equation and market index function to compute sales of stores in the outside option (y_{ot}), and, hence, total sales in the local market after policy changes at the store or in the local environment. Thus, the multi-

[45] All technical derivations are available from the authors upon request. Unlike the discrete choice specification, a constant expenditure specification allows consumers to buy more than one product (Verboven, 1996; Anderson and De Palma, 2006).

product sales equation can be written as $y_{i1t} = -\alpha_y y_{-i1t} + (1/\sigma)y_{ot} + T_{1t} + \mu_{1t}$, where term T_{1t} groups all store characteristics (labor, capital, inventory, productivity, and market characteristics) that are in equation (4), and $i = \{1, \cdots, np_1\}$ indexes the product categories of the store. The market share index equation can be written as $ln(\sum_{i=1}^{np_1} exp(y_{i1t})) - ln(y_{ot}) = \boldsymbol{\delta}_{1t}\boldsymbol{\rho} + \mu_{1t}$. We start with an initial value for y_{ot} denoted by $y_{ot}^{(0)}$. Then, we use the multiproduct equation to compute sales per product category $y_{i1t}^{(0)}$ using the fixed-point algorithm to solve the multiproduct sales system of equations (the number of equations is given by the number of categories). The computed sales per product category $y_{i1t}^{(0)}$ are used to obtain the next sales of the outside option $y_{ot}^{(1)}$, which are used to compute next period's sales per product category $y_{i1t}^{(1)}$. We repeat this process until $\|y_{i1t}^{(n+1)} - y_{i1t}^{(n)}\| < tol$ and $\|y_{ot}^{(n+1)} - y_{ot}^{(n)}\| < tol$, where tol is a numerical tolerance level, and n is the number of iterations. The same algorithm is applied if there are many stores in a market for which we observe their product categories.[46] There are cases when it is reasonable to assume that the sales of the outside option (y_{ot}) are unaffected by policy changes. If so, we can use only the multiproduct sales system of equations to compute sales per product category using the observed y_{ot}.

4 Results

This section presents the empirical results. First, we discuss the results of the estimated multiproduct sales-generating function, which include estimates of store productivity and demand shocks and how they evolve over time. Second, we explore the heterogeneity in store productivity and demand shocks and their role in explaining economies of scope and performance across stores in Section 4.1. We examine stores' optimal choices of the number of product categories, inventory per product, and demand functions for investment, labor (the number of employees) and inventory, which according to a store's dynamic optimization problem (5) are functions of the state variables. Third, we use the estimated model to perform counterfactual experiments for regional policy programs that target the determinants of economies of scope through subsidies to investment in technology and mentoring support (Section 4.2).

[46] The authors provide results of Monte-Carlo simulations upon request for a large number of products and stores (Maican and Orth, 2020). Julia simulations demonstrate a fast convergence of the algorithm.

Service generating function estimates. Table 5 shows the estimates of the multiproduct sales-generating function (equation (4)) by an OLS estimator and the nonparametric two-step estimator presented in Section 3. We estimate the multiproduct technology at the three-digit industry level and include five-digit subsector controls and fixed effects because the two-step estimator requires a large number of observations for consistency and some of the five-digit subsectors are small. This is less restrictive in retail than in manufacturing, where a production technology can be specific to a five-digit industry subsector. The two-step estimator controls for the endogeneity of store input choices (i.e., simultaneity) and allows us to identify store productivity separately from shocks to market share. If the two-step estimator is used, the coefficients of labor and inventory decrease from 0.786 (OLS) to 0.558 and from 1.036 (OLS) to 0.493, respectively.[47] The coefficient of capital increases from 0.059 (OLS) to 0.283 (the two-step estimator). These changes in the estimates are consistent with the production function literature following Olley and Pakes (1996), which suggests an upward bias for the coefficients of labor and inventory if one does not control for the correlation between inputs and productivity.

The estimated elasticity of demand for product substitution is 4.63. There is clear evidence of sales cannibalization and competition for limited shelf space among products in a store. Sales of a product category decrease when sales in other product categories increase. With the same resources, a 1 percent increase in sales of a product category reduces sales of other product categories by 0.856 percent. This finding is consistent with the profit maximization behavior of multiproduct firms (see Mundlak, 1964; Maican and Orth, 2020).[48] Our estimates also show that stores in markets with high population and population density sell more in each product category (i.e., the demand effect).

The results from the market share equation (8) clearly show that a store's market share increases in product variety (0.213). In other words, a wider range of products increases the market share. The magnitude of such an increase is sizable. For example, a store with a 30 percent local market share can increase its market share to 36 percent by adding one more product category. Income has a positive effect on consumers' utility function and, therefore, on

[47]To allow for comparisons across specifications, we present the results using the two-step estimator where coefficients are adjusted for the elasticity of substitution σ and the coefficient $\tilde{\alpha}_y$ of other product categories in the store.

[48]The coefficient of a store's other product categories includes information on the multiproduct service frontier, which influences the input coefficients and affects the productivity measure.

a store's market share.

Productivity and demand shocks. Using the estimated parameters from the sales-generating function, we recover productivity ω_{jt} and demand shocks μ_{jt} for each store and year. The heterogeneity of store productivity and demand shocks is informative because it drives the heterogeneity in sales across stores. Store demand shocks μ_{jt} have a larger variance than does productivity ω_{jt}. For productivity, a store in the 75th percentile has 27 percent greater productivity than a store in the 25th percentile. However, the store's demand shocks are approximately 50 percent higher for a store in the 75th percentile than for a store in the 25th percentile.

Table 6 shows the estimates of the processes of store productivity ω_{jt} and demand shocks μ_{jt}, i.e., equations (7) and (6). The figures in the second panel of Table 6 demonstrate that the persistence of the productivity process (0.85) is lower than that of demand shocks (0.92). The magnitude of persistence in productivity is similar to the findings in other studies in the productivity literature (e.g., Doraszelski and Jaumandreu, 2013 in manufacturing, and Maican and Orth, 2017 in retail).

In our model, demand shocks can affect store productivity, and the size of the impact depends on the level of store productivity. The results in Table 6 show that we reject the null hypothesis that the coefficients of μ_{jt} in the productivity process are equal to zero (*p-value*=0.000). Demand shocks have a positive impact on productivity, i.e., a one percent increase in μ_{jt} raises productivity by 0.013 percent on average. This finding suggests that retailers use information from the consumers' choice of products and stores to improve productivity, i.e., learning from managing demand. For example, stores with high-demand shocks have skilled and service-minded employees who help consumers during the shopping process. These high-ability employees use information from consumers to create appealing innovations that shift store productivity.

4.1 Product variety, demand for inputs, and market share

The solution of the store dynamic programming problem given by the Bellman equation states that the store's choices of the number of products, labor, inventory and investment are functions of state variables. The state variables that are used to make optimal choices are productivity (ω_{jt}), demand shocks (μ_{jt}), capital (k_{jt}), inventory in the beginning of the period (n_{jt}), and local market characteristics ($\mathbf{x}_{jt}$).[49]

The estimation framework controls for the impact of macro policies in different ways. First, we include year fixed effects and local market controls (i.e., population, population density, and income) in the two-step estimator used to recover productivity and demand shocks. Year fixed effects remove the impact of macro changes in a specific year from productivity and demand shocks. Local market covariates remove the effect of local market characteristics (that might include local policies) from store productivity and demand shock measures, which follow Markov processes. Second, we regress store choice variables on the states variables and add store fixed effects to control for other unobserved store factors correlated with productivity, demand shocks, and capital stock, e.g., aspects related to macro policies, location and logistics. For robustness, we use productivity and demand shocks in period $t-1$ to avoid possible endogeneity concerns related to productivity and demand shocks in period t.[50]

Product variety. To analyze the determinants of economies of scope, Table 7 shows the estimates of a store's product variety and diversification as functions of state variables using a linear specification that controls for store fixed effects. The changes in the number of product categories capture stores' adjustments in the extensive product-category margin.

Productivity improvements allow stores to offer a wider product variety. The results show that a 10 percent increase in productivity yields a 3 percent rise in the number of product categories. Stores that invest in technology offer more product categories; for example, to add an extra product category, an average store needs an approximately 3 percent increase in the stock of technology.

We observe that stores with high demand shocks offer a lower number of product categories. On average, a 10 percent increase in demand shocks reduces the number of product categories by 0.4 percent. Therefore, we find evidence of specialization for stores with high-demand shocks, i.e., it is costly for stores to keep the same quality and offer many product categories (there are diseconomies of scope). This finding is consistent with Bronnenberg (2015), who argues that price limits the demand for quantity and purchasing costs related to travel and waiting in

[49] In our capital stock measure, an investment decision made in the current period affects the capital stock in the next period.

[50] Online Appendix D presents these regression estimates, but we discuss the results in this section. While using previous productivity and demand shocks to estimate policy functions resolves possible endogenenity concerns, doing so is not entirely consistent with the store's dynamic optimization problem and underestimates current productivity improvements due to learning from demand.

checkout lines limit the demand for product variety. Stores with a large inventory at the end of the year reduce their number of product categories.

To evaluate the adjustments in intensive product-category margins, we use two measures of store diversification. The first measure is the Herfindahl index (HHI) calculated based on sales of product categories in the store. The second measure is the entropy of product sales that measures the extent to which a store's product sales are skewed toward the largest (main) products rather than the smallest.

Table 7 shows results for store diversification, i.e., how stores react from the perspective of the intensive margin to changes in store and market primitives. A one percent increase in productivity yields a decline of 7 percent in HHI in a store, i.e., a lower concentration in the store. Investments in technology also reduce concentration in the store. An increase in the stock of technology by one percent reduces concentration by 2 percent. On the other hand, an increase in demand shocks increases concentration, which is consistent with the results for the extensive margin.

The findings for entropy show the importance of productivity and demand shocks for diversification in a store. Highly productive stores have a lower entropy of product sales, which implies higher sales across all product categories. Specifically, entropy decreases by approximately 15 percent if productivity increases by one percent. In contrast, an increase in demand shocks by one percent raises entropy by 2 percent, which suggests that stores with high-demand shocks focus on their top-selling products. The results also indicate that stores with large end-of-year inventories have large entropy. In other words, stores characterized by top-selling products hold large inventory, which helps them avoid stock-outs.

Using previous productivity and demand shocks as covariates in regression specifications, we obtain the same sign of the impact of key state variables (i.e., productivity, demand shocks and capital) on the number of product categories, HHI in a store, and entropy (online Appendix D). The marginal effects of productivity and demand shocks on HHI in a store and entropy in product sales are of similar magnitudes. We observe that stores with high-demand shocks specialize even more. Thus, the negative marginal effect of demand shocks on the number of product categories is larger in absolute terms if previous demand shocks are used as a covariate. As we expect, the impact of previous productivity on product variety is smaller than that observed when using the current productivity because we omit the current productivity improvements due to learning from demand. In summary, we have shown that our results for the determinants of product variety are robust to the timing assumptions on the main state variables that appear as covariates in policy functions.

Inventory per product category and market share. Table 8 shows the determinants of the average inventory per product category before sales are realized ($log(A_{jt}/np_{jt})$) and the average inventory per product category after sales are realized ($log(N_{jt+1}/np_{jt})$). Higher productivity and demand shocks yield higher inventory per product category before sales, i.e., a higher demand for inventory. A one percent increase in store productivity (demand shocks) shifts the average demand for inventory per product category by approximately 0.05 percent (respectively, 0.06 percent). An increase in store productivity reduces inventory per product category after sales are realized. In other words, stores that sell more of a product category due to their high productivity remain with less inventory per product category after sales are realized. A 10 percent increase in productivity is associated with a 1.2 percent lower end-of-period inventory per product category. Stores with higher demand shocks have higher inventory per product category after sales are realized, which suggests that they eliminate stock-outs.

Table 8 also provides reduced-form results for the determinants of a store's local market share.[51] An increase in productivity or demand shocks yields a higher market share of stores in local markets. Productivity increases a store's market share substantially more than larger demand shocks – specifically, by 0.16 versus 0.01 percent. A positive effect of productivity and demand shocks on market share arises from two channels. First, stores that increase their productivity offer more products and sell more of each product. Second, stores with high-demand shocks focus on increasing sales of top products. Overall, stores with higher productivity and higher demand shocks achieve a higher market share.

The estimates, obtained using previous productivity and demand shocks as covariates to avoid possible endogeneity concerns, confirm the positive impact of demand shocks on inventory performance and market share. Moreover, the effects of revenue productivity on inventory demand and market share remain positive, and slightly larger than those observed when using the current productivity (online Appendix D).

[51] The number of product categories is a function of state variables, and thus we can write a store's market share using the index function as a function of state variables.

Demand for investment in technology and inputs. Table 9 shows the estimates of policy functions for investment demand in technology, labor demand, and the total inventory demand before sales as functions of state variables. Understanding these estimates is key for the analysis of stores' decisions over time. Panel A shows the linear specifications of policy functions, controlling for store fixed effects. Panel B shows a prediction of the observed data using a b-spline approximation and an OLS estimator. This specification is consistent with the nonlinear property of policy functions obtained by solving the Bellman equation. For all policy functions, b-spline approximations provide a good prediction of the observed data.

The findings in Panel A show that stores with high productivity and demand shocks invest more in technology. This result is consistent with the store's dynamic programming property used for identification in the framework of Olley and Pakes; i.e., the optimal investment demand increases with productivity.[52] A 10 percent rise in productivity increases the demand for investment by 2.4 percent on average. Demand shocks also increase a store's optimal investments. A 10 percent increase in demand shocks increases investments by 1 percent. These findings are consistent with the positively correlated trends of inventories and investments in new technology (Maican and Orth, 2018b).

In our model, labor demand is a central proxy variable in recovering productivity and demand shocks. Most importantly, industry facts emphasize that consumers' shopping experience (a part of demand shocks) depends on the employees in the store. We observe that the number of employees is increasing in productivity and demand shocks. As we expect, the impact of productivity on labor demand is larger than that of demand shocks. Furthermore, stores in markets with a large population and high income have more employees.

Higher productivity and demand shocks increase inventory demand a_{jt}. Inventory increases substantially more from productivity than from demand shocks. A 3.3 percent addition to the inventory before sales (a_{jt}) is the optimal response to a 10 percent rise in store productivity. The increase in inventory corresponding to a 10 percent rise in demand shocks is 0.1 percent. Store productivity thus plays a more important role in inventory than do demand shocks.[53]

[52] In this paper, investments in machinery and equipment are associated with investments in technology. For example, a new refrigerator includes innovations in both design and technology that saves space and costs and allows more products to be exposed efficiently.

[53] Because higher productivity and demand shocks increase a store's market power, these findings are consistent with those of Amihud and Mendelson (1989), who show that firms with greater market power hold larger inventories and have higher volatility in inventories.

As expected, stores that have large capital stock and that are located in markets with high population density hold larger inventories.

The findings, obtained using previous productivity and demand shocks as covariates to avoid possible endogeneity concerns, confirm the positive marginal effect of productivity that is larger than that of demand shocks on input demands for investment in technology, labor, and inventory (online Appendix D).

Summary of main results. Our policy function estimates suggest that productivity improvements result in an increase in the flow of products to consumers and allow stores to manage a wider product variety. Productivity as a main driver of product variety is closely linked to the study by Holmes (2001).[54] High-demand shocks, on the other hand, promotes specialization in fewer product categories. We show the role of productivity and demand shocks in deciding the optimal product category mix in the store.

4.2 Counterfactual experiments

We use the estimated model to perform counterfactual experiments for regional policy programs related to the determinants of economies of scope. A goal for policy makers in many countries is to minimize the discrepancies between rural and urban markets. In Sweden, the population in rural markets has decreased by two-thirds since 1985, which has led to lower purchasing power, tax income and service level over time (Statistics Sweden). As a result, the Swedish government has implemented regional development programs to improve commercial services and to support development in rural areas (see Section 2).

Policy programs in retail. Our experiments are directly grounded in policy programs such as *Stores in the countryside* and the *Rural Development Program* that aim to improve stores in rural areas by providing subsidies for investment in technology and assigning mentors to implement store performance enhancement actions, such as modernizing inventory and improving communication and activities in the store (Nordregio, 2011). Our main interest is in quantifying the impact of policy changes on the number of products (the extensive product-category margin), sales per product category (the intensive product-category margin), store-level sales

[54] The sales might decrease in the short run with the adoption of a new technology, but increase in the long run because consumers get used to that technology.

and market share. To gain knowledge on the design of policies that improve the retail landscape in rural areas, we explore differences between rural and urban markets.

We evaluate two types of support for stores. The first is a subsidy for investment in technology by a thirty percent increase in capital stock (the stock of machinery and equipment). This corresponds to an investment subsidy to stores, where the level of subsidy depends on the current stock of technology.[55] The second is a mentoring support program where stores improves learning from demand to foster productivity. We introduce mentoring by a thirty percent increase in the coefficient of the interaction term $\omega \times \mu$ in the productivity process, capturing a better use of information from consumers to improve productivity, and by a five percent increase in the average persistence of the productivity process. Higher productivity persistency means that new mentoring support will have a longer-lasting impact on future store sales. In our setting, how much stores learn from demand to generate innovations that increase productivity depends on stores' productivity level.

Table 10 shows results of the counterfactual experiments for stores in rural and urban markets. In CF_1, we provide investment subsidy and mentoring support for stores with productivity and technology (capital stock per employee) below median five-digit industry sector values. CF_2 considers only mentoring support for the same sample of stores as in CF_1. In CF_3, we provide investment subsidy and mentoring support for stores with demand shocks and technology (capital stock per employee) below median five-digit industry sector values.[56]

Focusing on incumbents, we compare store outcomes before and after a hypothetical policy change. The sign and size of short-run changes in stores' optimal decisions and outcomes from a counterfactual experiment are theoretically ambiguous and depend on store primitives. In our multiproduct model, changes in the economies of scope parameter, the revenue productivity and demand shock processes and the other state variables (e.g., technology stock) affect product variety (the extensive margin), sales per product category (the intensive margin), sales, input demand, market share and the local HHI.

Implementation of policy experiments. The counterfactual experiments require solving two systems of equations – the sales-per-product-category equations for each store and the lo-

[55]The impact of capital stock on sales remains robust if the change in the capital stock is endogenized using the estimated investment policy function.

[56]In online Appendix C, we discuss two additional experiments (CF_{1a} and CF_{2a}) that emphasize the benefits of maintaining high persistence in productivity and demand shocks under uncertainty.

cal market share index system – to obtain sales per product category and sales in the outside option after the policy change (see Section 3). Because we do not estimate the adjustment costs and our experiments do not imply changes in the store's cost structure, we do not need to solve stores' dynamic optimization problem. As discussed in Bajari et al. (2007), we use the estimated policy functions to calculate the number of product categories, demand for labor, and inventory demand after the policy change. This is possible because the new equilibrium, after the experiment, is consistent with the old equilibrium, and therefore the structure of policy functions does not change.

The policy experiments are implemented as follows. First, we use the estimated Markov processes to predict the changes in productivity and demand shocks, which affect the choice of product variety and input demand. We estimate the policy functions using a b-spline polynomial expansion in state variables (Bajari et al., 2007; Ryan, 2012). We predict only the changes in store inputs and state variables for each observation in the data using the estimated policy function and a simulation.[57] The number of product categories (the count variable) is predicted using a negative binomial regression.

Second, using the store's new inputs and state variables, we solve two systems of equations – pertaining to the multiproduct technology and the market share index function – to calculate the new sales per product category, store-level sales, and market shares. We solve the system of multiproduct sales equations for each store using a fixed-point iteration algorithm to obtain sales per product category and store-level sales (see Section 3 and online Appendix B). Using the recomputed sales per product category, we solve the market share index system for each local market to obtain the sales in the outside option, market share, and local concentration measures such as HHI.[58] We report the changes based on 100 simulations (note that productiv-

[57]Because we compute the optimal actions using the policy functions, the reported changes in variables are in the steady state after the experiment. One can use the policy function to simulate the short- and long-run impact of a policy change based on a forward simulation, which implies stimulating the evolution of the store beyond the years observed in the data. However, a good prediction outside the sample might be challenging to obtain if the state space is complex, as in our case, and the horizon is long (e.g., over five years). If the data allow, the researcher can include an exogenous policy change in the store's cost structure and solve the dynamic programming problem using a value function approximation to determine the optimal policies (Maican and Orth, 2020).

[58]We use an adjusted measure of HHI because our model does not endogenize entry and exit of stores. Therefore, our HHI measure is upward biased because it assumes that sales of the outside option are obtained from one store. However, we are interested in the sign of changes in HHI. We observe no sign differences when computing the changes in HHI using only stores in the sample and HHI that includes outside sales.

ity and demand shocks are stochastic).

Role of economies of scope. We begin by exploring the basic benefits of improving economies of scope in rural and urban markets. The main advantage of our multiproduct framework is that it provides an estimate of the degree of economies of scope in the store (parameter α_y). We implement this semicounterfactual as a fifteen percent decrease in the value of α_y while keeping the same number of product categories. While this simulated experiment is not directly related to observed policies in local markets, it is useful for understanding the contribution of economies of scope in driving the outcomes of a policy change. The median gain in stores' sales and sales per product category is 14 percent in rural markets and 12 percent in urban markets. In other words, the sales increase is 2 percentage points higher in rural than in urban markets if economies of scope are improved. This finding shows the importance of improving economies of scope in rural markets to raise sales and sales per product.

Policy support for low-productivity stores. CF_1 provides a subsidy to investment in technology and mentoring support for stores with low productivity and technology per employee. The results in Table 10 show that stores offer more product categories (the extensive margin) after the policy change. Product variety increases more for stores in rural than in urban areas. The median increase in the number of product categories is 3.17 percent in rural markets and 2.36 percent in urban markets. Stores not only offer a wider variety of products after the policy change but also sell more per product category (the intensive margin). Nevertheless, magnitudes are larger for stores in rural markets. The median increase in sales per category is 26.27 percent in rural locations and 22.63 percent in urban locations. More product categories and higher sales per category imply greater store-level sales. Stores in rural markets increase their sales by 30.05 percent at the median, whereas the corresponding increase for urban markets is 24.27 percent. Accounting for all stores in local markets, we observe that the median increase in market share is larger in rural markets. Changes in market shares and local market concentration (HHI) are, however, relatively modest in both market types.

Productivity improvements are an important underlying mechanism behind the greater product variety offered to consumers after the introduction of the policy. Productivity increases by 11.38 and 7.27 percent in rural and urban markets, respectively. A high dispersion in productivity responses indicates heterogeneous impacts of the policy on store productivity.

Enhanced productivity occurs together with higher demand for labor and inventory in both market types.

We compute the median net benefit from investment subsidies for technology (i.e., a rise in the capital stock) across local markets, while abstracting from the cost of mentoring support to improve productivity that is not possible to calculate. We define the subsidy's net benefit in a local market as the sum of gains in sales from raising the stock of technology less the (direct) cost of the investment subsidy under the assumption that there are no additional adjustment costs in implementing the subsidy. The median net benefit of the investment subsidy is 1.65 million SEK in rural markets and 1.98 million SEK in urban markets, and the dispersion is higher across rural areas.

Overall, a policy that combines both an investment subsidy and mentoring support based on learning from demand to increase productivity yields higher intensive and extensive product-category margins, store-level sales and market shares as well as positive net benefits. Targeting low-productivity stores allows consumers in rural areas to enjoy more product variety and encourages stores in rural locations to better utilize economies of scale and scope.

In CF_2 we provide only mentoring support while ignoring the investment subsidy for the same sample of stores as in CF_1. The results in Table 10 show that the intensive and extensive product-category margins increase, but the magnitudes of changes are smaller than in CF_1. The number of product categories (the extensive margin) increases 1 percentage point less if only mentoring support is allowed. Disregarding the investment subsidy has particularly strong implications for stores in rural locations. The increase in sales per product category and store-level sales in rural markets is 6 percentage points lower in CF_2 than in CF_1. The corresponding difference is only 2 percentage points in urban markets. Disregarding the investment subsidy also yields a smaller increase in market shares (0.54 versus 0.37 percent in rural markets).

We find that the investment subsidy, which increases the capital stock, encourages stores in rural markets to replace labor with capital. The labor demand of rural stores increases 2 percentage points more in CF_2 than in CF_1. There is no notable difference for stores in urban areas. Our findings thus suggest that stores in rural markets substitute labor by capital to a larger extent under the investment subsidy. A larger stock of technology also implies that the inventory demand of stores in rural markets increases 7 percentage points more in CF_1 than in

CF_2 (Holmes, 2001). The corresponding difference is only 3 percentage points in urban markets.

In summary, we find that an investment subsidy for technology and mentoring support to improve productivity play a crucial role in increasing product variety. Both a subsidy and mentoring support are needed for low-productivity stores to offer a wider product variety, especially in rural markets. Most importantly, the investment subsidy and mentoring support complement each other, yielding positive net benefits across local markets along with increases in sales, market share and employment that are more prominent in rural markets.

Policy support for low-demand stores. Experiment CF_3 provides a subsidy for investment in technology and mentoring support for stores with low-demand shocks and technology per employee. We thus implement exactly the same policy as in CF_1 but for low-demand stores instead of low-productivity stores. The findings in Table 10 show that the number of product categories, sales per category and store-level sales increase more in CF_3 than in CF_1. The larger magnitudes are driven by more substantial improvements in productivity. This indicates that it is easier for low-demand stores to increase their productivity after the policy change than it would be for low-productivity stores.[59] Stores with low demand are thus able to benefit relatively more than stores with low productivity from policies that promote economies of scale and scope.

Interesting patterns occur across market types. In contrast to CF_1, product variety, sales and productivity increase more in urban markets if the policy is directed to low-demand stores. The number of product categories increases by 6 percentage points more than in CF_1. Store-level sales increase by 16 percentage points more for stores in urban markets in CF_3 than in CF_1. Enhanced productivity and possibilities for utilizing economies of scope in urban areas result in a wider product variety and higher sales, market share and employment. Similar patterns are observed in rural markets, but the magnitudes are smaller. The median net benefit of a policy directed toward to low-demand stores is 4.45 million SEK in rural markets and 5.76 million SEK in urban markets. In other words, stores in urban markets benefit relatively more than do stores in rural markets.

Summary of experiments. The policy experiments suggest that a subsidy for investment in technology for low-productivity and low-demand stores increases the extensive and intensive

[59] Because the productivity process is nonlinear, how much productivity increases depends on the interaction between the current productivity and demand shock levels in our setting.

product-category margins. If the investment subsidy is complemented by mentoring support to improve learning from demand to foster productivity, the magnitudes of the effects on product variety, sales and market share increase further. A policy design for low-productivity stores results in a wider product variety offered to consumers and higher sales gains for stores in rural than in urban markets.

5 Robustness

This section discusses the robustness of results using alternative modeling specifications.

Estimation of the service generating function. In this paper, the labor and the cost of products bought are used as proxy variables to recover productivity and demand shocks. However, instead of labor demand, the investment demand function can be used to recover productivity. The estimation results remain robust if investment is used as a proxy; e.g., the estimated persistence in productivity and demand shocks is similar to our main results. Most importantly, productivity remains the main driver of a store's choices. We prefer the specification with labor demand because it uses all observations in the data and does not require positive investments.

The identification of the model uses the variables at $t-1$ as instruments (Ackerberg et al., 2007; Ackerberg et al., 2015). The estimates do not change if local market variables in the current period t are used as instruments. The persistence of productivity increases if the sales of other product categories in period t (y_{-ijt}) are used as an instrument. As we expect, this finding indicates that the moment condition based on y_{-ijt} does not hold and affects the identification of all parameters of the sales-generating function. For this reason, using previous sales of other products y_{-ijt-1} as an instrument is a better choice.

Relationship with input share estimators. Our model relates to methods that estimate input shares to analyze the relation between productivity and multiple products. In contrast, we use output shares and model economies of scope in the sales technology and do not require data on prices. As in De Loecker et al. (2016), we have separability in inputs and outputs in the production technology, and model firm/store productivity and not product-firm productivity.[60] In the retail context, it is difficult to define a meaningful measure of product-store productivity.

[60] See also Dhyne et al. (2016), Valmari (2016), Orr (2018).

Using the aggregation over inputs and outputs, we can show that there is a direct relationship between the input shares from a Cobb-Douglas technology at the product level and output shares of a transcendental technology. This relationship exists because both technologies use firm/store productivity; i.e., they do not need to aggregate product productivity.

Separating input allocations per product can be difficult in service industries. For example, different machinery and equipment are used to carry or store different product categories at the same time to increase efficiency. The separation of all inputs is not fully consistent with economies of scale and scope. Since our focus is on economies of scope and not on recovering product markups, a transcendental technology that uses observed output shares is preferable because it does not require additional assumptions that would be necessary to recover input shares (not observed in the data).

6 Conclusions

Retail businesses have changed drastically in recent years with substantial investments in new technologies, a sharp increase in warehouse clubs and a shift from products to services. This paper studies the determinants of economies of scope using a dynamic framework that models stores' incentives to hold variety based on their resources and the received demand shocks in local markets. We use the implications of the equilibrium behavior from the store's dynamic model to recover the store's key primitives and evaluate their role in driving the number of product categories (the extensive margin), sales per product category (the intensive margin), store-level sales and market shares. We estimate the model using novel data on a store's product categories, and inputs and outputs in retail sales of new goods in specialized stores in Sweden from 2003 to 2009. The estimated model is used to evaluate the impact of policy changes related to determinants of economies of scope on product variety and store performance.

Our model allows for economies of scale and scope and uses a multiproduct sales technology and information on local market structure to estimate store productivity and demand shocks. In our setting, the store's sales per product category and total sales are endogenous and are solutions to the system of multiproduct sales equations for each store. In the counterfactual experiments, we use the estimated model and the systems of equations for sales and market shares to compute the changes of product variety, sales and local concentration measures.

The empirical findings highlight the role of productivity and demand shocks for product variety in retail. Stores with high productivity and investment in new technology offer a wider variety of products and sell more of all product categories. Stores with high-demand shocks offer a smaller number of product categories and sell more of their top-selling product categories. Together, higher productivity and demand shocks increase store-level sales and market share. Improving economies of scope in the stores imply larger gains in store-level sales in rural than in urban markets.

Policy experiments show that regional development programs that introduce investment subsidies and learning support to foster productivity and promote economies of scope increase the number of product categories and sales. Investments in technology and mentoring support for low-productivity stores increase the extensive and intensive product-category margins, especially benefiting stores in rural markets. Providing the same policy support to low-demand stores leads to greater increases in intensive and extensive product-category margins, but the difference between rural and urban areas might widen because stores in urban locations benefit more due to their productivity advantage.

Our model on the supply side can be integrated with a more general demand framework that allows for rich substitution patterns between products if data on product-level prices are available, which can provide richer implications for consumer surplus in addition to gains in variety. Although our suggested modeling framework is applied to detailed data on retailers in this paper, our analysis has wide-ranging implications for many industries worldwide in which firms offer multiple products.

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Table 1: Descriptive statistics

Year	Sales		No. of employees		Capital stock		Cost of goods		Inventory end year	
	Q_{50}	IQR	Q_{50}	IQR	Q_{50}	IQR	Q_{50}	IQR	Q_{50}	IQR
2004	11.620	27.767	7	10	0.394	0.970	7.282	18.892	1.823	5.154
2005	11.207	20.463	7	7	0.392	0.979	6.865	13.378	2.060	4.050
2006	14.214	25.135	7	9	0.475	1.101	8.783	17.943	2.378	4.987
2007	11.193	23.452	7	9	0.435	1.129	6.572	15.191	1.990	5.040
2008	11.328	24.713	7	10	0.468	1.217	6.805	16.382	2.042	5.816
2009	11.417	24.818	7	10	0.522	1.283	6.785	15.840	2.162	5.572
Year	**No. of products**		**HHI product**		**Market share**		**HHI market**		**C4 market**	
	Q_{50}	IQR	Q_{50}	IQR	Q_{50}	IQR	Q_{50}	IQR	Q_{50}	IQR
2004	3	2	0.738	0.380	0.349	0.694	0.363	0.493	0.919	0.306
2005	4	2	0.495	0.333	0.339	0.635	0.364	0.437	0.918	0.286
2006	4	2	0.549	0.332	0.375	0.658	0.381	0.467	0.929	0.291
2007	4	3	0.601	0.364	0.372	0.626	0.361	0.427	0.925	0.272
2008	3	3	0.707	0.468	0.341	0.657	0.361	0.466	0.930	0.267
2009	3	3	0.655	0.448	0.378	0.665	0.393	0.448	0.941	0.238

NOTE: Sales (excl. VAT), capital stock, inventories, cost of goods, and wages are measured in millions of 2000 SEK (1 USD= 7.3 SEK, 1 EUR= 9.3 SEK). Q_{50} and IQR are interquartile ranges. Capital stock includes only machinery and equipment and is computed using the perpetual inventory method. The HHI product is the Herfindahl-Hirschman index for the product categories at the store level computed using sales. The HHI and C4 are the Herfindahl-Hirschman index and the four-store concentration ratio in a local market for a five-digit industry and are computed using sales.

Table 2: Correlations in the data and their evolution

Correlation (x,y)	2003	2004	2005	2006	2007	2008	2009
Extensive margin							
(Number of products, Capital stock per employee)	0.047	0.040	0.003	-0.019	0.074	0.067	0.121
(Number of products, Sales per cost of goods)	-0.030	-0.027	-0.003	-0.011	-0.017	-0.012	-0.015
(Number of products, Market share)	0.137	0.075	0.071	0.060	0.071	0.071	0.048
Intensive margins							
(Sales per product, Capital per employee)	0.204	0.250	0.111	0.029	0.090	0.136	0.119
(Sales per product, Wages per employee)	0.320	0.236	0.207	0.225	0.192	0.228	0.252
(Entropy of product sales, Inventory per product)	0.022	0.098	0.089	0.068	0.053	0.022	0.021
(Entropy of product sales, Sales per employee)	0.046	0.075	0.063	0.070	0.057	0.012	-0.061
Local market power							
(Market share, Entropy of product sales)	-0.071	-0.042	-0.076	-0.090	-0.065	-0.067	-0.038
(Market share, End-of-year inventory)	0.255	0.182	0.142	0.145	0.086	0.187	0.185
(Market share, Sales per employee)	0.194	0.059	0.071	0.123	0.084	0.093	0.111
(Market share, Capital stock per employee)	0.052	0.102	0.011	-0.025	-0.010	0.025	0.019

NOTE: Entropy measures store diversification in sales and is computed for each store j based on market share of each product category i inside store, i.e., $E_{jt} = \sum_i ms_{ijt} ln(ms_{ijt})$ (Bernard et al., 2011). A large measure of entropy suggests that the store focuses on top sales categories.

Table 3: Reduced-form: Effect of local competition, store's market share and margins on the number of product categories

	Number of product categories				HHI product category			
	Est.	Std.	Est.	Std.	Est.	Std.	Est.	Std.
Panel A: The effect of local competition								
HHI	-0.1400	0.4589			-0.0047	0.0253		
C4			-0.6704	0.7670			0.0941	0.0494
Log of capital stock	-0.0102	0.0689	-0.0102	0.0680	0.0125	0.0053	0.0125	0.0051
Store fixed effect	Yes		Yes		Yes		Yes	
Year fixed effect	Yes		Yes		Yes		Yes	
Adjusted R^2	0.0052		0.0057		0.0096		0.0104	
Panel B: The effect of store's market share and margins								
Log of store's market share	0.1087	0.1021			0.0141	0.0093		
Log of store's margin			-0.2755	0.1055			0.0451	0.0181
Log of capital stock	-0.0164	0.0645	-0.0223	0.0709	0.0118	0.0053	0.0136	0.0089
Store fixed effect	Yes		Yes		Yes		Yes	
Year fixed effect	Yes		Yes		Yes		Yes	
Adjusted R^2	0.0056		0.0065		0.0102		0.0114	

NOTE: HHI and C4 are the Herfindahl-Hirschman index and the four-store concentration ratio in a local market for a five-digit industry and are computed using sales. Store margins are proxied using the ratio (net sales - cost of goods)/net sales. The first difference estimator is used. Standard errors are clustered at the five-digit industry.

Table 4: Relationship between productivity and market share at the store level

	Log of labor productivity in period t			
	Static		Dynamic	
	Est.	Std.	Est.	Std.
Market share in period t	0.4048	0.1009		
Log of labor productivity in period $t-1$			0.8577	0.0289
Market share in period $t-1$			0.0453	0.0259
Store fixed effects	Yes		No	
Year fixed effects	Yes		Yes	
Adjusted R^2	0.0254		0.8167	

NOTE: Standard errors are clustered at the five-digit industry.

Table 5: Estimation of the multiproduct sales-generating function

	OLS		Two-step estimation	
	Estimate	Std.	Estimate	Std.
Log no. of employees	0.7866	0.0290	0.5582	0.0423
Log of capital	0.0599	0.0129	0.2833	0.0276
Log of inventory	1.0367	0.0212	0.4937	0.0237
Log of sales of other products	-0.8959	0.0098	-0.8562	0.0115
Log of sales outside option	-0.0055	0.0065	0.2240	0.014
Log of population	0.0233	0.0218	0.1396	0.036
Log of population density	0.0076	0.0151	0.1903	0.049
Log of income	34.7509	13.2213	0.9340	0.057
Log of income squared	-3.2989	1.2435	-0.0915	0.017
Coef. of no. of products (ρ_{np})			0.2137	0.0364
Elasticity of substitution			4.630	
Year fixed effect	Yes		Yes	
Sub-sector fixed effect	Yes		Yes	
No. of obs.	16,759		16,759	

NOTE: The dependent variable is the log of sales of a product category at the store level. Labor is measured as the number of full-time adjusted employees. Sales of other product categories are measured at the store level. *OLS* refers to an ordinary least squares regression. Two-step estimation refers to the extended Olley and Pakes (1996) estimation method presented in Section 3 (Maican and Orth, 2020). Reported standard errors (in parentheses) are computed using Ackerberg et al. (2012).

Table 6: Estimation of structural parameters: Productivity and demand shock processes

Productivity (ω_t) process			Demand shocks (μ_t) process		
	Estimate	Std.		Estimate	Std.
Productivity (ω_{t-1})	0.8540	0.0649	Demand shock (μ_{t-1})	0.8596	0.0224
Productivity squared (ω_{t-1}^2)	-0.0375	0.0181	Demand shock squared (μ_{t-1}^2)	-0.0195	0.0022
Productivity cubed (ω_{t-1}^3)	-0.0043	0.0015	Demand shocks cubed (μ_{t-1}^3)	-0.0005	0.0002
Prod.*Dem. shock ($\omega_{t-1} \times \mu_{t-1}$)	0.0946	0.0123			
Demand shock (μ_{t-1})	0.0172	0.0025			
Year fixed effects	Yes		Year fixed effects	Yes	
Sub-sector fixed effects	Yes		Sub-sector fixed effects	Yes	
Adjusted R^2	0.981		Adjusted R^2	0.792	
Coefficients of ω_{t-1} terms are zero	F-test	p-value			
	424.139	0.000			
Coefficients of μ_{t-1} terms are zero	F-test	p-value			
	27.713	0.000			
Persistence ($d\omega_t/d\omega_{t-1}$)	0.856		Persistence ($d\mu_t/d\mu_{t-1}$)	0.929	
Effect of demand shocks ($d\omega_t/d\mu_{t-1}$)	0.013				

NOTE: Productivity is estimated using the two-step estimation method in Section 3. Mean values are presented for the marginal effects.

Table 7: Impact of store and market characteristics on product category

Dependent variable	No. product categories (np_{jt})		HHI product categories in a store		Entropy of sales of product categories	
	Est.	Std.	Est.	Std.	Est.	Std.
Productivity (ω_t)	1.2058	0.1876	-0.0738	0.0094	-0.1568	0.0234
Demand shocks (μ_t)	-0.1647	0.0377	0.0116	0.0018	0.0228	0.0044
Log of capital (k_t)	0.3469	0.0978	-0.0162	0.0070	-0.0348	0.0130
Log of inventory (n_t)	-0.1218	0.0306	0.0060	0.0173	0.0231	0.0268
Log of population (pop_t)	-1.2601	0.2976	0.0309	0.0390	0.0746	0.0627
Log of population density ($popdens_t$)	0.4861	0.3686	-0.0101	0.0690	-0.0235	0.1090
Log of income (inc_t)	-2.6027	3.2836	-0.4403	0.1413	-0.7676	0.3607
Store fixed effects	Yes		Yes		Yes	
Year fixed effects	Yes		Yes		Yes	
Adjusted R^2	0.119		0.053		0.061	

NOTE: Productivity and demand shocks are estimated using the two-step estimation method in Section 3. Entropy is computed for each store j based on the market share of each product category i inside store, i.e., $E_{jt} = \sum_i ms_{ijt} ln(ms_{ijt})$. Store regressions control for the average wage.

Table 8: Determinants of inventory performance and store's market share

Dependent variable	Inventory per product before sales ($ln(A_{jt}/np_{jt})$)		Inventory per product after sales $ln(N_{jt+1}/np_{jt})$		Store's market share	
	Est.	Std.	Est.	Std.	Est.	Std.
Productivity (ω_t)	0.0483	0.0239	-0.1208	0.0255	0.1654	0.0120
Demand shocks (μ_t)	0.0599	0.0049	0.0550	0.0053	0.0132	0.0025
Log of capital (k_t)	0.0823	0.0177	-0.0119	0.0189	0.0945	0.0089
Log of inventory (n_t)	0.0212	0.0177	0.0229	0.0189	0.0517	0.0089
Log of population (pop_t)	0.6717	0.0876	0.5717	0.0934	-0.6770	0.0442
Log of population density ($popdens_t$)	-0.4953	0.0963	-0.3958	0.1026	-0.0884	0.0486
Log of income (inc_t)	0.0878	0.9702	1.1587	1.0338	1.9653	0.4901
Store fixed effects	Yes		Yes		Yes	
Year fixed effects	Yes		Yes		Yes	
Adjusted R^2	0.079		0.051		0.257	

NOTE: Productivity and demand shocks are estimated using the two-step estimation method in Section 3. Store regressions control for the average wage.

Table 9: Estimation of the investment, labor, and inventory demand functions

	Log of investment (i_t)		Log of labor (l_t)		Log of products and inventories (a_t)	
Panel A: Linear specifications						
Productivity (ω_t)	0.2456	0.0945	0.1029	0.0058	0.3368	0.0402
Demand shock (μ_t)	0.0342	0.0191	0.0084	0.0029	0.0129	0.0026
Log of capital (k_t)	-0.4541	0.0657	0.0614	0.0125	0.1694	0.0141
Log of inventory (n_t)	-0.1761	0.0786	0.0281	0.0071	-0.0195	0.0120
Log of population (pop_t)	0.6855	0.3112	0.2991	0.1654	0.3168	0.1209
Log of pop. density ($popdens_t$)	-0.5583	0.3289	-0.2161	0.1393	-0.3635	0.1214
Log of income (inc_t)	-0.7944	3.5450	0.6819	0.8813	-1.3234	1.8809
Store fixed effects	Yes		Yes		Yes	
Year fixed effects	Yes		Yes		Yes	
Adjusted R^2	0.093		0.171		0.343	
Panel B: Non-linear specification using b-splines of degree 5						
	Data	Prediction	Data	Prediction	Data	Prediction
25th Percentile	-2.9239	-2.6560	1.6094	1.7183	1.780	1.7754
50th Percentile	-1.6507	-1.8764	2.1972	2.2175	2.738	2.7188
Mean	-1.2917	-1.2917	2.5887	2.5887	2.945	2.9446
75th Percentile	-0.0092	-0.5624	2.9444	2.8759	3.771	3.7464
Year fixed effects	Yes		Yes		Yes	
Sub-sector fixed effects	Yes		Yes		Yes	
Adjusted R^2	0.667		0.929		0.717	

NOTE: The dependent variables are the log of investment in capital (i_t), the log of the sum between the inventories at the beginning of the year (n_t) and the cost of products bought during the year (a_t), and the log of inventories at the end of the year (n_{t+1}). All regressions include an intercept and control for the average wage. Productivity and demand shocks are estimated using the two-step estimation method in Section 3.

Table 10: Counterfactual experiments: Impact of investment subsidy in technology and mentoring support on stores in rural and urban markets

	Rural markets		Urban markets	
	Median	Std.	Median	Std.
CF_1: Investment subsidy in technology and mentoring support for stores with low productivity and capital stock per employee				
Change in store's number of products	3.17	6.46	2.36	5.81
Change in store's sales per product	26.27	8.97	22.63	15.11
Change in store's total sales	30.05	11.20	24.27	20.62
Change in store's market share	0.54	0.35	0.19	0.63
Change in market HHI	0.26	0.52	-0.34	0.89
Change in store's productivity	11.38	30.44	7.27	26.55
Change in store's labor demand	26.27	8.40	27.83	6.60
Change in store's inventory demand	37.00	20.12	36.90	20.00
Net benefit in local markets	1.65	6.06	1.98	3.11
CF_2: Mentoring support for stores with low productivity and capital stock per employee				
Change in store's number of products	2.18	7.10	1.80	7.33
Change in store's sales per product	21.63	12.08	20.25	18.07
Change in store's total sales	24.69	19.06	22.24	25.07
Change in store's market share	0.37	0.52	0.16	0.73
Change in market HHI	0.21	0.69	-0.29	1.33
Change in store's productivity	9.98	34.56	8.19	34.19
Change in store's labor demand	29.24	17.48	26.04	20.93
Change in store's inventory demand	29.29	26.26	33.40	27.37
CF_3: Investment subsidy in technology and mentoring support for stores with low demand and capital stock per employee				
Change in store's number of products	6.45	4.91	7.96	2.73
Change in store's sales per product	32.23	7.25	33.51	9.52
Change in store's total sales	39.57	9.74	41.81	11.82
Change in store's market share	0.38	0.37	0.63	0.43
Change in market HHI	0.53	0.33	0.09	0.91
Change in store's productivity	19.57	23.49	27.49	12.06
Change in store's labor demand	35.30	22.74	39.58	16.70
Change in store's inventory demand	39.44	30.01	47.61	16.64
Net benefit in local markets	4.45	6.08	5.76	14.48

NOTE: All figures, except net benefits, are in percentages. Net benefits are computed as total gain in sales minus the cost of subsidy in a local market and are in million SEK. In CF_1 and CF_3, investment subsidy in technology is implemented as an increase in capital stock by 30 percent. In $CF_1 - CF_3$, the mentoring support is implemented by setting the marginal impact of μ on ω to increase by thirty percent (i.e., coefficient of interaction term between productivity and demand in the productivity process) and an average increase in productivity persistency by five percent. CF_1 and CF_3 provide support for stores with productivity and capital stock per employee below the five-digit industry median values. CF_3 considers support for stores with demand shocks and capital stock per employee below the five-digit industry median values.

Online Appendix: Determinants of Economies of Scope in Retail

Florin Maican[1] and Matilda Orth[2]

Appendix A: Derivation of the sales-generating function

This appendix presents the derivation of the sales-generating function (4) using the multiproduct service technology (2) and a demand system. We follow the presentation of Maican and Orth (2020). The main aim is to develop a multiproduct sales function and identify its parameters. Separately identifying the coefficients of production technology and demand without price data is beyond the scope of this paper.

The multiproduct service technology is given by

$$\sum_{i=1}^{np_{jt}} \tilde{\alpha}_i q_{ijt} + \tilde{\alpha}_y Y_{jt} = \tilde{\beta}_l l_{jt} + \tilde{\beta}_k k_{jt} + \tilde{\beta}_a a_{jt} + \tilde{\omega}_{jt} + \tilde{u}^p_{jt}, \tag{1-a}$$

where q_{ijt} is the logarithm of quantity of product category i sold by store j in period t, Y_{jt} denotes total sales of store j in period t, l_{jt} is the logarithm of the number employees, k_{jt} is the logarithm of capital stock, a_{jt} is the logarithm of the sum of the inventory level in the beginning of period t (n_{jt}) and the products bought during period t, and $\tilde{u}^p_{jt}$ are i.i.d. remaining service output shocks. Variable np_{jt} denotes the number of products (categories) of store j.[3]

We use a CES demand system to obtain an expression for the logarithm of the price of product category i (p_{ijt}), i.e., $\mathrm{p}_{ijt} = -\frac{1}{\sigma}(q_{ijt} - q_{0t}) + \mathbf{x}'_{ijt}\frac{\tilde{\boldsymbol{\beta}}_x}{\sigma} + \frac{\sigma_a}{\sigma} a_{jt} + \frac{1}{\sigma}\tilde{\mu}_{ijt}$. Multiplying the logarithm of price by $\tilde{\alpha}_i$ and summing up over store's j product categories, we obtain the

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[3]As we mention in the main text, we have only information on product categories in the empirical application.

following expression:

$$\sum_{i=1}^{np_{jt}} \tilde{\alpha}_i p_{ijt} = -\frac{1}{\sigma}\sum_{i=1}^{np_{jt}} \tilde{\alpha}_i q_{ijt} + \frac{1}{\sigma}\sum_{i=1}^{np_{jt}} \tilde{\alpha}_i q_{0t} + \sum_{i=1}^{np_{jt}} \tilde{\alpha}_i \mathbf{x}'_{ijt}\frac{\tilde{\boldsymbol{\beta}}_x}{\sigma} + \frac{\sigma_a}{\sigma}\sum_{i=1}^{np_{jt}} \tilde{\alpha}_i a_{jt} + \frac{1}{\sigma}\sum_{i=1}^{np_{jt}} \tilde{\alpha}_i \tilde{\mu}_{ijt}. \quad \text{(2-a)}$$

The logarithm of sales per product category is $y_{ijt} = q_{ijt} + p_{ijt}$. To obtain an expression for sales per product category, we sum up the expressions (1-a) and (2-a), i.e., $\sum_{i=1}^{np_{jt}}[\tilde{\alpha}_i y_{ijt} + \left(1-\frac{1}{\sigma}\right)\tilde{\alpha}_y Y_{jt}] = \left(1-\frac{1}{\sigma}\right)[\tilde{\beta}_l l_{jt} + \tilde{\beta}_k k_{jt} + \tilde{\beta}_a a_{jt}] + \frac{1}{\sigma}\sum_{i=1}^{np_{jt}}[\tilde{\alpha}_i q_{0t}] + \sum_{i=1}^{np_{jt}}[\tilde{\alpha}_i \mathbf{x}'_{ijt}\frac{\tilde{\boldsymbol{\beta}}_x}{\sigma}] + \frac{\sigma_a}{\sigma}\sum_{i=1}^{np_{jt}}[\tilde{\alpha}_i]a_{jt} + \frac{1}{\sigma}\sum_{i=1}^{np_{jt}}\tilde{\alpha}_i\tilde{\mu}_{ijt} + (1-\frac{1}{\sigma})\tilde{\omega}_{jt} + (1-\frac{1}{\sigma})\tilde{u}^p_{jt}$. The logarithm of the aggregate quantity of the outside option q_{0t} can be written as $q_{0t} = \tilde{c}_{ij} q_{i0t}$, where $\tilde{c}_{ij} > 1$ and q_{i0t} is the logarithm of the quantity of product category i that is sold by stores in the outside option.[4] Thus, we can write $\sum_{i=1}^{np_{jt}}[\tilde{\alpha}_i q_{0t}] = \sum_{i=1}^{np_{jt}}[\tilde{\alpha}_i \tilde{c}_{ij} q_{i0t}]$. Using $q_{i0t} = y_{i0t} - p_{i0t}$, we obtain $\sum_{i=1}^{np_{jt}}[\tilde{\alpha}_i q_{0t}] = \sum_{i=1}^{np_{jt}}[\tilde{\alpha}_i \tilde{c}_{ij}(y_{i0t} - p_{i0t})]$. Because $\tilde{c}_{ij} > 1$, there exist $s_{ij} < 1$ and $c_j > 1$ such that $\sum_{i=1}^{np_{jt}}\tilde{\alpha}_i\tilde{c}_{ij} = c_j$ and $\sum_{i=1}^{np_{jt}} s_{ij} = 1$. Therefore, we obtain $\sum_{i=1}^{np_{jt}}[\tilde{\alpha}_i q_{0t}] = c_j(\sum_{i=1}^{np_{jt}} s_{ij} y_{i0t} - \sum_{i=1}^{np_{jt}} s_{ij} p_{i0t}) = c_j(\tilde{y}_{0jt} - \tilde{p}_{0jt}) = c_j y_{0jt} \equiv y_{ot}$, where $\tilde{y}_{0jt}$ are weighted sales of product categories of store j that are sold in the outside market, $\tilde{p}_{0jt}$ is a weighted price index, y_{0jt} are deflated sales of product categories of store j that are sold in the outside market, and y_{ot} denotes outside option sales. We measure y_{ot} by total sales of stores in the outside option. Most importantly, for any store j, we can write the term of the outside option in terms of total sales of the outside option in the multiproduct sales function. If there are no stores in the outside option, y_{ot} represents total sales in the market.

The next step is to regroup the remaining coefficients and determine how they are affected by σ. We denote $\beta_q \equiv 1/\sigma$, $\beta_l \equiv (1-\frac{1}{\sigma})\tilde{\beta}_l$ and $\beta_k \equiv (1-\frac{1}{\sigma})\tilde{\beta}_k$. As we mention in the main text, we are unable to identify the impact of inventory separately on demand and supply without additional assumptions. Therefore, we sum up the net impact of inventory on sales under parameter $\overline{\beta}_a$, i.e., we denote $(1-\frac{1}{\sigma})\overline{\beta}_a \equiv (1-\frac{1}{\sigma})\tilde{\beta}_a + \frac{\sigma_a}{\sigma}\sum_{i=1}^{np_{jt}}\tilde{\alpha}_i$. Furthermore, to shorten the notation, we denote $\beta_a \equiv (1-\frac{1}{\sigma})\overline{\beta}_a$. Because a_{jt} is part of both supply and demand equations, we are unable to identify $\tilde{\beta}_a$ and σ_a separately. In other words, we can identify only the net effect $\overline{\beta}_a$. In our case, $\mathbf{x}_{ijt}$ includes only market variables and therefore we denote $\overline{\boldsymbol{\beta}}_x \equiv \sum_{i=1}^{np_{jt}}\tilde{\alpha}_i\tilde{\boldsymbol{\beta}}_x$ and $\boldsymbol{\beta}_x \equiv \overline{\boldsymbol{\beta}}_x/\sigma$. We also denote by $\omega_{jt} \equiv (1-1/\sigma)\tilde{\omega}_{jt}$ a measure of revenue (sales) productivity, and refer to it as simple store productivity in what follows.

[4] Note that store j does sell only few product categories, and therefore $c_{ij} > 1$.

Additionally, μ_{jt} is a weighted sum of all unobserved product demand shocks at the store level, determined as $\mu_{jt} \equiv (1/\sigma) \sum_{i=1}^{np_{jt}} \tilde{\alpha}_i \mu_{ijt}$, and measures store j's specific demand shocks in period t, and u_{ijt}^p are i.i.d. remaining shocks to sales that are mean-independent of all control variables and store inputs. Using this notation we can write the multiproduct sales function as $\sum_{i=1}^{np_{jt}} [\tilde{\alpha}_i y_{ijt} + \left(1 - \frac{1}{\sigma}\right) \tilde{\alpha}_y Y_{jt}] = \beta_l l_{jt} + \beta_k k_{jt} + \beta_a a_{jt} + \beta_q y_{ot} + \mathbf{x}_{jt}' \boldsymbol{\beta}_x + \omega_{jt} + \mu_{jt} + u_{ijt}^p$.

The combination of the service technology and a simple CES demand yields an expression for the sales technology where the left-hand-side is a linear combination of sales per product category and the right-hand-side is a linear combination of store inputs, local demand shifters, store revenue productivity, and demand shocks. This relationship solves the aggregation problem across different products. How many output parameters $\tilde{\alpha}_i$ we can identify depends on the available data on products (categories) and the variation across stores. If there is large heterogeneity in products offered for sale across stores, we need to reduce the number of parameters $\tilde{\alpha}_i$ that can be identified. By choosing only stores that sell similar products, we induce a selection problem. As a result, even if we estimate many technology parameters, the overall inference of the empirical exercise might be biased. In our Swedish data, there is large heterogeneity in product categories stores offer for sale. Thus, since we solve the multiproduct aggregation problem across product categories using sales instead of quantity, we rewrite the linear expression for product sales to reduce the number of parameters. In other words, we focus on sales of product category i and sales of other product categories. To obtain an estimable product sales equation that includes the logarithm of sales of product category i, y_{ijt}, and the logarithm of sales of other product categories inside the store y_{-ijt}, we rewrite the linear sum of product category sales $\sum_{i=1}^{np_{jt}} \left[\tilde{\alpha}_i y_{ijt} + \left(1 - \frac{1}{\sigma}\right) \tilde{\alpha}_y Y_{ijt}\right] \equiv \alpha_i y_{ijt} + \alpha_y y_{-ijt}$. Using new transformations, we can rewrite the sales of product category i as[5]

$$y_{ijt} = -\alpha_y y_{-ijt} + \beta_l l_{jt} + \beta_k k_{jt} + \beta_a a_{jt} + \beta_q y_{ot} + \mathbf{x}_{jt}' \boldsymbol{\beta}_x + \omega_{jt} + \mu_{jt} + u_{ijt}^p, \tag{3-a}$$

which is the equation we estimate in the main text.

In summary, it is important to discuss a few aspects of identification of the multiproduct technology. First, we focus on developing a simple multiproduct setting that does not require detailed product data and that can be used to analyze trends and the impact of policies in

[5] We normalize $\alpha_i = 1$.

local markets. Second, we need product prices to identify the initial quantity weights $\tilde{\alpha}_i$ and variation in other product characteristics. Most importantly, in empirical settings, even if we have access to detailed product data and prices, we need data over a ong period of time to consistently identify $\tilde{\alpha}_i$ (solving a system of equations at the firm/store level). In our setting, the scope parameter α_y in the multiproduct sales-generating function (3-a) includes the sum of weights $\tilde{\alpha}_i$. In other words, parameter α_y provides information on the economies of scope in the store based on supply-side information (the multiproduct service frontier) and demand (elasticity of substitution).

Derivation of logit demand for homogeneous consumers. In what follows, we present the derivation of the well-known equation of logit demand for homogeneous consumers from a CES demand system (see, e.g., Anderson et al., 1987; Verboven, 1996; Anderson and De Palma, 2006; and Dube et al., 2020). This derivation helps provide an understanding of the form of the price equation (2-a). We assume that consumers are homogeneous and have CES preferences over the differentiated products and services $i \in \{1, \cdots, np_j\}$ of store j, and the utility function is given by

$$U(\{Q_{ijt}, \mathbf{x}_{ijt}, \mu_{ijt}, \mathbf{z}_{ijt}, \eta_{ijt}\}_{i=\overline{1,np_j}}) := \left(\sum_{i=1}^{np_j} \kappa(\mathbf{x}_{ijt}, \mu_{ijt}, \mathbf{z}_{ijt}, \eta_{ijt})^{\frac{1}{\sigma}} Q_{ijt}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, \tag{4-a}$$

where $\kappa(\mathbf{x}_{ijt}, \mu_{ijt}, \mathbf{z}_{ijt}, \eta_{ijt})$ is the kernel quality function (Dube et al., 2020). Terms $\mathbf{x}_{ijt}$ and $\mathbf{z}_{ijt}$ are the observed determinants of the intensive and extensive margins of the utility function when consumers buy product i. They might include common variables, and $\mathbf{z}_{ijt}$ includes at least one component that is not part of $\mathbf{x}_{ijt}$. Variables μ_{ijt} and η_{ijt} are determinants of the intensive and extensive margins of the utility and are unobservable by the researcher. The quality function $\kappa(\cdot)$ allows us to separate intensive and extensive margins and to accommodate a zero market share (Dube et al., 2020).

The optimization problem for the representative consumer is given by

$$\begin{aligned} &\max\nolimits_{Q_{ijt,i=\overline{1,np_j}}} \left(\sum\nolimits_{i=1}^{np_j} \kappa(\mathbf{x}_{ijt}, \mu_{ijt}, \mathbf{z}_{ijt}, \eta_{ijt})^{\frac{1}{\sigma}} Q_{ijt}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \\ &\quad s.t. \quad \sum\nolimits_{i=1}^{np_j} P_{ijt} Q_{ijt} = b_t \end{aligned} \tag{5-a}$$

The solution of this optimization problem gives us the demand function (Q_{ijt}) and the individual choice probability (π_{ijt}), which is the CES demand system with observed and unobserved product characteristics, i.e.,

$$Q_{ijt} = \frac{\kappa(\mathbf{x}_{ijt},\mu_{ijt},\mathbf{z}_{ijt},\eta_{ijt})P_{ijt}^{-\sigma}}{\sum_{h=1}^{np_j}\kappa(\mathbf{x}_{hjt},\mu_{hjt},\mathbf{z}_{hjt},\eta_{hjt})P_{hjt}^{1-\sigma}} b_t$$

$$\pi_{ijt} = \frac{\kappa(\mathbf{x}_{ijt},\mu_{ijt},\mathbf{z}_{ijt},\eta_{ijt})P_{ijt}^{-\sigma}}{\sum_{h=1}^{np_j}\kappa(\mathbf{x}_{hjt},\mu_{hjt},\mathbf{z}_{hjt},\eta_{hjt})P_{hjt}^{-\sigma}} \tag{6-a}$$

The elasticity of substitution σ is globally identified for the set of products with positive individual choice probabilities, i.e., $\pi_{ijt} > 0$. The reason is that system $\{\pi_{ijt}\}$ satisfies the connected substitutes condition provided by Berry et al. (2013), i.e., it is invertible.

The choice of the exponential kernel quality has key implications for the identification of the demand system. For $\mathbf{x}_{ijt} = \mathbf{z}_{ijt}$ and $\mathbf{z}_{ijt}$ being exogenous for all i, $\mu_{ijt} = \eta_{ijt}$, $\pi_{ijt>0}$ for all i, and we do not need any exclusion restriction to identify the demand system. In this case, the logarithm of the ratio of individual choice probabilities of product j and the outside option (or numeraire) if we normalize $\mathbf{x}_{0jt} = 0$, $\mu_{0jt} = 0$, and $\kappa(\mathbf{x}_{ijt}, \mu_{ijt}) = exp(\mathbf{x}'_{ijt}\boldsymbol{\beta}_x + \mu_{ijt})$ is given by[6]

$$ln(\pi_{ijt}) - ln(\pi_{0jt}) = -\sigma ln(P_{ijt}) + \mathbf{x}'_{ijt}\boldsymbol{\beta}_x + \mu_{ijt}. \tag{7-a}$$

Equation (7-a) is a logit demand system for homogeneous consumers, which can be written in terms of quantity

$$q_{ijt} - q_{0jt} = -\sigma p_{ijt} + \mathbf{x}'_{ijt}\boldsymbol{\beta}_x + \mu_{ijt}. \tag{8-a}$$

Appendix B: Economies of scope: Simulation of the model

In this section, we discuss using the estimated model to compute sales per product. We also highlight the channels that drive the increase in sales and the role of economies of scope. For simplicity of exposition, we consider a store j that offers three product categories. We can

[6] The reason is that $ln(\pi_{ijt}) - ln(\pi_{0jt}) = -\sigma ln(P_{ijt}) + ln(\kappa(\mathbf{x}_{ijt}, \mu_{ijt})) - ln(\kappa(\mathbf{x}_{0jt}, \mu_{0jt}))$.

rewrite the multiproduct sales of store j for each product category as a system of equations

$$\begin{aligned} lnY_{1jt} &= -\alpha_y ln(Y_{2jt} + Y_{3jt}) + T_{jt} + u^p_{1jt} \\ lnY_{2jt} &= -\alpha_y ln(Y_{1jt} + Y_{3jt}) + T_{jt} + u^p_{2jt} \\ lnY_{3jt} &= -\alpha_y ln(Y_{1jt} + Y_{2jt}) + T_{jt} + u^p_{3jt} \end{aligned} \tag{9-a}$$

where the second-to-last term in each equation is defined as $T_{jt} = (\beta_l l_{jt} + \beta_k k_{jt} + \beta_a a_{jt}) + \beta_q y_{ot} + \mathbf{x}'_{mt}\boldsymbol{\beta}_x + \omega_{jt} + \mu_{jt}$, i.e., it includes all store- and market-specific terms.[7] The system of equations (9-a) is nonlinear and satisfies the contraction mapping properties, i.e., it has a unique solution that can be found by using the fixed-point iteration algorithm. Total sales of store j are $Y_{jt} = Y_{1jt} + Y_{2jt} + Y_{3jt}$.

The estimation of multiproduct sales gives α_y and T_{jt}. A change in store characteristics (i.e., in T_{jt}) affects sales of all products in a store equally, and the magnitude of the effect depends on the scope parameter α_y. A change in market characteristics (part of T_{jt}) affects sales of a product in two stores differently depending on stores' characteristics and the scope parameter. The system of equations (9-a) is used to compute product sales after the introduction of a new product. It is important to note that in this example, the number of products is not endogenous, i.e., we do not model the cost of variety. Endogenizing the number of products by modeling the cost of variety has the advantage that a change in scope parameter α_y affects stores' optimal variety (the first-order condition of a store's dynamic optimization changes).

Table B.1 shows the changes in product sales after the introduction of a new product, a decrease in the scope parameter (i.e., α_y), and a change in store/market characteristics, considering a store with three product categories as an example. First, increasing the number of products from three to four yields a drop in sales per product and an increase in total sales (4.31 vs. 4.44). Thus, this finding shows the importance of economies of scope for a store, i.e., total sales increase if the store sells more products with the same resources. Second, a decrease in the scope parameter α_y from 0.85 to 0.80 results in an increase in total sales by 10 percent. For example, economies of scope are important if there is less competition between the products in the store. Third, an increase in term T_{jt} by 20 percent yields an increase in total sales by approximately 60 percent.

[7] Note that a store is unique, i.e., it exists only in one market.

Table B.1: Economies of scope: Model simulation example

	Product shocks	Model parameters		Log of sales				
		Scope param.	Store/market term	Prod. 1	Prod. 2	Prod. 3		Total
		α_y	T_{jt}	y_{1jt}	y_{2jt}	y_{3jt}	y_{4jt}	y_{jt}
Main specification	Yes	0.85	6.32	3.03	3.19	3.38		4.31
Main specification	No	0.85	6.32	3.09	3.09	3.09		4.19
Add a new product	Yes	0.85	6.32	2.83	2.96	3.10	3.25	4.44
Add a new product	No	0.85	6.32	2.91	2.91	2.91	2.91	4.29
Decrease α_y	Yes	0.80	6.32	3.14	3.30	3.48		4.41
Decrease α_y	No	0.80	6.32	3.20	3.20	3.20		4.30
Increase T_{jt} by 20%	Yes	0.85	7.58	3.71	3.87	4.06		4.99
Increase T_{jt} by 20%	No	0.85	7.58	3.78	3.78	3.78		4.87

NOTE: The product shocks are $u = \{0.1, 0.2, 0.3\}$ (three products) and $u = \{0.1, 0.2, 0.3, 0.4\}$ (four products).

Appendix C: Additional counterfactual experiments:

Trade-offs under uncertainty: Benefits of maintaining productivity and demand levels. Our framework models uncertainty in both productivity and demand shocks, which are stochastic processes. Policy experiments CF_{1a} and CF_{2a} in Table C.2 show the impact of improving the persistence of a store's demand shocks and productivity. This allows for a better understanding of the consequences of degradation in these key primitives, which can have negative consequences for stores if there are no resources to invest in technology. We implement CF_{1a} by considering a five percent increase in the coefficient of term μ_{jt-1} in the demand shocks' process. In CF_{2a}, we also add a five percent increase in the coefficient of term ω_{jt-1} in the productivity process.

The gains from a higher persistence in demand shocks drive specialization whereby stores sell fewer product categories and continue to experience high-demand shocks. Our findings show that store demand shocks increase by 6 percent in rural markets and by 13 percent in urban markets, i.e., the demand shock gain in urban markets is approximately double that in rural markets. If demand shocks increase, the number of product categories declines by 1 percent, whereas labor and inventories rise by 2-4 percent. The median sales per product category and store-level sales increase by 2-3 percent, whereas the market share increases slightly less in both markets. If higher demand shocks are associated with the quality of shopping, then the quality of shopping outweighs the decline in the number of product categories, suggesting that consumers benefit up to two times more in urban than in rural markets. Hence, consumers in urban markets benefit relatively more from specialization and demand improvements than consumers in rural markets.

A higher persistence in both productivity and demand shocks shows clear evidence of a mechanism whereby productivity improvements lead to considerably higher sales and a wider variety of goods that benefit consumers. The number of product categories increases by 4-5 percent in both markets. The median sales per product category and store-level sales increase by 10 percent and 14 percent, respectively. The difference in the growth in sales between the two types of markets is reduced, which shows the critical role of improving productivity in rural markets. The market share increases only slightly more than in CF_{1a}. Overall, consumers benefit substantially from the productivity channel due to the increase in variety and quality of shopping.

Increase in local market demand. Policy experiments CF_{3a} and CF_{4a} in Table C.3 investigate the impact of an exogenous increase in the aggregate local market demand. We consider a thirty percent increase in the average income and population in local markets and assume no changes in store productivity, demand shocks, and technology stock. Incumbents respond differently in their input choices to a higher local market demand, depending on their productivity and demand shocks. This leads to differences between stores and between rural and urban markets.

The findings in CF_{3a} show that a rise in the average income reduces the number of product categories by 2.5 percent. As a result, median sales per product increase in both markets. This suggests that an exogenous increase in market income influences both intensive and extensive product margins, leading to stores specializing in fewer products.[8] Median store-level sales decrease by 1 percent in rural markets and by 2 percent in urban markets. Specialization is thus more pronounced in urban than in rural markets. The dispersion in changes in store-level sales yields an increase in the median market share and HHI. That an increase in income results in lower labor demand and higher inventory before sales confirms the fact that stores specialize in fewer product categories in markets with higher purchasing power. This occurs because

[8] In recent years, there has been a well-documented increase in income inequality in many countries. Such a rise implies a shift from the middle-income class, which includes the main part of consumers of traditional retailers, to the high-income class containing individuals that might prefer saving to consumption (Goolsbee, 2020).

consumers with higher incomes become more sophisticated and prefer higher quality. Another reason is that stores face a higher labor cost and therefore reduce staffing.[9]

Experiment CF_{4a} quantifies the impact of an increase in the local market population. The results in Table C.3 show that a larger local market size reduces incumbents' number of product categories by 3.5 percent and increases sales per product category by 4-5 percent (at the median), i.e., stores specialize.[10] Store-level sales increase in the case of a larger population but not in that of a higher income in rural markets, where product demand might be limited. Policies that raise the aggregate demand are important for increasing store-level sales in rural regions. If income increases, consumers can choose products/stores with higher quality. Additionally, the importance of the aggregate demand is shown by a more substantial increase in labor and inventory demand in rural markets.

It is important to emphasize that the negative impact of rising local demand on store variety does not imply that aggregate product variety in the local market decreases. The reason is that we do not account for entry and how much product variety changes in the outside option.

Table C.2: Trade-offs under uncertainty – Benefits of maintaining productivity and experiencing demand shocks over time

	Rural markets		Urban markets	
	Median	Std.	Median	Std.
CF_{1a}: Higher demand shocks persistence over time				
Change in store's number of products	-0.85	1.39	-1.49	1.40
Change in store's sales per product	2.93	1.32	3.60	1.97
Change in store's total sales	2.17	1.63	2.17	0.83
Change in store's market share	0.59	0.76	0.88	1.01
Change in market HHI	1.00	0.62	0.05	2.13
Change in store's demand shocks	5.97	6.60	12.84	12.18
Change in store's labor demand	2.70	1.41	1.68	1.74
Change in store's inventory demand	3.76	1.88	1.90	2.72
CF_{2a}: Improve both productivity and demand shocks persistence over time				
Change in store's number of products	5.22	1.14	4.09	1.63
Change in store's sales per product	10.00	3.03	9.57	2.83
Change in store's total sales	14.24	2.67	13.93	2.72
Change in store's market share	1.06	1.05	1.23	1.25
Change in market HHI	1.04	0.78	0.20	2.54
Change in store's demand shocks	7.30	6.60	12.84	12.18
Change in store's productivity	26.54	5.47	25.86	4.94
Change in store's labor demand	0.48	1.64	0.01	1.84
Change in store's inventory demand	3.43	2.13	1.41	2.91

NOTE: All numbers are in percentages. There is a 5 percent change in the coefficient of μ_{jt-1} in the demand shocks process in CF_{1a} and CF_{2a}. In addition, there is a 5 percent change in the coefficient of ω_{jt-1} in the productivity process in CF_{2a}.

[9] While we model aggregate income effects, our model is limited in fully understanding the heterogeneous impact of changes in income on consumer preferences, which is also not the main aim of our paper.

[10] We only quantify the changes in incumbents' behavior and do not model entry and exit.

Table C.3: Role of demand in rural and urban markets

	Rural markets		Urban markets	
	Median	Std.	Median	Std.
CF_{3a}: Increase in average income				
Change in store's number of products	-2.49	0.001	-2.48	0.06
Change in store's sales per product	1.20	1.43	1.78	1.60
Change in store's total sales	-2.08	0.99	-1.06	0.73
Change in store's market share	0.43	0.30	0.52	0.35
Change in market HHI	0.47	0.18	0.28	0.61
Change in store's labor demand	-3.47	2.18	-1.26	1.86
Change in store's inventory demand	1.27	1.26	0.76	1.01
CF_{4a}: Increase in market size (population)				
Change in store's number of products	-3.49	0.001	-3.48	0.09
Change in store's sales per product	5.19	2.56	3.93	1.23
Change in store's total sales	1.19	1.87	-0.10	0.96
Change in store's market share	-0.30	0.16	-0.24	0.20
Change in market HHI	-0.58	0.38	-0.11	0.36
Change in store's labor demand	3.58	1.85	0.31	1.28
Change in store's inventory demand	0.57	4.44	0.07	1.07

NOTE: All numbers are in percentages. In CF_{3a}, the average income increases by 30 percent in all markets. In CF_{4a}, the population increases by 30 percent in all markets. There are no changes in stores' productivity, demand shocks, and capital stock.

Appendix D: Additional reduced-form specifications for the impact of states and market characteristics on product variety, inventory performance, and inputs and investment demand

This section of the appendix describes additional reduced-form specifications of the impact of a store's state space on product variety, store performance, inputs, and investment demand using previous productivity and demand shocks as covariates. These specifications in Tables D.4, D.5 and D.6 follow those in Tables 7 8, and 9 but use productivity and demand shocks in period $t-1$ instead of t to avoid possible endogeneity concerns related to productivity and demand shocks in period t. The specifications in the main text (i.e., Tables 7, 8 and 9) are entirely consistent with the store's dynamic optimization problem, and also include the store fixed effect. Store fixed effect controls for other unobserved factors at the store level that are correlated with productivity and demand shocks, e.g., factors related to location and logistics. The findings in Tables D.4, D.5 and D.6 demonstrates the robustness of those in Tables 7, 8 and 9.

Using previous productivity and demand shocks as covariates in the regression specifications, we observe no changes in the sign of the impact of key state variables (i.e., productivity demand shocks and capital) on the number of product categories, HHI in a store, and entropy (Table D.4). There are no significant changes in the marginal effects of productivity and demand shocks on HHI in the store and entropy in product sales. We observe that stores with high-demand shocks specialize even more. The negative marginal effect of demand shocks on product variety (the number of product categories) is larger in absolute terms if previous demand shocks are used as a covariate. As we expect, the impact of previous productivity on product variety is smaller than that observed when using the current productivity because we omit the current productivity improvements due to learning from demand.

The results in Table D.5 confirm the positive impact of demand shocks on inventory performance and market share. Moreover, the effect of revenue productivity on inventory demand and market share is positive (and larger than that observed when using the current productivity). The findings in Table D.6 confirm the positive marginal effect of productivity on input demands (labor, inventory, and investment in technology) that is higher than the effect of demand shocks.

Table D.4: Additional specifications for the impact of store and market characteristics on product category

Dependent variable	No. product categories (np_{jt})		HHI product categories in a store		Entropy of sales of product categories	
	Est.	Std.	Est.	Std.	Est.	Std.
Previous productivity (ω_{t-1})	0.8731	0.0510	-0.0652	0.0053	-0.1534	0.0104
Previous demand shock (μ_{t-1})	-0.2426	0.0186	0.0193	0.0019	0.0420	0.0038
Log of capital (k_t)	0.2112	0.0322	-0.0179	0.0033	-0.0407	0.0065
Log of inventory (n_t)	0.0245	0.0410	-0.0042	0.0042	-0.0089	0.0083
Log of population (pop_t)	-0.5440	0.0589	0.0426	0.0061	0.0910	0.0120
Log of population density ($popdens_t$)	0.0496	0.0394	-0.0001	0.0041	-0.0015	0.0080
Log of income (inc_t)	-0.4067	0.5616	0.0568	0.0584	0.1535	0.1148
Year fixed effects	Yes		Yes		Yes	
Sub-sector fixed effects	Yes		Yes		Yes	
Adjusted R^2	0.321		0.331		0.340	

NOTE: Productivity and demand shocks are estimated using the two-step estimation method in Section 3. Entropy is computed for each store j based on the market share of each product category i inside store, i.e., $E_{jt} = \sum_i ms_{ijt} ln(ms_{ijt})$. Store regressions control for the average wage. The intercept is included in all specifications.

Table D.5: Additional specifications for the determinants of inventory performance and store's market share

Dependent variable	Inventory per product before sales ($ln(A_{jt}/np_{jt})$)		Inventory per product after sales $ln(N_{jt+1}/np_{jt})$		Store's market share	
	Est.	Std.	Est.	Std.	Est.	Std.
Previous productivity (ω_{t-1})	0.2607	0.0207	0.0648	0.0210	0.3216	0.0184
Previous demand shock (μ_{t-1})	0.0927	0.0076	0.0779	0.0077	0.0649	0.0067
Log of capital (k_t)	0.2200	0.0126	0.1207	0.0128	0.1773	0.0112
Log of inventory (n_t)	0.4090	0.0153	0.6405	0.0155	0.1583	0.0135
Log of population (pop_t)	-1.5593	1.4669	-1.6357	1.4846	-3.7271	1.2995
Log of population density ($popdens_t$)	0.2818	0.1304	0.2809	0.1320	0.1263	0.1155
Log of income (inc_t)	0.2188	1.7069	-1.9396	1.7276	0.1995	1.5122
Year fixed effects	Yes		Yes		Yes	
Sub-sector fixed effects	Yes		Yes		Yes	
Adjusted R^2	0.857		0.856		0.781	

NOTE: Productivity and demand shocks are estimated using the two-step estimation method in Section 3. Store regressions control for the average wage. The intercept is included in all specifications.

Table D.6: Additional specifications for the estimation of the investment, labor, and inventory demand functions

	Log of investment (i_t)		Log of labor (l_t)		Log of products and inventories (a_t)	
Panel A: Linear specifications						
Previous productivity (ω_{t-1})	0.2275	0.0393	0.3044	0.0108	0.4228	0.0103
Previous demand shock (μ_{t-1})	-0.0129	0.0144	0.0233	0.0039	0.0228	0.0037
Log of capital (k_t)	0.5435	0.0265	0.3582	0.0068	0.2642	0.0065
Log of inventory (n_t)	0.4473	0.0338	0.2760	0.0087	0.4372	0.0083
Log of population (pop_t)	0.0370	0.0443	0.0178	0.0125	0.0367	0.0120
Log of pop. density ($popdens_t$)	0.0169	0.0299	0.0374	0.0083	0.0402	0.0080
Log of income (inc_t)	-0.0237	0.4125	0.2038	0.1190	-0.0206	0.1143
Year fixed effects	Yes		Yes		Yes	
Sub-sector fixed effects	Yes		Yes		Yes	
Adjusted R^2	0.093		0.919		0.947	

NOTE: The dependent variables are the log of investment in capital (i_t), the log of the sum between the inventories at the beginning of the year (n_t) and the cost of products bought during the year (a_t), and the log of inventories at the end of the year (n_{t+1}). All regressions include an intercept and control for the average wage. Productivity and demand shocks are estimated using the two-step estimation method in Section 3.