On the Complete Systems approach to Demand Analysis

N. Anders Klevmarken

The Industrial Institute for Economic and Social Research

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Preface

Demand analysis is one important area in applied economics and econometrics. Based on classical utility theory the complete system of demand functions forms an essential part in many econometric macro models and it is also a useful tool for demand analysis in its own right. Since the work by Ragnar Bentzel and collaborators in the 1950's a number of demand studies —in the complete systems approach as well as more specialized studies of particular markets and commodities—have been carried out at IUI. The present volume by Professor Anders Klevmarken summarizes his experience during the last ten years working in this area at IUI or in association with the institute.

Previous drafts of parts of this monograph have been presented at various conferences and chapter 3, although now somewhat revised, has previously been published in the Journal of Econometrics. The editor's consent to have it included in this volume is gratefully acknowledged.

In one way or another several persons contributed to this study. Fredrik Henell has done most of the data work. Per Högberg and Paul Olovsson provided very competent computational assistance. For constructive criticism and useful suggestions thanks also go to William Barnett, Claes Dolk, Gordon Fisher, Michael McAleer, Ed Palmer, Louis Phlips, Bo Sandelin and Henri Theil.

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Stockholm, in August 1980

Gunnar Eliasson

Contents

1 Introduction

The complete systems approach to demand analysis constitutes a joint analysis of the expenditures or consumption volumes of all those commodities which make up total private consumption. On a basis of a system of demand functions, mostly developed from classical demand theory, demand is explained by income and price changes. Total consumption is usually taken as the income variable, which implies that savings are exogenous. With very few exceptions, these models have been applied to aggregate time series and in practice they are used for forecasting and planning on a national level.[1] They are incorporated in most medium-term macro econometric models. One of the most wellknown is the model developed at the Department of Applied Economics, University of Cambridge; cf. Stone (1962). In Sweden, demand systems are used for medium-term forecasting and planning as part of a larger model; see Åberg (1971), Karlsson & Öberg (1976) and IUI (1979). A demand system is also included in the quarterly econometric model of Sweden STEP 1; Ettlin *et al.* (1979).

The development of true cost-of-living indices is another potential area where complete systems of demand functions might be used. A few attempts have been reported by e.g. Heien & Popkin (1971) and Christensen & Manser (1975).

Still another field where the same approach has been used is the analysis of portfolio choice. An example of a model which jointly determines the demand for consumer goods and the portfolio choice is found in Clements (1976).

The main advantage of the complete systems approach as compared to an analysis of each single commodity is the increased efficiency of the estimates. The aggregation, homogeneity and symmetry constraints of classical demand analysis add degrees of freedom to the analysis and in the estimation process, joint treatment of all commodities also makes it possible to take advantage of the correlation between commodities. Another useful property, in particular when the demand model is part of a larger model, concerns the aggregation constraint, which ensures that the sum of all commodity predictions is exactly equal to exogenous total consumption.

However, all demand systems are not consistent with utility theory. For instance in the early applied studies by Schultz (1938), Wold & Jurén

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[1] One exception is the study by Salvas-Bronsard (1978) based on a time series of cross-section household budget data from France.

(1952), Stone (1953) and Bentzel *et al.* (1957), the constant elasticity of demand model, or equivalently the double-log model, was used with great success. This model is still widely applied.

Some of the advantages of the single equation approach are that estimation is relatively less complicated, that specification errors in one equation are not transmitted to the estimates of other equations and that it is relatively easy to incorporate particular characteristics of the market for a commodity.

One of the first models within a utility framework to be applied was the linear expenditure system; see Stone (1954). Other early contributions were the addilog model in Houthakker (1960), the Rotterdam model in Theil (1965), and the work on almost additive preferences in Barten (1964). Excellent surveys of the development in this field of research include an article by Brown & Deaton (1972) and monographs by Phlips (1974), Powell (1974), Deaton (1974*a*) and Theil (1975), (1976). Thus, there is no need to attempt another review. We will instead focus on some of the problems inherent in this approach. As a matter of fact, the application of these models are burdened by both theoretical and practical problems. Some of these are related to aggregation of individuals and goods.

It is well known that some implications of demand theory, such as homogeneity and symmetry, do not in general hold on an aggregate level. The classical theory of demand was rejected in studies by Barten (1969) and Christensen, Jorgenson & Lau (1975).[2] In other studies specific assumptions about the utility function, i.e. additivity, have been tested without relinquishing the tenets of classical demand theory. For example, in Deaton (1974*a*), the hypothesis of an additive utility function was rejected.

In practice, however, data on the desired detailed commodity breakdown are scarce and price variation is usually insufficient to give well determined estimated demand responses to price changes. In order to estimate our models we thus need *a priori* restrictions to reduce the number of unknown parameters, and the only basis we have for choosing such restrictions is demand theory. In spite of the implications of previous results, one of our problems—since data are scarce—is to determine whether classical demand theory with and without its more restrictive assumptions, such as additivity, is good enough to be used in forecasting. This is analysed in Chapter 3.

As compensation for the data's low informational content, long time series are needed to obtain sharp tests. In the studies by Barten (1969) and

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[2] The properties of the conventional tests for homogeneity and Slutsky symmetry are at issue. In Laitinen (1978) and Meisner (1979) it is shown that these tests are biased towards rejection of the null hypothesis because a sample moment matrix of residuals is used rather than the true moment matrix of disturbances. This result, however, appears to hold only when the sample moment matrix is obtained from least-squares residuals, while the test would rather be conservative if it were based on maximum likelihood estimates of the moment matrix, see Klevmarken (1975 b).

Christensen, Jorgenson & Lau (1975) the sample period included 31 and 44 years respectively. Deaton's (1974*a*) study was also based on long time series covering the period 1900–1970, which, with the war years excluded, amounted to 48 years. This is longer than can usually be expected but, what is more important, the validity of these long time series may be questioned. Commodities covered by today's aggregates are vastly different from those included at the beginning of the century. There are not only changes in quality and substitution of new goods for old ones, but also services which used to be private consumption, that are now provided for by the public in many countries.

Everyone experienced in compiling expenditure and price series is aware of the great difficulties involved in obtaining not only consistent, but also reliable series (see Chapter 2). The consequent "ad hoccery" which pervades this kind of statistical work is annoying. In contending with these difficulties, it is perhaps not so surprising that some of the classical results of demand theory fail to gain statistical support whereas, on the other hand, the empirical results might be more favourable if higher quality data were available.

In most applications of complete systems of demand functions to aggregate data it is implicitly assumed that the demand functions are identified. This is not an altogether good assumption because in some countries and markets trade is controlled, supply is limited and an excess demand is created. This is particularly true with respect to the housing market. Powel (1974), for instance, gives some results for the linear expenditure system applied to Australian data and notes that "the exceptionally high estimated income elasticity for Housing is probably associated with the post-war elimination of demand back-logs and gradual dismantling of wartime controls" (p. 47). In Sweden rent controls have been in effect throughout the post-war period and have led to the build-up of large queues for low-priced flats, in particular during the 1960s. If a pure demand model is fitted to consumption data generated from such a market, the estimated elasticities will become mixtures of demand and supply elasticities and estimates for commodities other than housing will in general also be biased. In Chapter 4, a modest attempt is made to modify the linear expenditure system and incorporate these features of the housing market into the model.

There have been a few attempts to develop the originally static systems of demand functions into dynamic models. One approach is to delay the consumers' reaction to an income or price change—usually called habit formation. This is conveniently obtained in the linear expenditure system by making the originally constant subsistence level parameters functions of past consumption. This model has been analysed and applied in Pollak & Wales (1969), Pollak (1970), Dahlman & Klevmarken (1971) and Deaton & Wigley (1971). It is also included in the comparison in Chapter 3 of this study.

Another approach taken by e.g. Houthakker & Taylor (1966), Taylor (1968), Mattei (1971) and Phlips (1972 and 1974) is to incorporate so called state variables into the utility function. These variables are interpreted as either stocks of durable goods or measures of the persistency of habits. In this way habit formation and stock effects are introduced simultaneously. The dominant effect is then determined empirically. Houthakker, Taylor and Mattei developed their models from a utility function of the second degree in consumption volumes and state variables while Phlips used the Klein–Rubin–Geary utility function underlying the linear expenditure system. He made the subsistance level parameters a function of the state variables. A slightly different model was suggested in Dahlman & Klevmarken (1971). It is also a modification of the linear expenditure system but the consumer is assumed to maximize the utility yielded by the *services* obtained from the goods he purchases under a budget constraint on his expenditures. The stocks of durables are not explicitly introduced in the utility function. In this model the consumer's behaviour is also influenced by habits. The model was not tested empirically at the time it was suggested. This omission is now remedied in Chapter 5 which also includes a comparison with the Houthakker & Taylor and Phlips models.

Purchase decisions in all these models are allowed to be influenced only by past behaviour. There are no intertemporal tradeoffs. These are models of "myopic" utility maximization. However, in a series of papers in the beginning of the 1970s Lluch and his colleagues developed and applied the extended linear expenditure system.[3] In this model the consumer maximizes an intertemporal utility function of the Klein–Rubin–Geary type, subject to a wealth constraint. The result is a linear expenditure system extended by a macro consumption function, i.e. savings are made endogenous. But this model does not include the effects of accumulated stocks. It has been applied to aggregate time series and cross-section data, although the rewards from these studies do not quite match the theoretical sophistication.[4] However, longitudinal data, including data on wealth and stocks and the explicit introduction of stocks in the model might make this approach very fruitful. Given the limited scope of our data, such an approach has not been attempted in this study.

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[3] A key reference is Lluch (1973). Other references are given in Powell (1974) Chapter 6, where this model and an earlier model by Tintner are discussed extensively.

[4] Using a more pragmatic approach, Eliasson (1978) added a savings function to a demand system of the LES type in his simulation model MOSES.

2 Data

Time-series data on aggregate household consumption are generally not collected according to methods which are controlled in a statistical sense. On the contrary, numerous data sources of varying quality and more or less *ad hoc* methods are used to construct series of commodity expenditures and price indices. Since there is no general method of measurement, it is difficult to assess the quality of these time series. The best solution possible is to compare the resulting eastimates with estimates from other sources and indicate possible errors.

We might distinguish between two main approaches for estimating household expenditures at an aggregate commodity level. One is based on supply statistics, i.e. domestic production minus exports is added to imports to obtain total supply to the domestic market. The wholesale and retail trade markups are added in order to evaluate this supply at consumer prices. Finally, these figures are adjusted for stock changes and deliveries to sectors other than the household sector, which gives an estimate of household consumption. There are, of course, numerous difficulties and errors inherent in these computations. Statistical classification codes do not always coincide with the desired commodity definition. Reliable markup data are difficult to obtain, particularly on a time-series basis. They are not always adjusted for lower prices during sales, so that there is a tendency to overestimate the average markup. Markups may also differ considerably in different segments of the market and between domestically produced and imported goods. A major difficulty is encountered in estimating the amount of goods delivered to sectors other than the household sector. For instance, we experienced difficulties in estimating how much household equipment is delivered to the construction sector,[1] the public sector, restaurants, etc. As the "residual" is attributed to the household sector, there might be a tendency to overestimate the deliveries to this sector. It should also be mentioned that Swedish data are not adjusted for stock changes because there are no reliable statistics on stocks.

In the second approach, one or a few bench mark estimates are varied according to the rate of change in some time series to yield good estimates of both levels and changes. These bench marks are obtained from household budget studies, public or private market surveys or, for more recent

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[1] In Sweden flats and houses are usually equipped with stoves, refrigerators, washing-machines etc. by the builder. This equipment is left in the flat or house when a household moves.

Table 2.1 *Successively revised estimates in the Swedish national accounts of household expenditures on selected commodities (mill. Skr; current prices)*

Commodity	Source	1950	1955	1960	1965	1969
Housing	SM N 1971: 11	1,697	2,405	3,676	5,654	8,489
	SM N 1972: 93	3,798	5,377	8,221	10,755	15,139
Footwear	SM N 1971: 11	409	531	677	875	949
	SM N 1972: 93	409	531	677	903	1,081
Operations of motor vehicles and caravans	SM N 1971: 11	446	880	1,736	3,373	4,961
	SM N 1972: 93	395	729	1,391	2,729	4,060
Cab fares	SM N 1971: 11	201	230	352	580	884
	SM N 1972: 93	201	230	248	314	350

Note: SM is a shortform for the series of publications from the Swedish Central Bureau of Statistics called "Statistiska Meddelanden" (Statistical Tables).

years from input-output tables. Statistics used to interpolate between these bench marks include sales statistics, supply statistics and for some commodities the product of the number of units purchased and a price index. Of course, one single bench mark should not be extrapolated for a long period. The result might become a serious under- or overestimate of the long-run trend. A drastic example is revealed by earlier estimates of housing consumption in the Swedish national accounts. Up until 1971, they were based on a bench mark from 1946 which was varied by the production of new flats less demolition and adjusted for changes in the rent level. The old estimate in 1969 was Skr 8,489 millions. At that time a new household budget survey was carried out which showed much higher estimates. The same results were obtained independently when the weights of the consumer price index were revised. The estimates in the national accounts were adjusted accordingly and the new estimate for 1969 was Skr 15,139 millions, an increase of almost 80%! The two series for selected years are exhibited in Table 2.1. The differences are mainly due to underestimates in the old series of maintenance, repairs and the rental value of owner occupied houses. Imputed rents of secondary dwellings were not included in the old estimate, but have been incorporated into the new series. Table 2.1 also lists other examples of revised estimates. The decrease in the estimates of operations costs of vehicles was based on new lower estimates from the 1969 budget study. The estimates from the 1958 and 1969 budget studies were interpolated to produce the new series. Although these revisions are exceptional, they do indicate that errors in this kind of data can be much larger than the annual rate of change in the series.

Estimates from household budget surveys cannot be used as bench marks

Table 2.2 *Comparison of time series and budget estimates of consumer expenditures on selected commodities in 1969 (mill. Skr)*

Commodity	IUI estimates	Budget study estimates	Relative difference (%)
Coffee, tea and cocoa	856	755	−11.8
Soft drinks, lemonade and juice	907	614	−32.3
Beer, wine and liquor	3,962	1,718	−56.6
Tobacco	2,506	1,663	−33.6
Household equipment	732	777	+ 6.1
Purchases of vehicles	3,202	3,085	− 3.7
Maintenance and repairs of vehicles	4,534	4,086	− 9.9
Public transportation	1,907	1,261	−33.9

Source: Unpublished statistics from The Industrial Institute for Economic and Social Research (IUI), Stockholm, and *The Family Expenditure Survey 1969,* SM P1971: 9, the Swedish Central Bureau of Statistics.

for all commodities owing to the serious underestimates which are typical in regard to certain commodities. This is illustrated in Table 2.2 where the estimates used in this study—the IUI column—and those from the 1969 budget study are compared. Because of state monopolies on tobacco, wine and liquor, the sales statistics for these commodities, which form the basis of the IUI estimates are very good. The estimates from the budget study are much lower, however, which indicates a large negative bias. The same is also true for "soft drinks, lemonade and juice" and "public transportation". It is easy to forget to record these commodities in a household budget study because they are usually purchased separately and by more than one member of the household. Purchases of durables are major events in a household. They are usually well documented (warranties, instalment contracts, etc.) and thus more difficult to forget. These budget survey estimates are also in relatively close agreement with the IUI estimates based on supply and sales data.

Classical demand theory is based on the assumption of homogeneous commodities which do not change in quality. But in practice, we have to deal with quality changes, particularly when we work with aggregate commodities. It is reasonable to assume that changes in quality influence consumer preferences and stimulate increases in consumption. This effect should be analysed within the theoretical framework of demand analysis. There have been a few approaches in this direction, (see e.g. Lancaster, 1966 and 1971; King, 1975; and Theil, 1976) but no major advances in applied demand analysis. The main reason is probably a lack of time-series data on quality. This study contains only fragmentary information on quality changes and their effects on demand cannot be assessed explicitly.

Price increases may be decomposed into "pure" increases and increases

Table 2.3 *Comparison of price, volume and quality changes for selected durables (Index 1964=100)*

Year	Price index	Volume per unit index	Number of units index	Volume index
TV-sets				
1961	107	78	163	127
1964	100	100	100	100
1965	106	82	112	91
1970	91	126	183	230
1972	89	196	208	408
Vacuum cleaners				
1961	90	95	78	75
1964	100	100	100	100
1965	105	66	129	86
1970	110	98	155	152
1972	125	91	157	140
Washing machines				
1961	96	96	71	67
1964	100	100	100	100
1965	105	93	137	128
1970	120	72	145	104
1972	135	65	137	89

induced by improvements in quality. In accordance with classical demand theory the general practice in price index calculations is to attempt to eliminate the quality component. In theory, the size of this component depends on consumer preferences and would have to be estimated jointly with the other parameters of the model, but this procedure is not practical. When price data originate from the data base collected for the consumer price index or the like, it is usually possible to make adjustments similar to those made for this index, i.e. based on production costs. However, direct price observations were not available for some of the commodities in this study, so that implicit price indices have to be used. They include both components.

When quality adjusted price indices are used for deflation, a change in the consumption volume of a commodity will thus include a change in both the number of units and quality. It is the sum of these two components that we try to explain and forecast by income and price changes. In order to check the internal consistency of the data in general and the price indices in particular the ratio of the volume and the number of units consumed was computed for selected years and well defined durables. This ratio can be used as a measure of quality. The results are shown in Table 2.3.

With respect to TV sets, the increasing trend in the quality series is as could be expected. The jump in quality in the beginning of the 1970s can be explained by the introduction of colour TV. The estimates for vacuum cleaners do not indicate any increase in quality at all; a result which hardly agrees with what is commonly believed. The rate of price change for vacuum cleaners is probably slightly overestimated. The drop in quality in 1965 cannot be a true representation of reality. The price index for this year is not outside the trend, but the volume is unusually low, which might suggest an error in the estimate of expenditures for vacuum cleaners in 1965.[2] Finally, the quality series for washing machines is absurd indicating errors in one or all of the price, expenditure and number of units series.

These few examples illustrate not only the fragile quality of the data, but also how errors in the price index are transmitted to the volume estimates. In general, unless explicitly taken into account, these measurement errors will result in biased estimates of income and price elasticities. Cases with a positive as well as a negative bias could exist. The sign and magnitude of the bias would depend on the nature of the measurement errors, the properties of the model and the method of estimation. A few examples are offered in Theil (1979). For instance, when the rate of change in prices is overstated because technological improvements are ignored, and a Rotterdam model is used, it is shown that a downward bias is obtained in the expenditure elasticities for necessities and an upward bias for luxuries. This result was arrived at under the assumption of a common exponential error trend for each commodity. If the same strong assumption is used in conjunction with the following constant elasticity of demand model,

$$\ln(w_{it}) = \mu + (e_i - 1)\ln(y/p_t) + (E_{ii}+1)\ln(p_{it}/p_t) + \varepsilon_{it};$$

where e_i and E_{ii} are the expenditure and own price elasticities, respectively, for commodity i, it turns out that the estimated expenditure elasticity for necessities has an *upward* bias.[3] This example might indicate that the large

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[2] This kind of argument cannot be used *in absurdum* because all series would then become very smooth.

[3] Assume the following model,

$$S_{it} = \mu_i + \beta_i Z_t + \gamma_i X_{it} + \varepsilon_{it}; E(\varepsilon_{it}) = 0.$$

where

$$\begin{aligned} S_{it} &= \ln w_{it} \\ Z_t &= \ln(Y_t/p_t) \\ X_{it} &= \ln(p_{it}/p_t) \\ \beta_i &= e_i - 1 \\ \gamma_i &= E_{ii} + 1 \end{aligned}$$

p_{it} is not observed but only $p^*_{it} = p_{it}e^{\alpha t}$, for some unknown $\alpha > 0$. Thus the general deflator is also observed with the same error, $p^*_t = p_t e^{\alpha t}$. This implies that there is no measurement error differences between the estimated elasticities for the various demand models described in Chapter 3 of this study, as well as in previous studies, are at least partly explained by measurement errors. More comprehensive results are certainly desirable but they would require an extensive analysis based on more detailed knowledge of the nature of measurement errors in this context.

Before we leave the subject of price indices two additional problems should be mentioned briefly. The first concerns weights. In principle the weight system used in a price index depends on the parameters of the demand model and should be estimated jointly with these parameters, but again, this is not a practical procedure. In practice the choice of weights is usually of lesser importance as long as a chain index is used. Chain indices of the Laspeyres and Edgeworth types are used in this study.

The second problem is related to goods and services which are price subsidized. The most important commodities in this category are housing, medicine and the services of doctors and dentist. It has not been possible to

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in relative prices, but only in real income, i.e.

$$Z_t^* = Z_t - \alpha t.$$

Using observed variables the least-squares estimate of β_i *is*,

$$\hat{\beta}_i = \frac{\sum s_{it} z_t^* \sum x_{it}^2 - \sum s_{it} x_{it} \sum z_t^* x_{it}}{\sum z_t^{*2} \sum x_{it}^2 - \left(\sum z_t^* x_{it}\right)^2};$$

where lower case letters are deviations from the respective mean. If it is assumed that there is no trend in the logarithmic relative price, $\sum x_{it} t = 0$, and that the scale for t is chosen such that $\sum t = 0$, then

$$\hat{\beta}_i = \frac{\sum s_{it} z_t \sum x_{it}^2 - \sum s_{it} x_{it} \sum z_t x_{it} - \alpha \sum s_{it} t \sum x_{it}^2}{\sum z_t^{*2} \sum x_{it}^2 - \left(\sum z_t x_{it}\right)^2}.$$

Furthermore, if the rate of increase in the measurement error is less than twice the rate of increase in real income, i.e.

$$\alpha < 2\frac{\sum z_t t}{\sum t^2};$$

then $\sum z_t^{*2} < \sum z_t^2$. Since the expenditure shares for necessities would tend to decrease, i.e. $\sum s_{it} t < 0$, we would thus overstate the numerator and understate the denominator. The result is that β_i and thus the expenditure elasticity are overestimated for necessities. For luxuries $\sum s_{it} t > 0$ and there is thus no clear conclusion.

design a price index which corresponds exactly to the rules for granting these subsidies.[4] The gross price is assumed to be reduced by approximately the same factor for each commodity within an aggregate of subsidized commodities. This yields an index which is the product of an ordinary index and the ratio between the total amount of all subsidies in the current period and that in the base period.

Not all of the data used in this study are taken from the Swedish national accounts. There are several reasons for this. First, the national accounts do not contain consistent series for the whole post-war period. Second, a data base with a logical grouping of commodities was developed in previous studies at IUI by Bentzel (1957), Albinsson & Endredi (1966) and Dahlman & Klevmarken (1971). It also provides a good starting point for this study. Third, the commodity grouping and definitions of the SNA system applied in the national accounts are not always the best possible bases for a demand analysis. Our data are thus revised from Dahlman & Klevmarken (1971) and further aggregated for this study. In addition to the commodity grouping, a major difference from the national accounts is that expenditures are estimated net of subsidies and for cars and other vehicles gross of taxes. There are also differences in methods of measurement for some commodities.

In spite of the fact that considerable effort has been made to improve the quality of data, it is obvious that the lack of continuous direct measurements of consumer expenditures and the "ad hoccery" used to bridge this gap still leave ample room for errors. The examples of revisions and inconsistencies discussed in this chapter should no doubt give those who claim that measurement errors are of much less importance than specification errors in econometrics something to think about. Admittedly, recent work on annual input-output tables has increased the possibilities of obtaining estimates of household expenditures and consumer price indices which are consistent with other national accounts data and thus hopefully of higher quality. But a system is desirable which would permit quality evaluation of continuous and direct measurements of expenditures and prices. Further work on the consequences of measurement errors in this context would be equally desirable.

The time series used in this study are reproduced in the Appendix.[5]

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[4] The problem of designing a true cost-of-living index which includes subsidized commodities was treated in Klevmarken (1975 a).

[5] A more detailed commodity breakdown and an account of (in Swedish) the definitions and methods used are obtainable on request.

3 A Comparative Study of Complete Systems of Demand Functions

A number of comparative studies of demand systems, have been carried out, most of which are based on fit criteria, e.g. Parks (1969), Yoshihara (1969), Goldberger & Gamaletsos (1970 and 1973), Dahlman & Klevmarken (1971), Deaton (1974*a*), Theil (1975) and Lybeck (1976 and 1977). It is difficult to make an adequate summary of the results of these studies. Differences in model specification, estimation methods, definitions of commodities and data compilation may to some extent explain the sometimes conflicting results. However, in most studies, models which do not imply an additive utility function show a closer fit to the data than those in which such a function is implied. Thus, in a number of studies, variants of the Rotterdam system are found to be superior to, for instance, linear expenditure systems and direct and indirect addilog models; cf. Parks (1969), Deaton (1974*a*) and Theil (1975). The reported differences in fit between additive models such as the linear expenditure system, addilog models and the Rotterdam system with additivity enforced are smaller and inconsistent. It may also be noted that the constant elasticity of demand system or the double logarithmic model usually obtains a relatively high ranking by fit.

There are only a few studies of relative predictive performance. One example is the work by Theil (1975). His results on British data with four commodities show that his Rotterdam system with block independence performs better than the indirect addilog model and the linear expenditure system. The information inaccuracy measure for the linear expenditure system is even higher than for one of Theil's naive models.

A general finding is that the estimated elasticities are very sensitive to the model specification. Large differences are found between different models estimated on the same data. Results of a simulation study by Kiefer & MacKinnon (1976) show that estimates are likely to be biased and unreliable if the model is misspecified. In particular, they suggest that demand systems such as the linear expenditure system and the translog model cannot be expected to perform well when the data was not generated by these models.

In our study ten models—including two naive models—are applied to Swedish data and compared, using fit, predictive performance and sign and magnitude of estimated elasticities as criteria. In principle, predictions and estimated elasticities depend in more than a trivial sense on the commodity aggregation. In order to explore the aggregation effects each model is estimated twice on the same data, once for a four commodity breakdown and once for an eight commodity breakdown.

The major part of this study is carried out on data which cover food commodities. There are several reasons for partially limiting the study to food. First, the data on food are of a much higher quality than data for other commodity groups. Second food is nondurable and third, the food market functions relatively freely without the institutional restrictions that limit trade in, for instance, the housing market. This approach, whereby the major part of the study is confined to the demand for food rests on an implicit assumption of a weakly separable utility function. However, to obtain some reassurance that our results do not depend entirely on this assumption and to facilitate comparisons with other studies our ten models have also been estimated on data which include all goods.

3.1 Alternative Models

Two naive models serve as a reference base for evaluating the explanatory ability and predictive performance of the other eight models. Of course, neither of them can ever be a substitute for a demand model in simulations of the effects of economic policy. The particular choice of a naive model is somewhat arbitrary. Loosely speaking, expenditure shares are more stable than expenditures, volumes and rates of change in volume. A naive model may thus be more successful in explaining and forecasting expenditure shares than any of the other variables. The first model represents a trend in expenditure shares and the second a simple autoregressive structure, also in expenditure shares. Note that these models use twice as many parameters as there are commodities, i.e. one more than for instance the linear expenditure system. This means that they are only naive in an economic sense. As a special case, however, they include the no change extrapolation of budget shares, which implies homothetic preferences. With the exception of the last model, the remaining seven models have been chosen because they are among those most commonly applied. Although the constant elasticity of demand model does not satisfy the properties of classical demand theory, ever since Schultz (1938) and Wold (1952) it has become the best known and most widely used of all the models. One of its primary merits is the ease of estimation. To emphasize simplicity, one of the two versions estimated has no cross-price elasticities. The second version includes all price effects. The possibility of gaining degrees of freedom by local enforcement of the constraints of classical demand theory has not been used; cf. Byron (1970).

A more recent development with approximately the same advantages and disadvantages is the Rotterdam system; see Theil (1965). Both of these models may be regarded as approximations of an underlying classical demand model. Among the models which do satisfy the constraints of classical demand theory Stone's (1954) linear expenditure system is the most widely used; cf. Brown & Deaton (1972). It is easy to interpret and, although it involves non-linear estimation, it is not too difficult to estimate. It has also proved to be a good starting point for useful generalizations, one of which is included in this study, i.e. the linear expenditure system with habit formation; cf. Pollak & Wales (1969), Pollak (1970) and Dahlman & Klevmarken (1971).

Possible rivals to the linear expenditure system are the addilog models; see Houthakker (1960). But there is no conclusive evidence that the addilog models are superior to the linear expenditure system. In addition, they are more difficult to estimate and they have therefore not been used in this study.

A possible disadvantage of both the linear expenditures system and the addilog models is that they are derived from an additive utility function. This implies that the marginal utility of one commodity is independent of every other commodity, that there are no specific substitution effects and that the own price elasticity is approximately proportional to the income elasticity, as shown in Deaton (1974 *b*). In recent years more general models have been developed, one of which is the translog model; see Christensen, Jorgenson & Lau (1975) and Christensen & Manser (1977). The version used here is derived from an indirect utility function, the logarithm of which is of the second degree in the logarithm of the price-income ratios. Increased generality, however, is acquired at the expense of more difficult estimation which limits their use in applied work.[1]

In comparative studies by Brown & Deaton (1972), Deaton (1974 *a*) and Theil (1975), the authors argue that the left-hand variables of the competing models should be comparable. They should, for instance, all be expenditure shares or log changes in expenditure shares. If they do not meet either of these requirements, the measures of fit may by definition favour one model ahead of another. In our study, this recommendation is not followed in its entirety. Each model is preferred in its most commonly applied form, which implies that the stochastic structure varies from one model to another. However, some of the models which are not usually formulated as expenditure share equations have also been estimated in this form.[2]

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[1] This is not the place to analyse in detail the theoretical properties of the models used. The reader is referred to the references given and to the review article by Brown & Deaton (1972).

[2] This transformation is also motivated by heteroscedasticity in consumption expenditures and volumes. In Theil (1975), Chapter 5, it is also argued that autocorrelation could be removed by taking first differences but the simulations in Kiefer & MacKinnon (1976) show that this practice should be avoided.

The following notations are used in all of the models:

q_{it} demanded volume per head of commodity i in year t,
p_{it} price of commodity i in year t,
w_{it} expenditure share of commodity i in year t,
w^*_{it} $\frac{1}{2}(w_{it}+w_{it-1})$,
y_t total (food) consumption per head in current prices in year t,
p_t general price index (for food) in year t,
ε_{it} stochastic disturbance for commodity i in year t,
e_i income elasticity for commodity i,
E_{ij} (uncompensated) price elasticity,
n number of commodities.

α, β, c, π, $\varkappa$, σ are parameters. They do not necessarily have the same interpretation in all of the models. In the constant elasticity of demand systems, eqs. (3:3) and (3:4) below e_i and E_{ij} are also parameters. D is the logarithmic difference operator.

Results are reported for the following ten models:

Trend (Trend-w)

$$w_{it} = \alpha_i+\beta_i t+\varepsilon_{it};\quad i=1,\dots,n \tag{3:1}$$

Autoregressive model (Auto-w)

$$w_{it} = \alpha_i+\beta_i w_{it-1}+\varepsilon_{it};\quad i=1,\dots,n \tag{3:2}$$

Constant elasticity of demand system (CEDS-$\ln q$)

$$\ln(q_{it}) = \alpha_i+e_i\ln(y_t/p_t)+E_{ii}\ln(p_{it}/p_t)+\varepsilon_{it};\quad i=1,\dots,n \tag{3:3}$$

Note that all cross-price elasticities are assumed to equal zero.

Constant elasticity of demand system (CEDS-$\ln w$)

$$\ln(w_{it}) = \alpha_i+(e_i-1)\ln(y_t/p_t)+(E_{ii}+1)\ln(p_{it}/p_t)+\sum_{j\neq i}E_{ij}\ln(p_{jt}/p_t)+\varepsilon_{it};\quad i=1,\dots,n \tag{3:4}$$

Symmetry of price responses is not enforced. There are thus $n(n+2)$ parameters to estimate.

Linear expenditure system with habit formation (LESH-pq)[3]

$$p_{it}q_{it} = \alpha_i p_{it}q_{it-1}+\beta_i\left(y_t-\sum_k \alpha_k p_{kt}q_{kt-1}\right)+\varepsilon_{it};\quad i=1,\dots,n \tag{3:5a}$$

$$\sum_{j\neq i}\beta_i = 1; \tag{3:5 b}$$

Linear expenditure system with habit formation (LESH-w)[3]

$$w_{it} = \alpha_i p_{it} q_{it-1}/y_t + \beta_i \left(1 - \sum_k \alpha_k p_{kt} q_{kt-1}/y_t \right) + \varepsilon_{it}; \quad i = 1, \ldots, n \tag{3:6a}$$

$$\sum_i \beta_i = 1; \tag{3:6b}$$

Linear expenditure system (LES-w)[3]

$$w_{it} = c_i p_{it}/y_t + \beta_i \left(1 - \sum_k c_k p_{kt}/y_t \right) + \varepsilon_{it}; \quad i = 1, \ldots, n \tag{3:7a}$$

$$\sum_i \beta_i = 1; \tag{3:7b}$$

Rotterdam system (RD-w^*Dq)

$$w_{it}^* Dq_{it} = \mu_i Dq_t + \sum_j \pi_{ij} Dp_{jt} + \varepsilon_{it}; \quad i = 1, \ldots, n \tag{3:8a}$$

$$\sum_j \pi_{ij} = 0 \text{ for all } i; \tag{3:8b}$$

$$\pi_{ij} = \pi_{ji} \text{ for all } i \text{ and } j; \tag{3:8c}$$

$$\sum_i \mu_i = 1; \tag{3:8d}$$

Rotterdam system with intercept (RDI-w^*Dq)

$$w_{it}^* Dq_{it} = \varkappa_i + \mu_i Dq_t + \sum_j \pi_{ij} Dp_{jt} + \varepsilon_{it}; \quad i = 1, \ldots, n \tag{3:9a}$$

$$\sum_j \pi_{ij} = 0 \text{ for all } i; \tag{3:9b}$$

$$\pi_{ij} = \pi_{ji} \text{ for all } i \text{ and } j; \tag{3:9c}$$

$$\sum_i \mu_i = 1; \tag{3:9d}$$

Indirect translog model (ITRL-w)

$$w_{it} = \frac{\alpha_i + \sum_j \beta_{ij} \ln (p_{jt}/y_t)}{-1 + \sum_j \beta_{Mj} \ln (p_{jt}/y_t)} + \varepsilon_{it}; \quad i = 1, \ldots, n \tag{3:10a}$$

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[3] To conform with utility theory the following constraints should hold, $q_{it} - \alpha_i q_{it-1} > 0$ and $q_{it} - c_i > 0$ $\forall$ i,t for LESH and LES respectively. They were not, however, used in the estimation.

$$\beta_{Mj} = \sum_i \beta_{ij} \text{ for all } j; \tag{3:10b}$$

$$\beta_{ij} = \beta_{ji} \text{ for all } i \text{ and } j. \tag{3:10c}$$

In a first round of estimations, it was assumed that the error terms were contemporaneously correlated but not autocorrelated,

$$E(\varepsilon_{it}) = 0; \tag{3:11a}$$

$$E(\varepsilon_{is}\,\varepsilon_{jt}) = \begin{cases} \sigma_{ij} & \text{if } s=t \\ 0 & \text{if } s \neq t \end{cases} \tag{3:11b}$$

After examining the residual autocorrelation, a few models were reestimated under the assumption of autocorrelated error terms. With respect to the constant elasticity of demand system (CEDS-$\ln q$), one autocorrelation coefficient was allowed for each commodity,

$$\varepsilon_{it} = \varrho_i \varepsilon_{it-1} + u_{it}; \tag{3:12a}$$

but for those models which satisfy the budget constraint all commodities must have the same coefficient of autocorrelation, cf. Berndt & Savin (1975).

$$\varepsilon_{it} = \varrho\varepsilon_{it} - 1 + u_{it}; \tag{3:12b}$$

$$E(u_{it}) = 0; \tag{3:12c}$$

$$E(u_{is}\,u_{jt}) = \begin{cases} \lambda_{ij} & \text{if } s=t \\ 0 & \text{if } s \neq t \end{cases} \tag{3:12d}$$

Owing to the model specification and enforcement of the budget constraint, the contemporaneous error moment matrix in models (3:5)–(3:10) is singular. In each of these models, one equation is redundant and could be left out. In accordance with the estimation methods used, any equation could be dropped.

It was emphasized above that models (3:1)–(3:3) were included in this study because they are simple. This holds *a fortiore* in regard to estimation. Thus each equation was estimated by OLS, although other methods might have yielded some gain in efficiency in models (3:2) and (3:3). The Cochrane-Orcutt method was used to estimate model (3:3) with autoregressive errors. Model (3:4), the constant elasticity of demand system with all price effects included, was efficiently estimated by OLS.

The three versions of the linear expenditure system and the translog model were estimated by the quasi-maximum likelihood method,[4] and the

[4] The likelihood function for independently multivariate normally distributed error terms was maximized by using the Harwell Library Subroutines VA09AD and VA10AD—a quasi-Newton procedure.

two Rotterdam models by Zellner's iterative Aitken estimator (IZEF). The latter two estimation methods differ only computationally because, if convergent, IZEF will produce ML estimates; see Bradley (1973), Oberhof & Kmenta (1974), Charnes, Frome & Yu (1976) and Christensen & Manser (1977). No difficulty was experienced in obtaining convergence except in the case of the translog model with eight commodities. This model has therefore been estimated with two alternative assumptions about the error moment matrix. First, the model was estimated as specified above with coarse tolerance limits. In a second estimation round with fine tolerance limits, the moment matrix was fixed at the values obtained at convergence in the first round. In this way, convergence with fine tolerance limits could also be obtained. The results are reported below under the heading ITRL-w, Ω. The second version of the translog model was obtained by fixing the error moment matrix at the outset in the following way:

$$\sigma_{ij} = -\sigma^2 \bar{w}_i \bar{w}_j \tag{3:13a}$$

$$\sigma_{ii} = \sigma^2(1-\bar{w}_i)\,\bar{w}_i \tag{3:13b}$$

where $\bar{w}_i$ is the average expenditure share for commodity i and σ^2 is a constant. The heading used for results with this specification is ITRL-w, Ω_0.

3.2 Data

The sample period is 1950–1970. Two years, 1971 and 1972, are reserved for a comparison with *ex post* forecasts. Food data were originally obtained for 16 commodities, but they were grouped into 8 commodities for this study. Commodities which exhibited similar fluctuations in relative prices were grouped together. In order to study the effects of aggregation on estimated elasticities and predictions the eight commodities were further aggregated into four commodities. The four-commodity breakdown is related to the eight-commodity breakdown as shown below.

8 *Commodities*	4 *Commodities*
1. Flour, Bread, Potatoes and derivative products	A. Basic Supplies
2. Butter, Eggs, Suger, Spices	
3. Milk, Cream	
4. Vegetables	B. Vegetables and Fruit
5. Fruit, Berries, Ice cream, Chocolates and Sweets	
6. Meat and Pork, Cheese	C. Meat and Fish
7. Fish	

8. Restaurant Meals D. Restaurant Meals

Note that the last commodity, "restaurant meals", is the same at both aggregation levels.

As was mentioned in the Introduction, a second set of data has also been analysed. It includes all goods and services. The source is the same as reported in Chapter 2. These data were also aggregated into four commodities:

I. Food, Beverages and Tobacco
II. Housing Services
III. Clothing
IV. Other Goods and Services

The last commodity IV includes e.g. household equipment, purchases of automobiles, sports equipment, public transportation, telephone services, tourism, entertainment and other services.

In principle each model determines the proper method of aggregation. Price indices typically involve the unknown parameters. In this particular study, parameter estimates obtained from food data at the 8-commodity level could be used to estimate price indices for the 4-commodity level. In practice, however, this is usually not a feasible procedure, and we have opted in favour of the common procedure of aggregating expenditures by simple summation and using price indices of the Edgeworth and Laspeyres type. This makes our analysis of aggregation effects more realistic.

3.3 Results

The procedure for testing and discriminating between models should in principle be closely related to the purpose for which the model finally selected will be used. An aggregate model with only a few commodities would probably be sufficient for forecasting the business cycle, provided that the model would pick up the swings in the demand for durables which presumably are relatively important in the short run. As for medium-term forecasting and analysis of long-run economic policy it might be desirable to place more emphasis on the effects of relative price changes. For the purpose of computing cost-of-living indices, the effects of relative price changes would have to be modelled without distortions, which would e.g. rule out models based on additive utility functions. In this comparative study we would like to emphasize the general purpose of forecasting but not designate one particular application. Traditional measures of fit and testing procedures would thus be sufficient. But there is no obvious standard of comparison for our ten models. They are not nested, they do not have the same dependent variable and their stochastic properties differ. Since they are not nested, the procedure of testing one model against the others would not be straightforward. Although recent methods for testing nonnested hypotheses might have been attempted, our comparison of sample period evidence is based on simple descriptive goodness of fit statistics. This comparison could be made for expenditure shares, expenditure levels, volumes or relative changes in volume. The results reported below are based exclusively on comparisons of expenditure shares. The statistics used are information inaccuracies and coefficients of determination. In addition to sample period fit, the predictive performance for 1971 and 1972 is evaluated using the same statistics. As a third criterion for discrimination between the ten models the estimated elasticities are evaluated in terms of prior conceptions about their sign and size. Parallel results are presented for both data sets and both levels of aggregation. We conclude this chapter by discussing some results of the aggregation effects.

Before turning to these results, however, we may report that our results for the translog model permit tests of the symmetry restrictions (3:10c). As pointed out by Kiefer (1976), these tests can be interpreted as a test of either utility maximization or the functional form of the underlying indirect utility function. In his study Kiefer was not able to reject the hypothesis of symmetry, while in Christensen, Jorgenson & Lau (1975) the composite hypothesis that the β_{Mj} parameters in (3:10b) take the same values in each equation and that symmetry holds was rejected. Our results are also mixed. Using the four-commodity food data symmetry is not rejected by a likelihood ratio test ($\chi^2_{\mathrm{df}=6} = 7.8$ while the 5 percent critical value is 12.6), but it is rejected when the data including all goods are used ($\chi^2_{\mathrm{df}=6} = 27.6$).[5,6] This result is perhaps not so suprising given the mixture of durables, nondurables and services in the last data set. As already noted the classical theory of demand cannot be expected to give a good representation of the demand for durables and the consumtion of housing services in a partly controlled market.

3.3.1 Goodness of Fit

The first column in Tables 3.1, 3.2 and 3.3 lists average information inaccuracies for each model for the whole sample period. The second column shows the same statistic, but corrected for degrees of freedom; see Theil (1971) p. 649. The third column shows standard deviations for yearly information inaccuracies.

For the case of four food commodities the linear expenditure system with habit formation yields the closest fit to the data, while the ordinary linear expenditure system shows the worst fit. Its fit is even poorer than that of

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[5] The sample period is too short to give the same test meaning on the eight commodity aggregation level.

[6] Cf. footnote 2 to chapter 1 on page 10.

Table 3.1 *Average information inaccuracies; food items grouped into 4 commodities*

Model	$\bar{I}_{50-70}$	$\bar{I}^{A}_{50-70}$	$S_{I,\,50-70}$	I_{71}	I_{72}	$\bar{I}_{71-72}$
Trend−w	325	359	270	751	1,552	1,151
Auto−w	253	281	449	1,408	190	799
CEDS−ln q	245	283	217	128	306	217
CEDS−ln q−ϱ	144	189	156	777	295	536
CEDS−ln w	145	203	134	347	767	557
LESH−pq	153	170	128	660	118	389
LESH−w	145	161	107	713	206	459
LES−w	382	422	440	288	743	515
LES−w−ϱ	259	268	255	275	363	319
RD−w^*Dq	190	224	209	1,011	427	719
RDI−w^*Dq	175	219	189	1,170	614	892
ITRL−w	213	268	261	332	573	452
ITRL−w−ϱ	134	175	112	594	475	535

Note: $I_t = \sum_{i=1}^{4} w_{it} \ln (w_{it} / \hat{w}_{it}) \cdot 10^6$

$$\bar{I} = \frac{1}{T} \sum_{t=1}^{T} I_t$$

the "primitive" models.[7] The assumption of a constant subsistence level but not the theoretically restrictive assumption of an additive utility function thus appears to invalidate the fit of the linear expenditure system. The linear expenditure system with habit formation also shows the smallest yearly variation in fit. The performance of the constant elasticity of demand model is not much worse than that of the more sophisticated Rotterdam and translog models.

The eight-commodity grouping produces only a few changes in these results. LES still shows the worst fit, but LESH no longer takes a leading position. The translog model now shows a marginally better fit and the unrestricted constant elasticity of demand system has the best fit of all the models. This result is consistent with the preconception that there is more substitution between less aggregated commodities than between broad aggregates. Low frequency variation, however, is still important in the expenditure shares series, as shown by the relatively good fit of the simple autoregressive model. The results in these two tables also indicate that the constant elasticity demand system with a suitable number of nonzero cross-price elasticities is still a good alternative to more recent models.

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[7] For those models which do not satisfy the aggregation constraint, all predicted expenditure shares have been normalized to sum to unity. Otherwise the information inaccuracies for the double-log models and the "primitive" models would have increased.

Table 3.2 *Average information inaccuracies; food items grouped into 8 commodities*

Model	$\bar{I}_{50-70}$	$\bar{I}^{A}_{50-70}$	$S_{I,\,50-70}$	I_{71}	I_{72}	$\bar{I}_{71-72}$
Trend$-w$	818	904	506	2,540	3,490	3,015
Auto$-w$	448	498	437	1,576	356	966
CEDS$-\ln q$	555	648	500	1,300	1,699	1,500
CEDS$-\ln w$	155	310	112	1,985	4,571	3,278
LESH$-pq$	444	494	382	1,291	1,354	1,322
LESH$-w$	381	424	299	1,289	1,929	1,609
LES$-w$	1,076	1,189	1,059	8,465	1,651	5,058
RD$-w^*Dq$	372	509	384	1,573	1,027	1,300
RDI$-w^*Dq$	302	440	260	1,777	1,043	1,410
ITRL$-w$, Ω	313	420	238	2,306	1,624	1,965
ITRL$-w$, Ω_0	294	395	193	1,546	1,324	1,435

The results obtained from the data set including all goods are almost parallel to those obtained using only food commodities (Table 3.3). A minor difference is that the Rotterdam systems now fit as closely as the linear expenditure systems with habit formation, while the translog model fits almost as poorly as the ordinary linear expenditure system.

A comparison of information inaccuracies by commodities reveals approximately the same results, although the ranking of the models is not exactly the same for all commodities. For instance, with four food commodities the Rotterdam model explains "basic supplies" and "restaurant meals" relatively well, but comes out worse for "meat and fish". This is best explained by the linear expenditure system with habit formation and the translog model. As for the eight-commodity grouping, CEDS with no

Table 3.3 *Average information inaccuracies; all goods grouped into 4 commodities*

Model	$\bar{I}_{50-70}$	$\bar{I}^{A}_{50-70}$	$S_{I,\,50-70}$	I_{71}	I_{72}	$\bar{I}_{71-72}$
Trend$-w$	393	434	325	282	703	492
Auto$-w$	251	277	267	179	379	279
CEDS$-\ln q$	223	246	217	529	718	624
CEDS$-\ln q-\varrho$	129	156	154	569	535	552
CEDS$-\ln w$	128	179	147	298	230	264
LESH$-pq$	179	201	261	13	68	41
LESH$-w$	172	193	172	17	102	60
LES$-w$	294	331	229	476	467	472
LES$-w-\varrho$	228	263	189	352	213	283
RD$-w^*Dq$	179	209	272	144	17	79
RDI$-w^*Dq$	153	189	199	46	18	32
ITRL$-w$	254	320	286	426	1,429	927
ITRL$-w-\varrho$	154	201	205	55	40	48

Table 3.4 *Average information inaccuracies by commodity; food items grouped into 4 commodities*

Model	Commodity			
	A	B	C	D
Trend−*w*	77	124	58	156[a]
Auto−*w*	94	140	50	52[a]
CEDS−ln *q*	72	113	42	91[a]
CEDS−ln *q*−*ϱ*	45	82	25	35
CEDS−ln *w*	23	84	23	65
LESH−*pq*	52	67	28	53
LESH−*w*	46	64	27	51
LES−*w*	166	142	28	167
LES−*w*−*ϱ*	89	114	30	103
RD−*w***Dq*	24	122	47	44
RDI−*w***Dq*	21	114	45	41
ITRL−*w*	65	135	27	50
ITRL−*w*−*ϱ*	32	85	32	26

[a] The inaccuracy measures for Restaurant Meals and the first three models are not the same for both levels of aggregation, because the expenditure shares have not been standardized by the same factor.

restrictions, owing to its many parameters, comes out best for seven commodities. The translog model with an *a priori* fixed error moment matrix comes out best for one commodity and second best for three commodities. The Rotterdam system with intercepts fits second best for two commodities and the linear expenditure system with habit formation second best for one. The ranking is almost the same when evaluated by coefficients of determination.[8]

On the aggregate level almost all of the models fail to explain "vegetables and fruit" and similarly, at the disaggregate level they show a poor fit to the two commodities "vegetables" and "fruit, berries, ice cream, chocolates and sweets". Residual plots show that the commodity "vegetables" is systematically overestimated for the first half of the sample period and underestimated for the second. The reverse is true for "fruit, berries, ice cream, chocolates and sweets". There is no obvious explanation, however.

Parallel to previous experiences with Swedish data, all of the models fail to explain demand for "clothing" (Table 3.6). One possible explanation is offered by the peculiar features of the market for "clothing" after World War II. During and after the war there was almost no supply of textile materials and clothing to the Swedish market, and an unsatisfied demand

[8] To save space tables with coefficients of determination have not been reproduced, but they are available on request.

Table 3.5 *Average information inaccuracies by commodity; food items grouped into 8 commodities*

Model	Commodity							
	1	2	3	4	5	6	7	8
Trend$-w$	46	169	168	140	151	58	31	161[a]
Auto$-w$	49	77	50	67	130	62	29	53[a]
CEDS$-\ln q$	49	67	80	89	127	45	50	122[a]
CEDS$-\ln w$	19	10	8	28	65	8	9	27
LESH$-pq$	20	51	18	162	129	23	22	71
LESH$-w$	21	46	15	109	140	22	20	57
LES$-w$	131	98	316	73	369	34	40	162
RD$-w^*Dq$	31	14	25	64	151	51	32	57
RDI$-w^*Dq$	29	13	15	56	127	41	17	47
ITRL$-w$, Ω	22	30	11	73	113	36	23	48
ITRL$-w$, Ω_0	17	51	91	52	18	36	16	53

[a] See note to Table 3.4.

accumulated which did not result in purchases until the 1950s. This "stock effect" on demand is not incorporated in any of the models. It is worth noting that the ordinary linear expenditure system and the translog model also give a very poor fit for "food, beverages and tobacco". Another peculiar result is that all of the models show the closest fit for the commodity "other goods and services", a large proportion of which consists of purchases of consumer durables.

The only measures of autocorrelation available in this study are the

Table 3.6 *Average information inaccuracies by commodity; all goods grouped into 4 commodities*

Model	Commodity			
	I	II	III	IV
Trend$-w$	98	227	128	50
Auto$-w$	88	78	103	64
CEDS$-\ln q$	112	58	81	49
CEDS$-\ln q-\varrho$	51	16	69	33
CEDS$-\ln w$	72	14	48	43
LESH$-pq$	15	46	127	36
LESH$-w$	14	43	121	36
LES$-w$	104	147	111	23
LES$-w-\varrho$	88	25	144	34
RD$-w^*Dq$	56	24	118	30
RDI$-w^*Dq$	52	11	104	28
ITRL$-w$	156	24	101	71
ITRL$-w-\varrho$	49	10	109	26

Table 3.7 *Durbin-Watson test statistics; food items grouped into 4 commodities*

Model	Commodity			
	A	B	C	D
Trend−w	1.18	1.60	1.18	0.36
Auto−w	2.12	1.10	2.24	2.78
CEDS−ln q	1.03	1.42	1.86	1.24
CEDS−ln q−ϱ	1.39	1.91	1.84	1.50
CEDS−ln w	1.37	1.25	1.84	1.27
LESH−pg	1.96	1.85	1.63	1.29
LESH−w	2.21	2.24	1.36	1.34
LES−w	0.68	0.82	1.68	1.53
LES−w−ϱ	1.16	0.96	1.76	–
RD−w^*Dq	1.77	1.83	1.97	1.67
RDI−w^*Dq	1.86	1.89	2.04	1.64
ITRL−w	1.99	1.87	2.00	2.03
ITRL−w−ϱ	1.49	1.95	2.12	2.17

Note: The D–W statistics are calculated from the residuals of the estimated structure whether the left hand variable is an expenditure share or not.

The distributional properties of the D–W statistics are only partly known for the models analysed here.

Durbin-Watson test statistics exhibited in Tables 3.7–3.9. The ordinary significance limits for this autocorrelation test only apply to the first, third and fourth models. The distributional properties of the Durbin–Watson statistic are less well known for the nonlinear models, two of which also have the lagged dependent variable among the explanatory variables. Al-

Table 3.8 *Durbin-Watson test statistics; food items grouped into 8 commodities*

Model	Commodity							
	1	2	3	4	5	6	7	8
Trend−w	1.51	0.48	1.46	1.04	0.33	1.45	1.37	0.36
Auto−w	1.91	2.08	0.95	1.78	0.77	2.31	2.40	2.52
CEDS−ln q	0.66	1.30	1.31	1.72	0.85	1.79	1.29	1.25
CEDS−ln w	1.78	1.91	1.37	1.79	2.09	3.14	1.92	1.89
LESH−pg	1.77	0.40	0.61	2.13	1.29	1.94	2.33	0.85
LESH−w	1.98	0.63	0.94	2.35	1.57	2.12	2.44	1.04
LES−w	0.39	1.43	0.41	1.11	0.15	1.75	0.90	0.35
RD−w^*Dq	1.83	1.79	1.92	1.23	1.87	1.99	1.81	2.02
RDI−w^*Dq	1.74	1.93	1.98	1.57	1.78	2.03	2.22	2.23
ITRL−w, Ω	1.56	1.32	1.53	1.56	1.89	1.74	1.45	–
ITRL−w, Ω_0	1.58	1.33	1.61	1.58	1.65	1.84	1.76	–

See note to Table 3.7.

Table 3.9 *Durbin-Watson test statistics; all goods grouped into 4 commodities*

Model	Commodity			
	I	II	III	IV
Trend$-w$	0.97	0.36	0.97	1.26
Auto$-w$	2.19	1.22	2.64	2.35
CEDS$-\ln q$	0.62	0.48	1.28	1.29
CEDS$-\ln q-\varrho$	1.88	2.54	2.12	1.72
CEDS$-\ln w$	1.43	2.13	2.57	1.61
LESH$-pq$	2.43	2.00	2.66	2.45
LESH$-w$	2.59	2.52	2.83	2.33
LES$-w$	0.39	0.35	1.31	1.55
LES$-w-\varrho$	0.89	1.54	1.72	–
RD$-w^*Dq$	2.25	1.57	2.77	–
RDI$-w^*Dq$	2.33	2.57	3.01	–
ITRL$-w$	0.73	0.81	1.47	0.87
ITRL$-w-\varrho$	2.39	3.07	3.03	2.38

See note to Table 3.7.

though no rigorous test can be applied to all of the models, several low Durbin–Watson values indicate autocorrelation and possible specification errors. Reestimation of all of the models would be unnecessary, but three models were selected for estimation under the specification (3:12a–d) with the four-commodity food data and with the data including all goods. This limited analysis would at least indicate whether fit and predictions would improve and the extent to which the estimated elasticities would depend on the error specification. The three models were the constant elasticity demand system without cross-price elasticities, the ordinary linear expenditure system and the translog model. The results for these three models are given along with those obtained without an autoregressive error structure.

Table 3.10 *Estimated autocorrelation coefficients*

Data	Model/Commodity					
	CEDS$-\ln q-\varrho$				LES$-w-\varrho$	ITRL$-w-\varrho$
	A	B	C	D		
Food items	0.8680	0.4342	−0.0193	0.9900	0.4631 (0.1054)	0.8674 (0.0574)
	I	II	III	IV		
All goods	0.7422	0.9328	0.2209	0.1726	0.3594 (0.0880)	0.9340 (0.0300)

As might be expected, there is an improvement in fit. The first and third of these models now show the same good fit as the linear expenditure system with habit formation while the ordinary expenditure system still fits poorly.

The estimated autocorrelation coefficients for the linear expenditure system and the translog model are all significantly different from zero but the autocorrelation is approximately twice as strong in the translog model as in the linear expenditure system (Table 3.10). The results for the constant elasticity of demand model indicate that there are also large differences between commodities. Nondurables and necessities show the strongest autocorrelation. Assumption (3: 12b) would thus be too restrictive.

3.3.2 Prediction

Good fit for the sample period does not necessarily imply good predictions. The last three columns of Tables 3.1–3.3 show the information inaccuracy arising between observed and predicted expenditure shares for 1971 and 1972. These predictions were made using observed values for prices and total expenditures per head.

In regard to food share predictions with four commodities, the constant elasticity of demand model with no cross-price elasticities shows the smallest average information inaccuracy. The predictions from the Rotterdam systems and the naive models are relatively poor, while the linear expenditure systems and the translog model take an intermediate position. However, there are large differences in predictive ability for the two years, and this makes it difficult to compare the models in this respect. Somewhat more attention should perhaps be paid to the results for 1971, because this was an exceptional year. For the first time since World War II, total private consumtion declined (−1 percent in constant prices). Total food consumption also declined by one percent. For this year the simple double-log model with no cross-price effects performs much better than any of the more recent system models. However, the translog model and the linear expenditure system also predict relatively well. The predictive ability of the latter model is improved with an autoregressive error structure, but the opposite holds for the constant elasticity of demand system with no cross-price elasticities and the translog model.

The same comparison for the eight-commodity grouping shows that the autoregressive model has the smallest average information inaccuracy. This good fit is probably exceptional, however, owing to the very good predictions for 1972. The Rotterdam system (RD-wDq) and the linear expenditure system with habit formation (LESH-pq) show the second best predictions. Except for the trend model, the constant elasticity of demand model with no restrictions and the ordinary linear expenditure system, which all give poor predictions, the differences in predictive ability are small.

If we again pay particular attention to the results for 1971 we find that the two linear expenditure systems with habit formation and the constant elasticity of demand model with no cross-price elasticities give the best predictions, closely followed by the translog model (ITRL-w, Ω_0), the Rotterdam model (RD-w^*Dq) and the autoregressive model.

It is also interesting to note, for both levels of aggregation, that the many parameters in the constant elasticity of demand model with no restrictions and in the Rotterdam model with intercepts do not guarantee better predictions than those obtained from the corresponding more restrictive models.

All of the conclusions reached so far about relative predictive performances do not hold for the commodities including all goods. The linear expenditure systems with habit formation still give very good predictions but the two Rotterdam systems and the translog model *with* autoregressive errors now do so as well, while all of the other models give relatively poor predictions.

3.3.3 Comparison of Elasticities

The estimated elasticities can also be used as a basis for comparison. For instance, we may investigate whether the compensated own price elasticities are negative for all models and commodities, as is suggested by economic theory. We should also expect expenditure elasticities for luxury food items to be higher than those for non luxuries. "Meat and fish" and "restaurant meals" should thus be more expenditure elastic than "basic supplies", and "food, beverages and tobacco" and "housing services" should be less elastic than "clothing" and "other goods and services".

Table 3.11 shows that only two models give an estimated own price elasticity for "basic supplies" with the right sign. These are the linear expenditure system with habit formation and the translog model without autoregressive errors. However, the estimated standard errors are relatively large for the Rotterdam and the CEDS models.

Table 3.12 exhibits several positive estimates of compensated own price elasticities for the eight-commodity grouping. For instance, the constant elasticity of demand model and the two Rotterdam systems have positive elasticities for "flour, bread, potatoes and derivative products". The Rotterdam systems also give positive elasticities for "fish" and "restaurant meals". All of the models except the Rotterdam system with intercepts and the indirect translog model, give a positive estimated price elasticity for "butter, eggs, sugar and spices". The nonnegative estimates for the ordinary linear expenditure system and the constant elasticity of demand system with no cross-price effects are not due to random fluctuations, but in most other cases the estimated standard errors are large.

There are also a few positive but insignificant estimates of own price elasticities for nonfood commodities. All estimates of the compensated price elasticities of the ordinary linear expenditure system with an auto-

Table 3.11 *Income and price elasticities (1960); food items grouped into 4 commodities*

Model	Income elasticities Commodity				Compensated own-price elasticities Commodity			
	A	B	C	D	A	B	C	D
CEDS−ln q	−0.787	2.115	1.273	2.348	0.823	−0.573	0.008	−0.492
	(0.154)	(0.473)	(0.143)	(0.585)	(0.516)	(0.225)	(0.044)	(0.079)
CEDS−ln $q-\varrho$	0.112	2.476	1.300	0.167	0.361	−0.323	−0.008	−0.276
	(0.198)	(0.479)	(0.123)	(0.342)	(0.248)	(0.226)	(0.057)	(0.195)
CEDS−ln w	0.175	2.094	0.847	2.034	0.542	−0.220	−0.338	−0.528
	(0.148)	(0.523)	(0.202)	(0.527)	(0.258)	(0.312)	(0.128)	(0.217)
LESH−pq	0.211	2.355	1.513	0.418	−0.031	−0.341	−0.206	−0.062
	(0.082)	(0.270)	(0.164)	(0.308)	(0.016)	(0.087)	(0.053)	(0.072)
LESH−w	0.142	2.536	1.625	0.114	−0.028	−0.493	−0.895	−0.018
	(0.055)	(0.211)	(0.147)	(0.260)	(0.018)	(0.070)	(0.043)	(0.090)
LES−w	−0.662	2.224	1.196	4.223	0.263	−0.483	−0.242	−0.633
	(0.141)	(0.245)	(0.131)	(0.429)	(0.057)	(0.043)	(0.039)	(0.053)
LES−$w-\varrho$	−0.868	2.732	1.514	3.403	0.297	−0.359	−0.212	−0.500
	(0.288)	(0.636)	(0.291)	(0.692)	(0.076)	(0.053)	(0.055)	(0.172)
RD−w^*Dq	0.400	2.320	1.181	0.648	0.199	−0.344	−0.173	−0.083
	(0.143)	(0.537)	(0.242)	(0.420)	(0.110)	(0.222)	(0.113)	(0.206)
RDI−w^*Dq	0.412	2.417	1.156	0.530	0.100	−0.190	−0.231	−0.142
	(0.140)	(0.542)	(0.244)	(0.428)	(0.122)	(0.290)	(0.124)	(0.308)
ITRL−w	0.760	1.077	0.857	2.017	−0.152	−0.821	−0.368	−0.531
ITRL−$w-\varrho$	0.642	1.761	1.064	0.931	0.007	−0.254	−0.335	−0.144

Note: Elasticities for the dynamic models LESH−pq and LESH−w are one-period elasticities. (Assymptotic) standard errors—in parenthesis when available—are estimated conditional upon observed shares and volumes.

regressive error structure are positive. They might represent a local maximum of the likelihood function, but alternative initial values gave the same results, which seems to suggest that this model is inferior. Parallel to the results obtained with food data, price responses are generally small. One exception is the constant elasticity of demand system with all cross-price elasticities included. Multicollinear price variables, however, make the estimates of this model very uncertain.

All of the models give a higher estimated income elasticity for "meat and fish" than the corresponding elasticity for "basic supplies". The same is not true, however, for "restaurant meals". While the ordinary linear expenditure system gives an estimated income elasticity of 4.2, the linear expenditure system with habit formation and the Rotterdam models suggest that this commodity is inelastic. Also, according to the translog model, the

Table 3.12 *Income elasticities (1960); food items grouped into 8 commodities*

Model	Commodity 1	2	3	4	5	6	7	8
CEDS−ln q	0.039	−1.062	1.018	4.356	1.368	1.289	1.697	2.350
	(0.421)	(0.401)	(0.317)	(0.660)	(0.661)	(0.152)	(0.280)	(0.584)
CEDS−ln w	0.274	−0.268	1.242	1.514	1.806	0.521	1.442	1.700
	(0.377)	(0.343)	(0.336)	(1.034)	(0.854)	(0.212)	(0.549)	(0.505)
LESH−pq	0.629	0.027	0.447	2.178	1.931	1.491	0.807	0.714
	(0.104)	(0.027)	(0.027)	(0.205)	(0.193)	(0.119)	(0.211)	(0.089)
LESH−w	0.463	−0.001	0.461	1.471	2.410	1.561	1.066	0.389
	(0.099)	(0.037)	(0.036)	(0.162)	(0.176)	(0.130)	(0.216)	(0.114)
LES−w	1.133	−1.148	−0.568	4.633	2.165	1.198	1.541	1.380
	(0.126)	(0.144)	(0.123)	(0.364)	(0.188)	(0.091)	(0.199)	(0.096)
RD−w^*Dq	0.197	0.607	0.653	1.603	2.496	1.233	1.109	0.444
	(0.356)	(0.236)	(0.435)	(1.221)	(0.833)	(0.352)	(0.721)	
RDI−w^*Dq	0.211	0.634	0.647	1.603	2.396	1.246	1.191	0.455
	(0.368)	(0.286)	(0.366)	(1.210)	(0.825)	(0.335)	(0.574)	
ITRL−w, Ω	0.047	3.697	1.401	0.191	1.565	0.933	1.686	1.377
ITRL−w, Ω_0	0.323	3.431	1.245	0.307	1.358	0.853	1.035	1.706

Compensated own-price elasticities (1960)

Model	Commodity 1	2	3	4	5	6	7	8
CEDS−ln q	0.342	0.460	−0.559	−1.046	−0.981	−0.463	−0.087	−0.781
	(0.114)	(0.141)	(0.107)	(0.145)	(0.216)	(0.064)	(0.233)	(0.079)
CEDS−ln w	−0.214	0.466	−0.202	−0.586	0.134	−0.830	0.010	−0.703
	(0.126)	(0.255)	(0.157)	(0.376)	(0.586)	(0.123)	(0.514)	(0.429)
LESH−pq	−0.198	0.013	−0.163	−0.661	−0.562	−0.359	−0.283	−0.216
	(0.030)	(0.012)	(0.007)	(0.075)	(0.064)	(0.027)	(0.081)	(0.035)
LESH−w	−0.157	0.029	−0.176	−0.503	−0.670	−0.390	−0.404	−0.127
	(0.037)	(0.015)	(0.013)	(0.069)	(0.055)	(0.033)	(0.084)	(0.046)
LES−w	−0.314	0.308	0.147	−1.134	−0.583	−0.353	−0.382	−0.366
	(0.022)	(0.019)	(0.012)	(0.013)	(0.026)	(0.022)	(0.010)	(0.012)
RD−w^*Dq	0.054	0.196	−0.097	−0.556	−0.526	−0.302	0.372	0.223
	(0.145)	(0.127)	(0.138)	(0.263)	(0.276)	(0.131)	(0.354)	
RDI−w^*Dq	0.013	−0.187	−0.147	−0.636	−0.374	−0.328	0.159	0.311
	(0.156)	(0.135)	(0.145)	(0.274)	(0.446)	(0.131)	(0.326)	
ITRL−w, Ω	−0.169	−0.807	−0.674	0.486	−0.089	−0.343	0.004	−0.097
ITRL−w, Ω_0	−0.138	−0.628	−0.334	0.823	−0.033	−0.329	−0.129	−0.150

See note to Table 3.11.

Table 3.13 *Income and price elasticities (1960); all goods grouped into 4 commodities*

Model	Income elasticities Commodity				Compensated own-price elasticities Commodity			
	I	II	III	IV	I	II	III	IV
CEDS−ln q	0.267	0.437	0.342	0.682	−0.849	0.143	−0.306	0.013
	(0.048)	(0.014)	(0.055)	(0.042)	(0.311)	(0.122)	(0.247)	(0.712)
CEDS−ln $q-\varrho$	0.216	0.227	0.367	0.683	−0.072	0.020	−0.335	−0.340
	(0.047)	(0.086)	(0.055)	(0.037)	(0.304)	(0.096)	(0.233)	(0.647)
CEDS−ln w	0.859	0.961	0.722	1.224	−1.429	−3.440	2.419	0.210
	(0.046)	(0.034)	(0.087)	(0.039)	(0.091)	(0.759)	(1.015)	(1.475)
LESH−pq	0.603	0.167	1.780	1.667	−0.011	−0.021	−0.114	−0.029
	(0.176)	(0.130)	(0.564)	(0.172)	(0.008)	(0.016)	(0.041)	(0.019)
LESH−w	0.673	0.073	1.910	1.612	−0.021	−0.015	−0.130	−0.038
	(0.181)	(0.162)	(0.605)	(0.211)	(0.010)	(0.017)	(0.040)	(0.021)
LES−w	0.356	0.952	0.953	1.701	−0.023	−0.180	−0.206	−0.134
	(0.041)	(0.041)	(0.070)	(0.016)				
LES−$w-\varrho$	0.299	0.921	1.143	1.721	0.066	0.184	0.248	0.173
	(0.047)	(0.033)	(0.137)	(0.027)	(0.035)	(0.103)	(0.153)	(0.097)
RD−w^*Dq	0.543	0.796	0.782	1.652	−0.058	−0.111	−0.639	0.194
	(0.124)	(0.107)	(0.378)	(0.093)	(0.177)	(0.075)	(0.282)	
RDI−w^*Dq	0.671	0.077	1.980	1.592	0.007	−0.068	−0.657	0.136
	(0.270)	(0.179)	(0.853)	(0.206)	(0.201)	(0.057)	(0.277)	
ITRL−w	0.750	0.818	0.747	1.437	−0.823	−0.053	−0.136	−0.183
ITRL−$w-\varrho$	0.607	0.397	1.635	1.571	0.026	−0.084	−0.543	0.086

See note to Table 3.11.

estimated income elasticity of "basic supplies" is 0.76, while two other models, the constant elasticity of demand model and the linear expenditure system, indicate that this commodity is inferior.

The results for eight commodities show roughly the same pattern. In all of the models "fruit, berries, ice cream, chocolates and sweets" are expenditure elastic, and in all but one—the translog model—"vegetables" are elastic. With few exceptions, this is also true for "meat, pork and cheese" and "fish". The estimates for the linear expenditure systems with habit formation and the Rotterdam models indicate that the commodity "restaurant meals" is inelastic while the other models give elasticities well above unity. Almost all of the models indicate that the three subcommodities which comprise "basic supplies" are either inelastic or inferior. The exceptions are the ordinary linear expenditure system ("flour, bread, potatoes and derivative products"), the constant elasticity of demand model with no restrictions ("milk and cream") and the translog models ("butter, etc." and "milk and cream").

As for the data set including all goods, the estimated income elasticities conform to a general pattern better than the food expenditure elasticities do. As expected, demand for "food, beverages and tobacco" and "housing services" is inelastic, while demand for "clothing" and "other goods and services" is elastic. For this data set, however, there are also exceptional estimates and major differences between the models. Divergent estimates are perhaps most noteworthy for "clothing". The difficulties in estimating a demand function for this commodity were already noted above.

The introduction of an autoregressive error structure eliminates some but not all of the sign inconsistencies of price and income elasticities and it also creates some new ones. For instance, using the constant elasticity of demand system, the estimated income elasticity of "basic supplies" becomes positive although small and the price elasticity of "other goods and services" becomes negative, while the price elasticities of "basic supplies", "food, beverages and tobacco" and "other goods and services" for the translog model have the wrong sign.

The erroneous signs of the price elasticities given by the linear expenditure system have already been mentioned. There are also some additional changes which are considerable, although they do not involve a change in sign; see e.g. the estimates for "restaurant meals". We can thus conclude that the estimated elasticities do depend on the error structure, but the sign inconsistencies remain even with an autoregressive error structure.

The comparisons of elasticities in Tables 3.11–3.13 show that none of the models conforms exactly to the *a priori* expected signs and magnitudes of the elasticities. The finding in previous studies, that estimates of elasticities crucially depend on the model used, is also confirmed. Our eight models give a vastly different interpretation of food data and there are also major divergencies in the interpretation of nonfood data. If the demand models and estimation methods used are not all equally sensitive to measurement errors, this might be at least a partial explanation for these large differences in estimated elasticities, but if this is not the case, the choice of model may become decisive for forecasting and policy.

3.3.4 On the Effects of Aggregation

This study offers a possibility of demonstrating a few effects of commodity aggregation in numerical terms. Aggregation has been analysed for linear models—see, for instance, Theil (1954), Lütjohann (1974) and Chipman (1975, 1976 and 1977)—but it is an almost unexplored field for nonlinear models. Consistent aggregation is exceptional in linear models, i.e. consistent aggregation requires unrealistically simple micro relations. However, if

Table 3.14 *Information inaccuracies for predictions obtained by aggregation from 8 to 4 commodities*

Model	I_{71}	I_{72}	$\bar{I}_{71-72}$
Trend$-w$	735	1,535	1,135
Auto$-w$	1,008	189	599
CEDS$-\ln q$	105	366	235
CEDS$-\ln w$	611	2,784	1,697
LESH$-pq$	493	285	389
LESH$-w$	553	236	395
LES$-w$	3,038	257	1,645
RD$-w^*Dq$	1,099	614	856
RDI$-w^*Dq$	7,045	880	3,963
ITRL$-$w, Ω	103	109	106
ITRL$-w$, Ω_0	121	201	161

aggregation is inconsistent there are in general no macro parameters, i.e. the pseudo macro parameters specified depend on the aggregation process and the variables involved. There is no reason to believe that the situation would generally be more favourable for non-linear models.

The estimated elasticities are thus *in general* not independent of the level of aggregation. The colums for "restaurant meals" in Tables 3.11 and 3.12—note that commodities D and 8 are identical—show that they *in fact* depend on the grouping. With respect to some models—for instance, the ordinary linear expenditure system and the translog model—the differences in estimated elasticities are large.

There is also an aggregation effect on the goodness of fit as revealed for "restaurant meals" by Tables 3.4 and 3.5. A comparison of fit between the two levels of aggregation does not have any meaning except for those models which satisfy the budget constraint. All these models, except the ordinary linear expenditure system, show a closer fit to observed expenditure shares for "restaurant meals" when the other commodities are aggregated. But there are no large differences in fit.

In order to investigate whether predictive performance depends on the level of aggregation, the eight predicted expenditure shares have been aggregated to expenditure shares for the four commodities A–D, for each model and year. Using the same information inaccuracy measure as before, they are then compared to observed expenditure shares and to those predicted by the aggregate models. Table 3.14 shows that among the disaggregate models, the translog model, the constant elasticity of demand model with no cross-price elasticities and the linear expenditure system with habit formation give the best predictions. This was also the result obtained when the models were estimated on aggregate data (Table 3.1). There are no great differences in prediction accuracy due to the level of aggregation for the best models, but differences do arise for some of the inferior models. There is no unique indication, however, that a disaggregate analysis would be superior or vice versa.

3.4 Conclusions

The process of compiling a general ranking list of our ten models would necessarily involve subjective evaluations because the ranking order depends on the criteria and data used. However, a few conclusions may be drawn. If we were to classify our ten models with goodness of fit as the only criterion, the naive models and the ordinary linear expenditure system would be classified as inferior, the constant elasticity of demand model with no cross-price elasticities and the translog model, both without autoregressive errors, as intermediate and the other models as superior. If yearly forecasting ability is also taken into consideration the same grouping is obtained, except that the two constant elasticity of demand models would now both be classified as intermediate. The Rotterdam systems are on the borderline between the intermediate and the superior groups. One conclusion is thus that demand models tend to be superior to naive models. If we would also like to base our choice on the expected sign and magnitude of estimated elasticities, the linear expenditure system with habit formation emerges as the only model which performs rather well according to all three criteria and with all three data sets. The indirect translog model *with* an autoregressive error structure has only been tried on the two data sets with four commodities, where this model has given relatively good results. But it is more difficult to estimate than the linear expenditure system, particularly for models with many commodities.

Given these criteria, and contrary to previous studes—cf. Deaton (1974 *b*)—no severe measurement distortions are found which are due to the assumption of additivity. The translog model and the Rotterdam system, which are non-additive models, are not found to be superior to the linear expenditure system with habit formation. However, our results also show that none of the models is obviously best. Aggregate time series do not provide enough information to allow sharp discrimination between models and, in addition, it has been shown that the estimated structures do depend on the level of aggregation. Thus, the stability of these models should be analysed further in both time and commodity dimensions. This analysis should *inter alia*, be based on predictions for a period longer than two years, and include a comparison of not only expenditure shares but also consumption volume and relative changes in volume.

4 Demand for Housing and Other Commodities under Rent Controls. An Application of the Linear Expenditure System

4.1 Introduction

It is usually implicitly assumed that the demand functions in a complete system are identified. This is not always a realistic assumption. In many countries rent controls and other regulations have been enforced on the housing market to keep housing costs low. This has typically created an excess demand as evidenced by queues for housing. In this situation when the demand model is fitted to expenditure data we will obviously not obtain estimates of a demand function for housing but rather of a supply function or a mixture between the two. The identification problem may be solved by using data on the magnitude of the excess demand or, if these data are not available, by specifying the supply side and the effects of rent controls in the model. This section is based on an idea presented in Klevmarken (1974). Our aim is not an analysis of the housing market *per se*. If it had been, the complete systems approach would not have become out primary choice. This analysis constitutes a modest attempt to reduce the possible bias in estimates of a complete system of demand functions which may arise when the effects of rent control are not taken into account. Our approach has to be rather simple because in itself, estimation of a complete system of demand functions is not a straightforward matter.

4.2 The Swedish Housing Market

In Sweden rent controls on flats have been in effect since 1942. At the outset they were only intended as a temporary arrangement during the war. But housing costs did not decrease after the war as expected; they increased instead and impeded the abolishment of rent controls. Several studies of the Swedish housing market have been carried out. They deal with the effects of a housing policy which, in addition to rent controls, included a controlled credit market, interest subsidies, government loan guarantees and direct subsidies to households; see Bentzel *et al.* (1963), Rydenfelt (1971) and (1972), Lindbeck (1972) and Du Rietz (1977).

Table 4.1 *Excess demand for housing*

Year	Excess demand in percent of		
	number of appartments	number of rooms	housing expenditures
1945	2.6	5.1	7–13
1960	3.9	9.2	10–16
1965	7.6	12.5	17–23
1970	2.7	2.3	5–10
1975	−0.3	0.7	5–10

Source: IUI:s långtidsbedömning 1976, page 99 and Bilaga 4 Table 4:12, Industriens Utredningsinstitut, Stockholm 1976.

Rent controls resulted in an excess demand, particularly in the large cities. It is difficult to obtain good measures of its magnitude. The number of persons registered for a flat according to the queuing system is a poor measure because there was no charge to join the queue. Moreover, the increased difference in rent levels between old and new flats induced by the control system stimulated people who already had a flat to join the queue to acquire an even less expensive place to live. Thus the size of the queue overestimates the excess demand, particularly at the end of the period when households had adjusted to the queuing system. Most economists, however, would say that excess demand reached a peak around 1965 and that the market was almost in balance during the first half of the 1970s. The pressure exerted by long housing queues led to allocation of major resources to housing construction.[1] Combined with increases in rent levels —within the limits of the control system—this meant that rent controls could gradually be eliminated. By 1972, only 43 cities and 13–14 percent of the total number of flats were still included. Table 4.1 shows some of the most recent estimates of the excess demand for housing. These estimates are from a study by Du Rietz (1977).

4.3 A Model for Demand under Rent Controls

In order to forecast future demand for housing and other commodities, our model has to be estimated on data from a period with excess demand for housing, whereas in the forecasting period there will most likely be more or less a balanced housing market. Thus our predictions cannot be based on

[1] Only 44,000 new flats were built in 1950, but by 1970, when housing construction reached a maximum, the number had climbed to 110,000. However, the net increase to the housing market was considerably less because, in addition to demolition a large share was transferred from the housing market to the uncontrolled market for offices and other business uses. The estimate of the annual net increase during 1966–1970 in Rydenfelt (1972) is 61,000.

the assumption that the effects of public housing policy will be the same during the forecasting period as during the sample period.

In order to tackle this problem, we modify the linear expenditure system with habit formation to incorporate the fact that throughout the sample period, consumption of housing services was determined by supply rather than demand. The linear expenditure system with habit formation has previously shown itself (Chapter 3) to be one of the best models for forecasting in spite of its additivity property, which in this particular application will prove quite useful. We assume that consumption of housing services slowly adjusts to "desired" consumption, which is defined as consumption without rent controls. We begin by defining this concept in more detail and then specify the adjustment mechanism.

Suppose that in the hypothetical situation where there are no controls, and comsumption is allowed to adjust freely to given prices, the average consumer seeks to maximize the following utility function

$$U_t = \sum_{i=1}^{n} \beta_i \log (q^*_{it} - \alpha_i q_{it-1}) \tag{4:1}$$

subject to the budget constraint

$$y_t = \sum_{i=1}^{n} p_{it} q^*_{it} \tag{4:2}$$

where q^*_{it} is the demanded volume of commodity i at the given price p_{it}, q_{it-1} is the volume actually consumed in the previous period. As in Chapter 3 y_t is income (total consumption), β_i the marginal propensity to consume commodity i and α_i a parameter which indicates the persistence of consumption habits. This model is illustrated in Figure 4.1 for two commodities. At the given prices a maximum will be found at the point $(q^*_1; q^*_i)$ in Figure 4.1. The demand functions are

$$p_{it} q^*_{it} = \alpha_i p_{it} \, q_{it-1} + \beta_i \left(y_t - \sum_{k=1}^{n} \alpha_k p_{kt} \, q_{kt-1} \right) + u_{it} \tag{4:3}$$

where u_{it} is a random error term with zero expectation added to the function.

If the supply of commodity 1 is completely inelastic and equal to q^0_{1t} and if prices are controlled and cannot adjust, the optimal consumption combination now becomes $(q^0_1; q_i)$, i.e., the consumer seeks to maximize utility subject to the constraint that q_{1t} equals q^0_{1t}. Thus, the utility function

$$U^0_t = \beta_1 \log (q^0_{1t} - \alpha_1 q_{1\,t-1}) + \sum_{i=2}^{n} \beta_i \log (q_{it} - \alpha_i q_{it-1}) \tag{4:4}$$

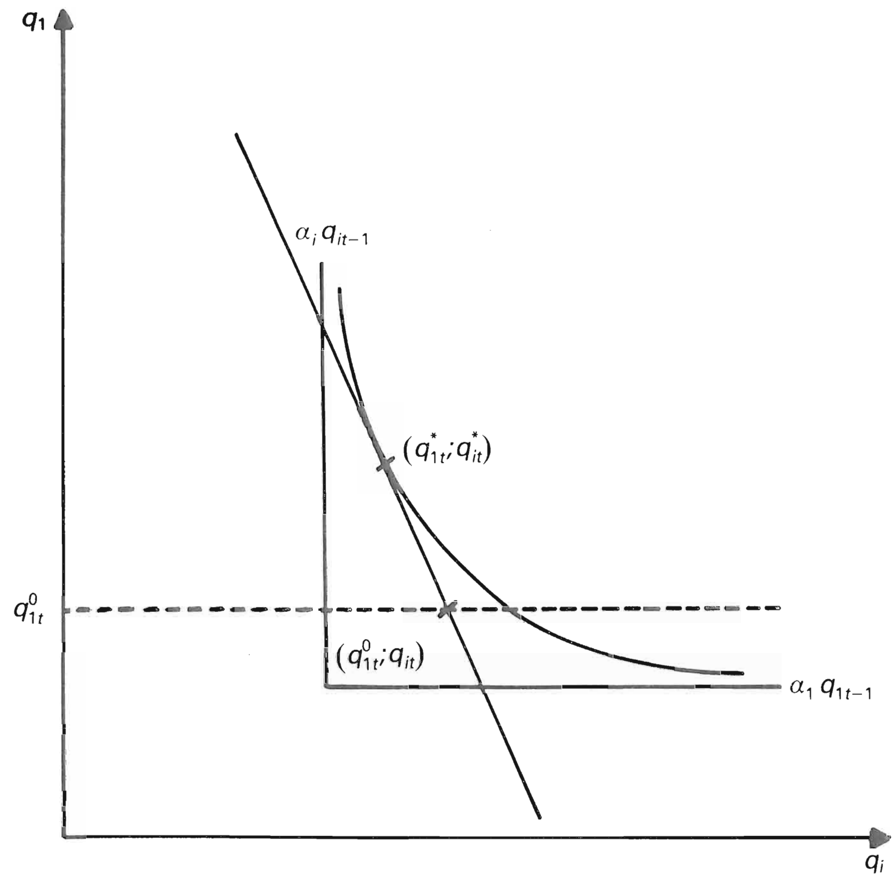

Figure 4.1 *Demand under price control.*

is maximized with respect to $q_{2t}, \ldots, q_{nt}$ subjetc to the constraint

$$y_t - p_{1t} q^0_{1t} = \sum_{i=2}^{n} p_{it} q_{it} \tag{4:5}$$

This gives the following expenditure functions

$$p_{it} q_{it} = \alpha_i p_{it} q_{it-1} + \frac{\beta_i}{1-\beta_1}\left(y_t - p_{1t} q^0_{1t} - \sum_{k=2}^{n} \alpha_k p_{kt} q_{kt-1} \right) + \varepsilon_{it} \quad i = 2, \ldots, n \tag{4:6}$$

to which random errors ε_{it} with zero expectation have again been added. To distinguish between demanded volumes when there are no price controls and consumed volumes subject to controls of p_{1t}, we used q_{it} in eqs. (4:4)–(4:6) rather than q^*_{it}.

In this model, consumers' habits are assumed to adjust to the volumes

Table 4.2 *Estimates of conditional expenditure functions*

Commodity	Conditional estimates				Unconditional estimates			
	α_i	$\beta_i/(1-\beta_1)$	R_i^2	D–W	α_i	$\beta_i/(1-\beta_1)$	R^2	D–W
Food, beverages and tobacco	0.9831 (0.0152)	0.2267 (0.0465)	0.9991	1.90	0.9799 (0.0207)	0.2196 (–)	0.9987	2.43
Clothing	0.9335 (0.0469)	0.1973 (0.0402)	0.9891	2.70	0.9221 (0.0565)	0.1866 (–)	0.9839	2.66
Remainder excl. housing	0.9764 (0.0321)	0.5761 (0.0413)	–	–	0.9590 (0.0461)	0.5937 (–)	0.9995	2.45

Note: The distributional properties of the D–W statistics are unknown for these models.

actually consumed subject to price controls and not to the hypothetically demanded volumes in a situation without controls. This seems to be a reasonable assumption. If housing expenditures are kept down by rent controls and an inelastic supply, then the induced "supernumerary" income can be used for increased purchases of food, clothing, durables, etc. Households will probably get used to the higher consumption standards linked to these commodities and find it difficult to reduce them. This assumption implies two concepts of excess demand. One is $q_{1t}^{*}-q_{1t}^{0}$, conditional only on current prices, income and last year's consumed volumes which is the excess demand in the short run. The other is the difference between the demanded volume if controls had *never* been introduced and the controlled volume. This difference depends on all past prices and incomes.

Of course the model can easily be generalized to include more than one controlled commodity, but in this study we assume only one, housing.

If there is no information about the magnitude of excess demand (in the short-run) or about the supply function, it is not possible to identify and estimate any of the demand functions (4:3), although the conditional functions (4:6) can be estimated. The results in Table 4.2 were obtained under the assumption of contemporaneous, but not autocorrelated errors ε_{it}. The nonlinear iterative Aitken procedure of the TSP program was used to estimate the model. The data were the same as those previously used for the four commodities "food, beverages and tobacco", "housing", "clothing" and "remainder".[2] The estimates of eq. (4:6) are listed on the lefthand side of Table 4.2 under the heading "conditional estimates". By way of comparison the table also shows the estimates obtained when eq. (4:3) are fitted to all four commodities under an assumption of no rent controls ("unconditional estimates"). The two sets of estimates differ only slightly,

[2] In this chapter "housing" is labeled commodity No. 1.

which indicates that omission of rent controls does not seriously affect the estimates for nonhousing commodities. This result obviously cannot be generalized to other models. In particular a model based on a nonadditive utility function might give a different result.

If information about the magnitude of excess demand were available, it could be used to identify and estimate the demand function for housing and thus the whole model as well. The observed q_{1t} and the measure of excess demand would give an estimate of $q^*_{1t.}$ Given the estimates of the α_i-parameters in the conditional model α_1 and β_1 could then be estimated from (4:3), i.e.

$$p_{1t}q^*_{1t}=\alpha_1(1-\beta_1)p_{1t}q_{1t-1}+\beta_1\left(y_t-\sum_{k\neq 1}\hat{\alpha}_k p_{kt}q_{kt-1}\right)+v_{1t} \tag{4:7a}$$

where

$$v_{1t}=u_{1t}+\beta_1\sum(\hat{\alpha}_k-\alpha_k)p_{kt}q_{kt-1.} \tag{4:7b}$$

The efficiency of this method depends *inter alia* on the correlation between u_{1t} and ε_{it}.

Unfortunately, there are no reliable annual estimates of the excess demand for Sweden. For reasons already mentioned we cannot use data on the number of people registered in the queues. The best estimates available appear to be those obtained by Du Rietz (1977) and reproduced in Table 4.1. Previous Swedish and international estimates were used to make an expert judgement about the income and price elasticities. These were then used to predict demand. The difference between these predictions and observed supply then served as Du Rietz' estimate of excess demand. This estimate gives at best a rough idea of the trends during the sample period. The utmost we can do is to investigate the sensitivity of the estimates of the demand function for housing to alternative assumptions about excess demand. To this end, the figures in the middle column of Table 4.1 were interpolated and used to inflate the observed consumption volumes for housing.[3] The resulting estimates are shown in Table 4.3 in the row designated ''observed excess demand''. As compared to the estimates of the linear expenditure system fitted to all four commodities—''unconditional estimates''—these results are theoretically less plausible. The marginal propensity to consume is negative and the habit parameter exceeds one even more than the ''unconditional estimate''. The standard errors of the estimates, however, are large enough to suggest that this might be the result of chance.

[3] The last column was not used because Du Rietz indicates that these estimates are not very reliable.

Table 4.3 *Estimates of the demand for housing*

Model	α_1	β_1	λ_0	λ_1	λ_{21}	λ_{22}	R^2	D–W
Unconditional estimates	1.0160 (0.0092)	0.0338 (0.0264)	–	–	–	–	0.9993	2.00
"Observed" excess demand	1.1645 (0.1011)	−0.0159 (0.0233)	–	–	–	–	0.9865	0.27
(4:9b):I	0.4951 (5.1360)	0.3715 (0.8796)	95.2824 (94.2707)	0.1327 (0.2684)	−34.4414 (57.6695)	0.0001 (0.0010)	0.9999	–
(4:9b):II	0.7958 (3.6374)	0.2877 (0.5025)	72.1038 (75.6592)	0.1731 (0.2491)	−20.2696 (53.6095)	–	0.9999	–
(4:9b):III	0.6512 (5.1122)	0.3277 (0.8019)	49.2657 (40.0091)	0.1294 (0.2590)	–	0.0001 (0.0009)	0.9999	–
(4:9b):IV	0.8334 (3.9298)	0.2901 (0.5442)	45.2799 (21.4682)	0.1585 (0.2428)	–	–	0.9999	–
(4:9b):V	0.9971	0.0957 (0.0400)	64.0400 (57.0979)	0.6358 (0.2099)	−10.6596 (41.9040)	−0.0002 (0.0007)	0.9999	–
(4:9b):VI	1.0106	0.0202 (0.0095)	40.4645 (15.7945)	0.5619 (0.2238)	–	–	0.9999	–

Note: The estimates of the unconditional model were obtained by a ML-procedure while model (4:9b) was estimated by a non-linear least-squares minimization.

Alternatively, the model may be identified if the supply function is properly specified. Suppose that the *change* in supplied housing volume depends on lagged excess demand and possibly also on other variables, which, say χ, capture changes in housing policy and other exogenous changes in the housing market.

$$q_{1t}-q_{1t-1}=\lambda_0+\lambda_1(\overset{*}{q}_{1t-1}-q_{1t-1})+\lambda_2\chi_t+w_{1t;} \tag{4:8}$$

After premultiplying by p_{1t} and substituting the r.h.s. of eq. (4:3), lagged one year, for $p_{1t-1}\overset{*}{q}_{1t-1}$ we obtain

$$p_{1t}q_{1t}-p_{1t}q_{1t-1}=\lambda_0 p_{1t}+$$

$$\lambda_1\frac{p_{1t}}{p_{1t-1}}\left[\alpha_1 p_{1t-1}q_{1t-2}+\beta_1\left(y_{t-1}-\sum_{k=1}^{n}\alpha_k p_{kt-1}q_{kt-2}\right)+u_{1t-1}-p_{1t-1}q_{1t-1}\right]+$$

$$\lambda_2 p_{1t}\chi_t+p_t w_{1t;} \tag{4:9a}$$

$$q_{1t}=(1-\lambda_1)q_{1t-1}+\lambda_1\alpha_1(1-\beta_1)q_{1t-2}+\frac{\lambda_1\beta_1}{p_{1t-1}}\left(y_{t-1}-\sum_{k=2}^{n}\alpha_k p_{kt-1}q_{kt-2}\right)+$$

$$\lambda_0+\lambda_2\chi_t+\lambda_1\frac{u_{1t-1}}{p_{1t-1}}+w_{1t.} \tag{4:9b}$$

Although not in a wholly efficient way, this equation can be estimated conditionally on the estimates $\hat{\alpha}_2, \hat{\alpha}_3, \ldots, \hat{\alpha}_n$ of eq. (4:6). Without going too deeply into the mysteries of the Swedish housing market, this equation was estimated using two χ-variables, the ratio of a building cost index and the price index for housing services and the number of new flats built with government loans. Although nonprofit organisations are responsible for a large share of housing investments, it is assumed that the investment activities of private entrepreneurs are not negligible for the supply of housing services and that the ratio variable will capture some of their willingness to invest. We would thus expect this variable to have a negative effect. The number of new flats built with government loans is supposed to catch the degree of involvment of public housing policy.[4] In a situation with excess demand, this variable is determined by the amount of resources the government is willing to allocate for investments in housing. The effect of this variable should thus be positive.

Although the signs of the estimated parameters are as expected and the fit is excellent the results are discouraging because the estimates are so poorly determined. It is impossible to draw definite conclusions about any of the parameters. In order to investigate whether this is due to the two χ-variables they were successively deleted from the model, but this did not lead to any major improvement.

λ_0 is interpreted as a trend increase in the supply of housing services. The estimates range from 40 to 95, while as a comparison the average increase in housing services in constant 1964 prices during the sample period was close to Skr 300 million. A λ_1-parameter of 0.15 would mean that 15 percent of the excess demand would be eliminated each year. Then too, this figure might be somewhat high. The point estimates of the habit parameter, however, are on the low side. The estimates of the marginal propensity to consume can be expected to be higher than previously and this is also confirmed. The estimates for model (4:9b) in Table 4.3 would imply an expenditure elasticity of approximately 1.5 which is high, but not unrealistically so in this model. Such a result would also imply that the marginal propensities to consume and expenditure elasticities previously obtained for the other three commodities would be reduced by approximately 70 percent. Unfortunately this is only speculation. The estimates are too unreliable and the specific point estimates do not satisfy the theoretical properties of the model because they give an excess supply and not an excess demand.[5]

In a final attempt to increase the precision of the estimates and extricate some information from the sample, the model was constrained to show a

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[4] Effects of changes in the rules for obtaining a government loan and in the average size and quality of flats have not been considered.

[5] Demand is estimated from eq. (4:3) and observed expenditures are set equal to supply.

balanced market in 1970, the last sample year. This gives α_1 as a known function of β_1 and thus one parameter less to estimate. The results for two variants of the model, (4:9b):V and (4:9b):VI, are shown in Table 4.3. The estimates of α and β are now much closer to the unconditional estimates. The estimates of λ_1 are very high indicating a quick reaction to excess demand. The estimated excess demand is small and shows a more or less random fluctuation around zero. This result casts some doubts on the model, although some of the λ-coefficients are significantly different from zero.

Perhaps less aggregated data would yield more reliable estimates and a more clear-cut test of the model. One obvious defect of the present study is that the controlled sector of the housing market is not separated from the free sector. An attempt was made to obtain separate expenditure estimates for owner occupied houses, since this portion of the market is less affected by control and government regulations. But as it turned out, such estimates could not be achieved because it was impossible to obtain sufficiently long consistent time series. We can only conclude that much more abundant data are needed to single out the effects of a regulated housing market.

5 Demand for Durables in the Complete System Approach

We now go one step further in the specification of a dynamic demand model and discuss attempts to incorporate stocks of consumer durables into systems of demand functions. Previous studies are briefly reviewed and a model originally suggested in Dahlman & Klevmarken (1971) is further developed, estimated and compared to other models.

These models all assume "myopic" utility maximization, i.e. the planning horizon is not extended beyond the present period, which might seem very restrictive. However, Hadar (1971) has shown that an intertemporal utility maximization can be collapsed into a one-period problem.[1] Although the intertemporal utility functions and the corresponding generalized budget constraint have not been derived for the models discussed below, the existence of a corresponding intertemporal problem might, as in Clements (1976), simply be assumed.

5.1 A Brief Review of Previous Studies

An important idea in the pioneering work of Houthakker & Taylor (1970) is that stock adjustment behaviour and inertia due to habits can be treated analogously. They suggest, for instance, that "the consumer has built up a psychological stock of smoking habits" (p. 10). Houthakker & Taylor introduce a "state variable" into their model which is interpreted as a stock of durables or a stock of habits or possibly a mixture of the two. The dominant component is determined by the sign of the estimated effect on purchases. If this effect is positive, habit formation is dominant if it is negative, stock adjustment is dominant.

The demand model of Houthakker & Taylor is derived from a quadratic utility function in which state variables as well as flows appear as arguments. When this function is maximized subject to an ordinary budget constraint and to the assumption of constant depreciation rates, the following estimating equations—after some transformations—are obtained

$$q_{it} = K_{i0} + K_{i1} q_{it-1} + K_{i2} \lambda_t p_{it} + K_{i3} \lambda_{t-1} p_{it-1} + u_{it}; \quad i = 1, \ldots, n \tag{5:1}$$

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[1] A more accessible proof of Hadar's theorem by Alan Powell is found in Clements (1976), Appendix 1.

The K's are functions of the original parameters only. As in the preceding chapters λ_t is the marginal utility of income (total expenditures). Because λ_t is not observable, (5:1) is not readily estimated, although Houthakker & Taylor suggested a nonlinear estimation procedure (to which we return later on).

An almost identical approach is adopted by Mattei (1971). There are minor differences in the specification of the utility function and the rates of depreciation, but Mattei arrives at the same estimating equation (5:1).

The "state variables" concept is also used in Phlips (1972 and 1974), but he derives his model from the Stone-Geary utility function, i.e. his model is a dynamic version of the linear expenditure system. Phlips assumes that the "minimum required quantities" of the Stone-Geary utility function are linearly related to the state variables s_i. The utility function to be maximized is thus

$$U = \sum_{i=1}^{n} \beta_i \log (q_i - c_i - \alpha_i s_i); \tag{5:2}$$

where β_i, c_i and α_i are parameters. The resulting demand functions are awkward expressions in observed variables and Phlips chose to approach the estimation problem in the same way as Houthakker & Taylor. Phlips' estimation equation is,

$$q_{it} = k_{i0} + k_{i1} q_{it-1} + k_{i2} (\lambda_t p_{it})^{-1} + k_{i3} (\lambda_{t-1} p_{it-1})^{-1} + u_{it}. \tag{5:3}$$

the k_i's are derived parameters and the remaining symbols preserve their previous meaning. The only difference from eq. (5:1) is that the λp-variables are replaced by their reciprocals.

Both models were estimated iteratively according to the following scheme:

(i) Select a sequence of start values for λ_t, $t=0, 1, \ldots, T$. Since the marginal utility of income is a decreasing function of income, $\lambda_t = 1/y_t$ might serve as the initial choice.

(ii) Given these start values, apply LS on the estimting equation to obtain estimates of the K- or k-parameters, respectively.

(iii) New values of λ_t are calculated recursively using a formula derived from the budget constraint and the necessary marginal condition for maximal utility. As this formula is recursive, new estimates on λ_0 cannot be obtained. For this reason and since the λ-parameters are only identified up to a scale factor, it is suggested that $\lambda_0 = 1$ in all iterations.

(iv) The procedure is repeated until the budget constraint is satisfied, i.e.

$$\left| y_t - \sum_i p_{it} \hat{q}_{it} \right| < g y_t \quad \text{for all } t; \tag{5:4}$$

where g is a convergence limit, for instance 10^{-3}.

The convergence properties of this procedure have not, to our knowledge, been analysed, but there are some observations related in the studies cited above. If p in (5:1) and (5:3) is replaced by p/y and thus λ by λy, convergence is reported to be relatively rapid. Mattei (1971) used only eight iterations. In Taylor & Weiserbs (1972) it is suggested that the speed of convergence depends to a large extent on the initial choice of λ_t.

Even less is known about the distributional properties of the estimates. If convergence is attained, i.e. if the budget constraint is satisfied, then it can be shown that the moment matrix of the residuals is singular. This implies a correlation across equations which is not accounted for in the estimation. A GLS method could be tried as a fifth step in the estimation procedure.

These two models have been compared in Taylor & Weiserbs (1972) and Phlips (1974). The dynamic version of the linear expenditure system shows a smaller number of implausible elasticities and rates of depreciation. It also provides better forecasts than the model of Houthakker & Taylor. In addition the estimates of the latter model imply the presence of satiation which is reflected by a sharp drop in λ at the end of the sample period. These results indicate that the dynamic version of the linear expenditure system is the most preferable model.

5.2 A New Dynamic Version of the Linear Expenditure System

A slightly different dynamic version of the linear expenditure system was suggested in Dahlman & Klevmarken (1971). In this model consumption and purchases are explicitly separated and accumulation of stocks of durables is not regarded as an analogue to habit formation. Although the introduction of state variables might seem elegant and convenient it should be recognized that consumers have to accumulate stocks of durables because these come in "large" units, while habit formation is a psychological process which depends partly on the previous experience of the consumer himself and other consumers in his neighbourhood. It is possible, of course, to develop consumption habits with respect to the services of a durable good, but it is less obvious that habits should accumulate and wear off in the same way as stocks of durables.

In the following, consumption of commodity i is denoted by q^*_{it}. In each year t the consumer is assumed to maximize the following utility function

$$U_t = \sum_{i=1}^{n} \beta_i \log (q^*_{it} - a_i q^*_{it-1}) \tag{5:5}$$

subject to the budget constraint

$$\sum_{i=1}^{n} p_{it} q_{it} = y_t \tag{5:6}$$

and the definition of consumption

$$q_{it}^{*} = \delta_i s_{it-1} + \theta_i q_{it}. \tag{5:7}$$

There are thus two components of consumption volume. One is consumption out of the stock at the end of the previous period and the other is consumption out of present purchases. In order to simplify the model, we assume that the depreciation rates δ_i and θ_i are constants. This assumption probably lacks realism at the micro level. It implies that the consumer can only change his consumption by changing his purchases. But this assumption is less harmful at an aggregate level. For nondurables both coefficients will be 1 and no stock will accumulate. For values less than 1 it is reasonable to assume that $\theta_i < \delta_i$ as long as all purchases are not made in the beginning of each period. Given these assumptions, stocks are accumulated according to the following expression

$$s_{it} = (1-\delta_i)\, s_{it-1} + (1-\theta_i)\, q_{it}; \tag{5:8}$$

which can be transformed to

$$s_{it} = (1-\theta_i) \sum_{r=0}^{\infty} (1-\delta_i)^r q_{it-r}. \tag{5:9}$$

In Phlips' model the subsistence level is defined by the accumulated stocks; in our model the subsistence level is defined by stocks with two years' lag and expenditures lagged one year. Stocks lagged one year also enter the utility function through current consumption volumes.

Utility maximization gives—after some manipulations—the following expenditure functions,

$$p_{it} q_{it} = \left\{ \alpha_i - \frac{\delta_i (1-\theta_i)}{\theta_i} \right\} p_{it} q_{it-1} + \frac{\delta_i (\alpha_i + \delta_i - 1)}{\theta_i} p_{it} s_{it-2} +$$

$$\beta_i \left[y_t - \sum_{k=1}^{n} \left\{ \left[\alpha_k - \frac{\delta_k (1-\theta_k)}{\theta_k} \right] p_{kt} q_{kt-1} + \frac{\delta_k (\alpha_k + \delta_k - 1)}{\theta_k} p_{kt} s_{kt-2} \right\} \right] \tag{5:10}$$

Committed purchases are functions not only of purchases in the preceding period but also of all previous purchases since they determine the stock variables. It is easier to interpret these functions verbally if they are transformed into consumption functions, but first let us analyse the short-run purchase behaviour. The short-run purchase propensity is β_i and the corresponding elasticity is thus

$$e_{i0} = \beta_i / w_i \tag{5:11}$$

where w_i is the expenditure share.

The short-run own price elasticities are

$$E_{ii0} = -1 + (1-\beta_i)\left\{\left[\alpha_i - \frac{\delta_i(1-\theta_i)}{\theta_i}\right]\frac{q_{it-1}}{q_{it}} + \frac{\delta_i(\alpha_i+\delta_i-1)}{\theta_i}\frac{s_{it-2}}{q_{it}}\right\} \qquad (5{:}12)$$

$$= -1 + (1-\beta_i)\left\{\alpha_i q^*_{it-1} - \delta_i s_{it-1}\right\} / \theta_i q_{it;}$$

and the cross price elasticities are

$$E_{ik0} = -\beta_i\left\{\left[\alpha_k - \frac{\delta_k(1-\theta_k)}{\theta_k}\right]\frac{p_{kt}q_{kt-1}}{p_{it}q_{it}} + \frac{\delta_k(\alpha_k+\delta_k-1)}{\theta_k}\frac{p_{kt}s_{kt-2}}{p_{it}q_{it}}\right\}$$

$$= -\beta_i\frac{p_{kt}}{p_{it}}\left\{\alpha_k q^*_{kt-1} - \delta_k s_{kt-1}\right\} / \theta_k q_{it;} \qquad (5{:}13)$$

These expressions indicate that the purchase elasticity is the same as in the ordinary linear expenditure system with habit formation while the price elasticities involve stocks. However, if $\theta_i = \delta_i = 1$ they reduce to the corresponding expressions for a model without stocks. A commodity will be price elastic only if consumption out of the stocks at the beginning of the period exceeds committed consumption, which is not a very likely event. It also follows that a price elastic commodity is a gross substitute for all other commodities. It is easily shown that these short-run elasticticities satisfy the aggregation, homogeneity and symmetry constraints.

By using definition (5:7), the system of purchase equations (5:10) can be transformed into a system of consumption equations.

$$p_{it}q^*_{it} = \alpha_i p_{it} q^*_{it-1} + \beta_i\left\{\theta_i y_t + \sum_{k=1}^{n}\frac{\theta_i}{\theta_k}\delta_k p_{kt} s_{kt-1} - \sum_{k=1}^{n}\frac{\theta_i}{\theta_k}\alpha_k p_{kt} q^*_{kt-1}\right\} \qquad (5{:}14)$$

In this model stocks add to the resources for consumption and they thus contribute to "supernumerary income", the term within brackets. The ratio θ_i/θ_k may be interpreted as an "exchange rate". We also find that the short-run marginal propensity to consume is $\beta_i\theta_i/p_{it}$.

The following relations between the consumption and purchase elasticities also follow from eq. (5:7)

$$e^*_{i0} = \frac{\partial q^*_{it}}{\partial y_t}\frac{y_t}{q^*_{it}} = \theta_i e_{i0}; \qquad (5{:}15)$$

$$E^*_{ik0} = \theta_i E_{ik0}; \qquad (5{:}16)$$

These results hold only in the short run, but we are also interested in the long-run behaviour of the model. If there exists an equilibrium, i.e., if there is a state in which consumption equals purchases and stocks are constant, for given values of y_t and p_{it}, say y_0 and p_{i0}, then these equilibrium values can be derived.

Assume $\quad q_{it} = q_{iL}; \quad t = 1, 2, \ldots \qquad (5{:}17)$

and $\quad s_{it} = s_{iL}; \quad t = 1, 2, \ldots \qquad (5{:}18)$

which inserted into (5:7) gives

$$s_{iL} = \frac{1-\theta_i}{\delta_i} q_{iL}. \tag{5:19}$$

By substituting (5:19) into (5:10) we obtain

$$p_{i0}\, q_{iL} = \frac{\alpha_i + \theta_i - 1}{\theta_i} p_{i0}\, q_{iL} + \beta_i \left(y_0 - \sum_k \frac{\alpha_k + \theta_k - 1}{\theta_k} p_{k0}\, q_{kL} \right) \tag{5:20}$$

$$\text{Let } a_i = \frac{\alpha_i + \theta_i - 1}{\theta_i}; \qquad i = 1, \ldots, n \tag{5:21}$$

eq. (5:20) can then be rewritten in matrix form as

$$\begin{Bmatrix} p_{10}\, q_{1L} \\ p_{20}\, q_{2L} \\ \vdots \\ p_{n0}\, q_{nL} \end{Bmatrix} = \begin{Bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{Bmatrix} y_0 + \begin{Bmatrix} (1-\beta_1)\, a_1 & -\beta_1\, a_2 & \ldots & -\beta_l\, a_n \\ -\beta_2\, a_2 & (1-\beta_2) a_2 & \ldots & -\beta_2\, a_n \\ \vdots & \vdots & \vdots\vdots\vdots & \vdots \\ -\beta_n\, a_1 & -\beta_n\, a_2 & & (1-\beta_n)\, a_n \end{Bmatrix} \begin{Bmatrix} p_{10}\, q_{1L} \\ p_{20}\, q_{2L} \\ \vdots \\ p_{n0}\, q_{nL} \end{Bmatrix} \tag{5:22a}$$

or in a more compact notation,

$$X = By_0 + AX \tag{5:22b}$$

This is a system of linear equations which can be solved for X. The equilibrium values are

$$X = (I-A)^{-1} By_0 \tag{5:23}$$

The long-run demand functions thus turn out to be Bergson functions; Bergson (1936). That is, all long-run income elasticities are unity, all own price elasticities minus one and all cross price elasticities zero.

Experience from previous studies shows that Bergson functions are not an acceptable description of short-run consumer behaviour. However, as an ultimate goal in the long run, they cannot be dismissed as unrealistic. This property certainly lends a degree of stability to the model and what matters is how quickly and on what path the long-run values are attained.[2]

[2] In principle the proportional habit formation could be generalized to a linear habit formation, but it is doubtful if this more general model could be successfully estimated with the present data, see below.

These results were obtained under the assumption that the model would converge towards an equilibrium. Eqs. (5:10) and (5:14) are such complex difference equations that it is difficult to show analytically that this assumption holds. At the end of this chapter, the convergence properties are demonstrated numerically for parameter values selected according to the estimates obtained below.

5.3 Estimation and Empirical Results

The demand functions (5:10) are not readily estimated because stocks are not directly observed. Houthakker and Taylor, Mattei and Phlips had the same problem. We will, however, prefer an estimation approach which differs from those used in previous work. This involves augmenting the quality of our data set by new data.

The data used are primarily the same as described in Chapter 2 and Chapter 3, Section 3. They are aggregated into four commodities, "food, beverages and tobacco", "housing services", "clothing" and "other goods and services". The sample period begins in 1950, but it can be extended back to 1931 for the same four aggregate commodities. However, the quality of the data for the period 1931–1949 is lower than for the post-war period. The old data should not be used merely by adding them to the data set beginning in 1950. We prefer to use the same sample as before, but to incorporate the series of consumption volumes which can be derived for the period 1931–1949 as exogenous information in order to calculate stock values. Another reason for not extending the sample period further back than 1950 is that structural changes in demand probably occurred during and after the war.[3]

If data on consumed volumes were available for a long time period and if depreciation rates were known, then stocks could be calculated from eq. (5:9). As an approximation it is now suggested that only 17 terms be included in the sum on the r.h.s. of the equality sign. For reasonable values of δ_i, additional terms would add little to the stock estimates. It would thus be possible to obtain stock estimates for 1948 and later years. The depreciation parameters θ_i and δ_i are not known, however, and have to be estimated jointly with all of the other parameters.

Relatively simple assumptions were made about the stochastic error terms added to eq. (5:10). Contemporaneous correlations across equations were permitted and it was recognized that the error moment matrix is singular because of the budget constraint, but neither heteroscedasticity nor autocorrelation was assumed. Along with the additional assumption of normally distributed errors the likelihood is thus a function of all α_i, β_i, θ_i and δ_i as well as of the second order moments, conditional on total

[3] Data do not permit a formal test with the present model.

expenditures, all prices and consumption of each commodity in constant prices before 1950. The likelihood function was programmed using the generalized inverse suggested in Deaton (1975) and maximized by a quasi-Newton procedure from the Harwell subroutine library (VA06AD). Estimates of asymptotic standard errors were obtained from the information matrix. The first two commodities, "food, beverages and tobacco" and "housing services", do not include durable goods and thus, for these commodities the θ_i and δ_i parameters equal 1. As a check on the model, it is also estimated without this constraint. The third commodity "clothing" is a durable, presumably with a relatively high rate of depreciation. The fourth commodity is a mixture of major durables, nondurables and services. The share of durables can be estimated at approximately 35 percent. If we assume that 1 percent of the stocks of major durables such as cars or refrigerators remains after 10 years, which is equivalent to a rate of depreciation of 0.37, the average depreciation rate for the fourth commodity would become 0.78.

Since all purchases are not made at the beginning of a year, the model allows for a depreciation rate of current purchases θ_i which differs from the depreciation rate of stocks δ_i. If $\theta_i=1$, then also $\delta_i=1$ and if θ_i is close to 0 the same should be true for δ_i, but in the interval (0,1) $\theta_i<\delta_i$. This suggests that one parameter could be made a function of the other which would add a few degrees of freedom. One possibility is to assume

$$\theta_i = \delta_i^2; \qquad i = 1, \dots, n \tag{5:24}$$

The model (5:10) was estimated both with and without this additional assumption.

Estimates of four model versions are shown in Tables 5.1 and 5.2. When the θ_i-parameters were not constrained to the δ_i-parameters implausible estimates of δ_i were obtained. Only δ_{IV} is less than one. All of the estimates are very unreliable, however. The estimates did not improve when no stocks of the first two commodities were assumed to accumulate. However, when θ_i was constrained to the square of δ_i the results were in better aggrement with *a priori* conceptions (Table 5.2). The standard errors of the estimates of the depreciation rates are still uncomfortably high, but all point estimates are now less than one and the estimates for the last two commodities are lower than those for the first two. When δ_{I} and δ_{II} are constrained to 1, the estimated standard errors drop somewhat and the point estimates of δ_{III} and δ_{IV} are quite realistic. Except for commodity II, housing services, the habit parameter estimates are less than one and show only insignificant differences. The high estimate of α_{II} might be the result of a misspecification (cf. Chapter 4), but the hypothesis that α_{II} is less than one cannot be rejected in this model either. The estimates of the marginal propensities to consume are all plausible. The implied elasticities differ only

Table 5.1 *Estimates of the dynamic demand model (5:10) with unconstrained θ_i*

Parameter or statistic	Commodity I	II	III	IV	Commodity I	II	III	IV
α_i	0.992	1.020	0.947	1.008	0.985	1.013	0.933	0.999
	(0.013)	(0.008)	(0.038)	(0.025)	(0.016)	(0.011)	(0.042)	(0.031)
β_i	0.185	0.035	0.244	0.535	0.189	0.047	0.226	0.538
	(0.062)	(0.030)	(0.066)	(0.046)	(0.061)	(0.031)	(0.062)	(0.051)
θ_i	0.885	0.897	0.668	0.542	1.0	1.0	0.728	0.568
	(0.178)	(0.069)	(0.125)	(0.207)	–	–	(0.133)	(0.249)
δ_i	1.223	1.741	1.169	0.433	1.0	1.0	1.179	0.405
	(0.820)	(0.133)	(0.290)	(0.285)	–	–	(0.367)	(0.305)
I_i	48	26	184	27	46	31	170	32
$R^2_{i,w}$	0.947	0.903	0.554	0.987	0.949	0.882	0.590	0.985
DW_i	1.67	1.68	1.23	2.00	1.95	1.92	1.47	2.33
w_{i71}	0.337	0.181	0.084	0.398	0.337	0.181	0.084	0.398
$\hat{w}_{i,71}$	0.339	0.181	0.082	0.397	0.339	0.180	0.083	0.398
$w_{i,72}$	0.339	0.171	0.085	0.406	0.339	0.171	0.085	0.406
$\hat{w}_{i,72}$	0.342	0.169	0.085	0.405	0.341	0.168	0.085	0.405
$I_{i,71/72}$	5	5	9	0	3	10	2	0

Note: $R^2_{i,w} = 1 - \sum_{t=50}^{70} (w_{it} - \hat{w}_{it})^2 \Big/ \sum_{t=50}^{70} (w_{it} - \bar{w}_{it})^2$

slightly from previous results as we indicated in Table 5:3. This table shows estimated elasticities for the model version with θ_i constrained to the square of δ_i and the first two δ_i *a priori* set at 1. As compared to the income elasticities in Table 3.13 for the linear expenditure system with habit formation, the elasticity for food, beverages and tobacco is now lower, while the elasticities for housing services and clothing are somewhat higher. The compensated price elasticities are as low in absolute value as those in the ordinary LESH model. Since the consumption elasticities differ from the purchase elasticities by the factor θ_i the former are lower for the last two commodities which include durables.

In spite of the increased number of parameters as compared to an ordinary linear expenditure system with proportional habit formation the fit is not quite as good (cf. Tables 3.3, 3.6, 5.1, 5.2 and 5.4). But all of the models fit the data closely and it is hardly possible to distinguish between them according to fit criteria. The forecasting performance is extremely good—better than for any of the models analysed in Chapter 3. The Durbin-Watson statistics, which can only be used descriptively for a dynamic model such as this, are all comfortably high.

We also tried to estimate a model version in which θ_i was constrained to equal δ_i. This attempt was not very successful. It was difficult to find a maximum of the likelihood function and the point which was finally accept-

Table 5.2 *Estimates of the dynamic demand model (5:10) with* $\theta_i = \delta_i^2$

Parameter or statistic	Commodity I	II	III	IV	Commodity I	II	III	IV
α_i	0.987	1.020	0.943	0.987	0.985	1.012	0.939	0.982
	(0.018)	(0.090)	(0.046)	(0.045)	(0.018)	(0.011)	(0.042)	(0.040)
β_i	0.182	0.033	0.217	0.569	0.180	0.049	0.211	0.560
	(0.065)	(0.032)	(0.063)	(0.053)	(0.062)	(0.030)	(0.058)	(0.052)
θ_i	0.914	0.758	0.656	0.689	1.0	1.0	0.696	0.748
δ_i	0.956	0.871	0.810	0.830	1.0	1.0	0.834	0.865
	(0.107)	(0.097)	(0.092)	(0.126)	–	–	(0.088)	(0.116)
I_i	47	30	173	32	47	31	170	33
$R^2_{i,w}$	0.947	0.887	0.585	0.985	0.947	0.883	0.592	0.985
DW_i	1.75	1.61	1.49	2.15	1.93	1.92	1.64	2.26
w_{i71}	0.337	0.181	0.084	0.398	0.337	0.181	0.084	0.398
$\hat{w}_{i,71}$	0.340	0.182	0.083	0.396	0.340	0.180	0.083	0.397
$w_{i,72}$	0.339	0.171	0.085	0.406	0.339	0.171	0.085	0.406
$\hat{w}_{i,72}$	0.342	0.170	0.085	0.404	0.342	0.168	0.085	0.405
$I_{i,71/72}$	5	1	4	1	5	10	2	0

Note: $R^2_{i,w} = 1 - \sum_{t=50}^{70} (w_{it} - \hat{w}_{it})^2 \Big/ \sum_{t=50}^{70} (w_{it} - \bar{w}_{it})^2$

ed as a maximum gave estimates which were implausible in an economic sense. For instance, the rate of depreciation for clothing was as low as 0.007! The fit, however, was the best obtained for any model, $I_{50\text{–}70}$ was 149. These results prompted us to look for local maxima, but without success. We also checked the model version where $\theta_i = \delta_i^2$ and without any additional constraint on δ_i, for local maxima. But the maximization routine converged to the same point from three different starting points after 200–300 iterations.

Table 5.3 *Income- and compensated own-price elasticities (1960)*

Elasticity	Commodity I	II	III	IV
Purchase elasticities				
e_{i0}	0.513	0.241	2.082	1.627
E_{ii0}	−0.005	−0.025	−0.125	−0.031
Consumption elasticities				
e^*_{i0}	0.513	0.241	1.449	1.217
E^*_{ii0}	−0.005	−0.025	−0.087	−0.023

Note: These elasticities were calculated from the estimates of the model version with $\theta_i = \delta_i^2$ and $\delta_1 = \delta_2 = 1$.

Table 5.4 *Goodness of fit for all commodities*

Model version	$\bar{I}_{50-70}$	R^2_w	R^2_{pq}	I_{71}	I_{72}	$\bar{I}_{71-72}$
$\theta_i = \delta_i^2$						
δ_i unconstrained	229	0.9471	0.9989	18	20	19
$\delta_1 = \delta_2 = 1$	228	0.9469	0.9989	15	38	26
θ_i unconstrained						
δ_i unconstrained	232	0.9474	0.9990	26	25	26
$\delta_1 = \theta_1 = \delta_2 = \theta_2 = 1$	226	0.9477	0.9990	11	32	22

Note: $R^2_w = 1 - \sum_{i=I}^{IV} \sum_{t=50}^{70} (w_{it} - \hat{w}_{it})^2 \Big/ \sum_{i=I}^{IV} \sum_{t=50}^{70} (w_{it} - \bar{w}_{it})^2$

$$R^2_{pq} = 1 - \sum_{i=I}^{IV} \sum_{t=50}^{70} (p_{it} q_{it} - \widehat{p_{it} q_{it}})^2 \Big/ \sum_{i=I}^{IV} \sum_{t=50}^{70} (p_{it} q_{it} - \overline{p_{it} q_{it}})^2$$

The long-run properties of the model were derived given the assumption that it converged towards equilibrium. In order to investigate whether this assumption holds, a few deterministic simulations were performed, based on the estimates in Table 5.2. The results from two of these simulations are exhibited in Table 5.5 and Figure 5.1. All of the simulations were started from the year 1970 and run for 500 years. The volumes, stocks and expenditure shares simulated using the estimates in Table 5.2 for the model version with no stock accumulation of the first two commodities—model version A in Table 5.5 and the broken curves in Figure 5.1—do not tend towards long-run equilibria, but follow explosive paths. The reason is that α_2 is greater than one; cf. Pollak (1970). If the value of α_2 is changed to 0.98—model version B in Table 5.5 and the unbroken curves in Figure 5.1—the model converges, but rather slowly. It does not begin to come close to the long-run values until after 100–150 years. Similar results were also obtained with other parameter values. The dynamic and long-run properties of the model thus depend on good estimates of the habit parameters. However, if the model is to be used for short or medium-term forecasting its long-run behaviour is only of secondary interest.

As Phlips' model has been considered superior to the model developed by Houthakker & Taylor a comparison between the results reported in this chapter and the corresponding results for Phlips' model would be of interest. However, in spite of repeated attempts to estimate Phlips' model we were not very successful. The iterative method used by Phlips did not converge.[4] We also tried our own approach. The expenditure functions of

[4] Phlips had the same experience with Belgian data. He was only able to estimate the model when leisure was one of the commodities (personal communication).

Table 5.5 *Simulated long-run behaviour of model (5:10)*

No of years after 1970	Volumes by commodity				Stocks by commodity[a]	
	I	II	III	IV	III	IV
Modelversion A						
0	2.8134	1.4946	0.8215	3.4496	0.0254	0.0675
10	3.7538	2.4540	0.7234	4.0878	0.2330	0.9413
20	3.5126	2.8609	0.5761	4.0695	0.1847	0.9395
30	3.2682	3.3091	0.4728	3.9690	0.1512	0.9184
100	1.3697	8.1011	−0.0216	1.5698	−0.0037	0.3765
200	−3.7380	24.7175	−1.4474	−8.5132	−0.4439	−1.9192
300	−17.3743	70.6978	−5.3944	−36.9099	−1.6624	−8.3878
Modelversion B						
0	2.8134	1.4946	0.8215	3.4496	0.0254	0.0675
10	3.9071	1.8322	0.8300	4.4497	0.2630	1.0149
20	3.8320	1.6234	0.7596	4.8040	0.2394	1.0988
30	3.7620	1.4515	0.7185	5.0870	0.2258	1.1657
100	3.4516	0.8624	0.6621	6.0430	0.2069	1.3911
200	3.3179	0.6970	0.6609	6.3433	0.2065	1.4618
300	3.2894	0.6752	0.6610	6.3935	0.2066	1.4737
∞	3.2820	0.6719	0.6611	6.4040	0.2407	1.8688

Modelversion A: Simulations based on the estimates in Table 5.2 with $\theta_1=\theta_2=1$. Thus, $\alpha_2=1.01228$; no stocks of commodities I and II accumulate.
Modelversion B: Ditto, but $\alpha_2=0.98$.
[a] Lagged 2 years.

Table 5.6 *Estimates of a modified Phlips' model*

Parameter or statistic	Commodity			
	I	II	III	IV
α_i	0.755	0.719	0.059	0.236
	(0.186)	(0.151)	(0.029)	(0.100)
β_i	0.207	0.015	0.184	0.593
	(0.049)	(0.030)	(0.044)	(0.052)
δ_i	0.777	0.697	0.007	0.222
	(0.187)	(0.156)	(0.005)	(0.125)
I_i	38	30	84	38
$R^2_{i,w}$	0.9582	0.8851	0.8048	0.9820
DW_i	1.79	1.60	1.86	1.88
$w_{i,71}$	0.337	0.181	0.084	0.398
$\hat{w}_{i,71}$	0.339	0.182	0.083	0.396
$w_{i,72}$	0.339	0.171	0.085	0.406
$\hat{w}_{i,72}$	0.340	0.170	0.085	0.405
$I_{i,71/72}$	2	1	5	1

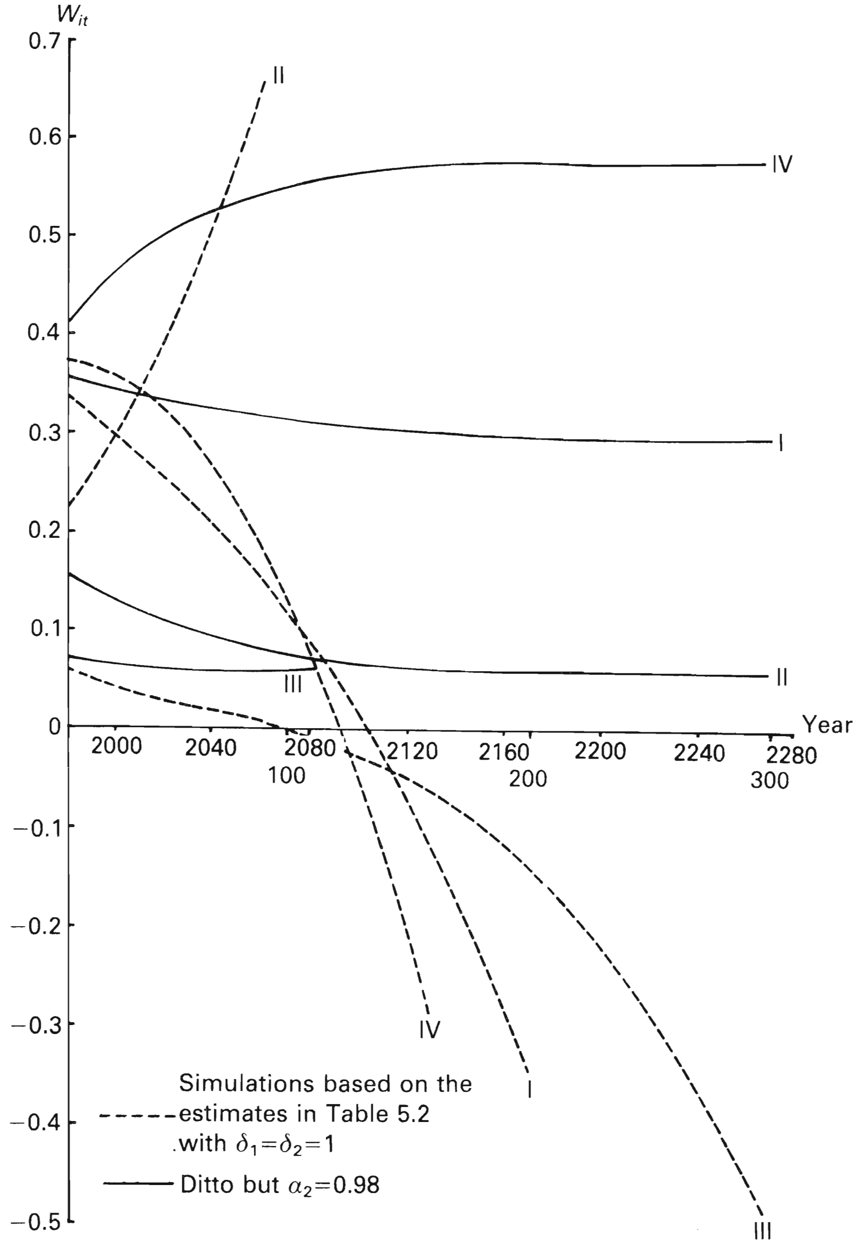

Figure 5.1 *The dynamic and long-run behaviour of model (5:10).*

Phlips' model can be written as functions of the state variables in a form similar to eq. (5:10)

$$p_i q_i = c_i p_i + \alpha_i p_i s_i + \beta_i \left(y - \sum_k c_k p_k - \sum \alpha_k p_k s_k \right). \tag{5:25}$$

All attempts to estimate these functions, with s_i approximated by both the average of the stock estimates for two adjacent years and the stock estimate with a lag of one year, were unsuccessful. It was not possible to find a maximum of the likelihood function. We were not able to estimate the model unless all c_i were set at zero and the state variables were defined as in eqs. (5:8) and (5:9). Since these additional constraints make Phlips' model very similar in structure to model (5:10) with $\delta_i = \theta_i$ it does not come as a surprise that the estimates of the two models are almost the same. The fit is excellent and the predictions are at least as good as for model (5:10). The overall inaccuracy is 149 for the sample period and 14 for the prediction period. However, not all of the parameter estimates conform to *a priori* expectations. All the estimated depreciation rates are very small, particularly for commodity III. The estimates of the state parameters α_i are all positive, which indicates that habit formation dominates the stock effect. The point estimates for the first two commodities, for which there is no stock effect, are much lower than the corresponding estimates of model (5:10). The marginal propensities to consume agree quite well with previous results. The estimated asymptotic standard errors for $\hat{\alpha}_i$ and $\hat{\delta}_i$ in particular are much higher in Phlips' model than in model (5:10).

6 Concluding Remarks

Throughout this study we have frequently alluded to the problems which arise—as in other analyses based on aggregate data—due to the low information content of the data. The estimates become uncertain and tests lack in power. The likelihood function sometimes exhibits such flat surfaces that we encounter numerical difficulties in mazimizing it. It thus becomes very difficult to discriminate between rival models. In order to alleviate this difficulty, the model evaluation was based not only on fit criteria but also on predictive performance and an evaluation of the parameter estimates in relation to prior economic conceptions. The difficulties in collecting meaningful and reliable data, as discussed in Chapter 2, might create a pessimistic attitude towards the application of sophisticated econometric models and methods. Indeed, improved data is a high priority issue and some improvements have already been achieved which do not justify too much pessimism. The results reported in Chapter 3 also indicate that models based on economic theory do perform better than naive models.

It was also shown in Chapter 3 that the linear expenditure system with habit formation was one of the best models according to all criteria, while the ordinary linear expenditure function was one of the worst. Both models originate from an additive utility function. Other nonadditive models did not produce better results than the linear expenditure system with habit formation, which indicates that the assumption of additivity might not be too critical. This result contrasts with those of other studies, but is should be kept in mind that these studies have normally not included additive models with habit formation. It might be argued, however, that our result is obtained because only relatively aggregate commodities were used and that further disaggregation might give different results. Nevertheless, it is interesting to note that in this analysis, the assumption of additivity is equally good at all three levels of aggregation. The methods used to group goods into commodities might also contribute to the high ranking of the linear expenditure system with habit formation. When close substitutes and goods with parallel price changes are grouped together, substitution between commodities is reduced. This leads to commodities, which show a relatively low variation in expenditure shares and prices over time.

Our results certainly do not imply that an additive model would adequately represent the effects of a major change in relative prices, but they do imply that for the modest and gradual changes normally observed, an additive model might perform well. Furthermore, this study indicates that there is not enough price variation in our aggregate data to estimate more general nonadditive models. Although the assumption of an additive utility function might introduce a specification error, the resulting systematic error component in the forecasts is compensated by less random fluctuation.

In the present state of the art, numerical and statistical problems make it very difficult to estimate a general demand system for a large number of commodities. In practice, strong *a priori* constraints such as additivity—see e.g. Deaton (1975)—or those of a utility tree would have to be imposed. The latter approach might seem more attractive. It implies that the consumer first allocates total expenditures to a few major aggregates, each corresponding to a branch of the utility tree, and then distributes the resulting subtotal to the commodities of each aggregate. The allocation within an aggregate only depends on the relative prices of those commodities which belong to that particular aggregate. The computational problem is thus reduced substantially since a demand system can be estimated independently for each aggregate. It is also possible, at least in principle, to use a more general demand model at the detailed level of aggregation where substitution between commodities is more likely to be strong. Furthermore, the submodels used do not have to belong to the same family.

This approach was used in a recent study based on Swedish national accounts data for 60 commodities, see Flood and Klevmarken (1980). A linear expenditure system with proportional habit formation was estimated for ten major aggregates. The indirect translog model was then used to explain demand in six of these aggregates, the linear expenditure system with habit formation in two and models not explicitly based on utility functions in the remaining two aggregates. The results, although preliminary, are somewhat contradictory. On the aggregate level, a good fit was obtained and the estimated elasticities had the expected signs and magnitudes, but for some commodities on the disaggregate level a few atheoretical results were obtained. For instance, for the aggregate commodity "vegetabilia" which included three commodities, "bread and other grain products", "fruits and vegetables other than roots" and "potatoes and other roots" the estimated compensated own price elasticities were significantly positive for the first two commodities. In order to check whether this result might depend on the particular model used, i.e. the indirect translog model, estimates were also obtained for a Rotterdam system. The same anomaly, however, also appeared for this model. On more than one occasion it was difficult to find a maximum of the likelihood function for the ITRL model and the estimates were not always plausible. In one case, "furniture and home appliances", an optimum could not be found at all for the ITRL model, whereas LESH gave a good fit and reasonable estimates. It is possible that these results are unique for the particular data set used, but more likely, they indicate that attempts with more general models to capture at least some of the substitution between disaggregate commodities will meet with more difficulties than expected, merely owing to the size of the problem.

Our results also suggest that the most important issue in demand analysis is *not* that of finding the most general nonadditive utility function which yields convenient demand functions, but rather how the constraints imposed on demand by particular characteristics of each commodity market can be used to the best advantage. It is of course beneficial if this can be done within the framework of a complete system so that the constraints of the classical demand theory are not put aside. In this study attempts were made to incorporate two particular commodity and market characteristics, i.e. imbalance in the housing market and stocks of consumer durables.

As a substitute for reliable data on the magnitude of excess demand for housing a model for the supply of housing services was suggested. In its simplest form, this model did not use any more data than the corresponding demand model and given the existing quality of the data, it is perhaps no great surprise that there was poor support for this model. It is difficult to improve on an R^2 of 0.998! Another contributing factor might be that the data did not permit a separation between the controlled and the non-controlled segments of the market.

The models analysed in Chapter 5 include stocks of durables and habit formation, i.e. the effects of past behaviour, but the planning horizon is limited to one time period. From a theoretical point of view, it is of course desirable to generalize these models in the framework of intertemporal utility maximization; but from an empirical point of view this will probably not become very rewarding until more ample data are available. Several models already fit to data closely and the numerical problem of finding optimal estimates is still an obstacle for the highly nonlinear models with many parameters.

In spite of these difficulties the results in this chapter show that it is possible to obtain meaningful estimates of the effects of stocks and habits on demand. There is also an improvement in predictive ability. Although total private consumption in Sweden declined in 1971 for the first time since World War II, the predictions are almost perfect. However, a thorough analysis of this surprisingly good result would require more information about the distributional properties of the predictions. It has also been shown that the effects of habits can be separated from those of stocks. But this result is based on an economic interpretation of the point estimates rather than on a formal test.

Although desirable, it might be difficult to include explanatory factors unique to a commodity or market in a system of complete demand functions. The properties of a classical demand model might be violated. The more disaggregated the data, the more desirable—but also the more difficult

—it becomes to include exogenous variables other than income and prices. The complete systems approach can never replace a detailed analysis of particular commodities and markets. Given the present state of the art and scarcity of data, the complete systems approach is probably best suited for forecasting and policy evaluation on an aggregate level.

Appendix A

Time-series data, Parameter estimates

Tables

Table A.1. *Price indices and expenditure series for all goods grouped into 4 commodities, 1931–1972*

	Price indices by commodity (1964=100)				General price index (1964=100)	Expenditures by commodity (Current prices; Mill. Skr)				Total private consumption	Population (thousands)
Year	P_{I}	P_{II}	P_{III}	P_{IV}	P	$P_{I}Q_{I}$	$P_{II}Q_{II}$	$P_{III}Q_{III}$	$P_{IV}Q_{IV}$	Y	
1931	21.1	54.6	34.1	33.6	31.9	2,637	1,958	763	1,866	7,224	6,162
1932	20.2	54.6	33.2	33.6	31.3	2,551	1,958	696	1,789	6,995	6,190
1933	21.1	53.4	32.4	34.6	31.8	2,456	1,950	667	1,803	6,876	6,212
1934	21.1	52.3	33.2	34.6	31.7	2,525	1,986	782	1,949	7,242	6,233
1935	22.1	52.3	32.4	34.6	31.6	2,715	2,042	822	2,131	7,710	6,251
1936	22.1	52.3	32.4	34.6	31.6	2,864	2,073	898	2,302	8,137	6,267
1937	23.0	52.3	34.1	35.6	32.5	3,071	2,144	979	2,410	8,603	6,285
1938	24.0	52.3	35.0	36.7	33.4	3,220	2,216	994	2,680	9,110	6,310
1939	25.0	53.4	35.9	36.7	34.1	3,479	2,310	1,101	2,816	9,706	6,341
1940	29.8	59.0	43.1	40.7	39.2	3,831	2,508	1,133	2,631	10,102	6,371
1941	34.6	62.3	53.0	45.8	44.4	4,244	2,588	1,257	2,938	11,028	6,406
1942	39.4	63.5	57.5	49.9	48.5	4,511	2,743	1,305	3,266	11,825	6,458
1943	39.4	64.6	58.4	51.9	49.3	4,799	2,732	1,315	3,568	12,413	6,523
1944	40.3	64.6	57.5	51.9	49,6	5,364	2,796	1,451	3,841	13,452	6,597
1945	40.3	63.5	57.5	53.0	49.8	5,572	2,901	1,789	4,043	14,305	6,674
1946	40.3	63.5	57.5	54.0	50.0	6,380	3,207	2,281	4,666	16,535	6,764
1947	42.2	65.7	58.4	54.0	51.4	6,817	3,420	2,465	5,146	17,848	6,842
1948	46.1	66.8	60.2	59.1	55.0	7,397	3,622	2,736	5,787	19,541	6,925
1949	46.1	67.9	61.1	60.1	55.5	7,592	3,759	2,672	5,857	19,880	6,986
1950	48.0	66.8	65.6	60.1	56.8	8,129	4,031	2,570	6,404	21,134	7,042
1951	55.0	71.1	80.7	67.7	64.4	9,220	4,472	2,992	7,418	24,102	7,099
1952	61.3	72.2	83.1	73.1	69.2	10,541	4,801	2,796	8,091	26,229	7,151
1953	62.5	74.6	80.0	73.3	70.0	10,737	5,066	3,081	8,596	27,480	7,192
1954	63.8	74.8	80.9	73.3	70.6	11,214	5,420	3,050	9,403	29,087	7,235
1955	66.8	76.6	77.8	75.0	72.5	11,889	5,742	3,262	9,908	30,801	7,290
1956	71.4	84.4	78.2	77.4	76.5	12,790	6,477	3,421	10,444	33,132	7,341
1957	74.7	87.4	79.6	80.9	79.7	12,996	6,878	3,557	11,493	34,924	7,393
1958	77.6	92.9	80.6	83.8	82.9	13,694	7,637	3,601	12,379	37,311	7,436
1959	77.8	95.6	81.3	84.8	82.9	13,965	8,044	3,775	13,474	39,258	7,471
1960	82.4	96.9	85.1	88.0	87.3	14,762	8,501	4,247	14,429	41,939	7,498
1961	84.5	98.6	95.0	90.0	90.1	15,652	8,898	4,580	15,619	44,749	7,542
1962	91.0	97.7	96.9	94.1	93.9	17,123	9,103	4,944	17,041	48,211	7,581
1963	97.0	98.0	98.7	96.8	97.3	18,357	9,440	5,422	18,915	52,134	7,628
1964	100.0	100.0	100.0	100.0	100.0	19,662	9,922	5,946	21,230	56,760	7,695
1965	105.9	104.9	104.2	104.8	105.1	21,365	10,875	6,239	23,696	62,175	7,773
1966	112.8	116.7	108.7	110.8	112.3	23,356	12,496	6,339	25,291	67,482	7,843
1967	117.1	119.4	112.4	116.0	116.6	25,053	13,091	6,813	27,515	72,472	7,893
1968	119.1	122.1	112.6	117.6	118.4	26,187	13,838	6,902	30,004	76,931	7,932
1969	122.4	127.0	112.5	120.7	121.7	27,560	15,068	7,432	32,627	82,687	8,004
1970	130.7	135.0	116.5	126.6	128.5	29,715	16,305	7,734	35,291	89,045	8,081
1971	142.3	138.6	126.7	137.0	138.0	31,943	17,187	7,935	37,659	94,724	8,115
1972	153.5	136.5	133.2	144.1	144.7	34,773	17,541	8,691	41,632	102,637	8,129

Table A.2. *Price indices and expenditure series for food items grouped into 4 commodities, 1950–1972*

Year	Price indices by commodity (1964=100) P_A	P_B	P_C	P_D	General price index (1964=100) P	Expenditures by commodity (Current prices; Mill. Skr) $P_A Q_A$	$P_B Q_B$	$P_C Q_C$	$P_D Q_D$	Total food consumption Y	Population (thousands)
1950	49.0	60.7	40.1	38	48	2,656	1,150	1,662	729	6,197	7,042
1951	57.1	67.3	47.4	44	56	3,088	1,103	1,961	843	6,995	7,099
1952	63.3	74.9	56.6	49	63	3,417	1,339	2,290	951	7,997	7,151
1953	64.5	72.1	58.8	51	64	3,446	1,378	2,293	996	8,113	7,192
1954	64.2	71.8	57.1	53	64	3,448	1,493	2,392	1,023	8,356	7,235
1955	67.2	78.6	61.8	55	68	3,604	1,579	2,597	1,073	8,853	7,290
1956	71.8	82.4	69.6	59	71	3,863	1,683	2,833	1,123	9,502	7,341
1957	71.1	86.7	72.1	63	73	3,805	1,716	2,857	1,175	9,553	7,393
1958	75.6	82.2	73.9	66	75	4,111	1,753	3,016	1,208	10,088	7,436
1959	76.4	82.5	74.4	69	76	4,235	1,771	3,061	1,230	10,297	7,471
1960	81.4	88.2	78.2	77	81	4,381	1,911	3,213	1,340	10,845	7,498
1961	83.7	87.7	82.4	82	84	4,515	1,998	3,512	1,444	11,469	7,542
1962	92.5	94.8	86.9	91	91	4,987	2,298	3,760	1,554	12,599	7,581
1963	98.0	99.2	94.8	98	97	5,272	2,425	4,098	1,723	13,518	7,628
1964	100.0	100.0	100.0	100	100	5,408	2,618	4,517	1,743	14,286	7,695
1965	107.2	104.1	106.6	110	107	5,763	2,908	4,805	1,977	15,453	7,773
1966	113.7	108.4	113.0	120	113	6,183	3,148	5,123	2,187	16,641	7,843
1967	118.3	112.1	117.0	132	118	6,507	3,378	5,492	2,465	17,842	7,893
1968	119.7	111.2	119.9	135	120	6,604	3,427	5,738	2,658	18,427	7,932
1969	123.8	116.5	123.5	141	125	6,853	3,545	5,996	2,935	19,329	8,004
1970	133.7	119.3	133.3	155	133	7,423	3,709	6,516	3,188	20,836	8,081
1971	144.4	133.0	143.5	173	145	8,274	4,107	6,847	3,230	22,458	8,115
1972	162.0	142.4	158.0	189	160	9,142	4,500	7,415	3,689	24,746	8,129

Table A.3. *Price indices and expenditure series for food items grouped into 8 commodities, 1950–1972*

Year	Price indices by commodity (1964=100)								General food price index (1964=100)
	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P
1950	42	40	68	63	43	40	47	38	48
1951	49	43	76	75	48	46	55	44	56
1952	57	49	84	81	52	56	60	49	63
1953	59	47	81	82	52	58	63	51	64
1954	59	46	81	81	52	56	63	53	64
1955	62	54	87	82	58	61	66	55	68
1956	64	70	86	86	68	70	68	59	71
1957	64	72	91	84	70	72	73	63	73
1958	70	66	87	81	79	73	79	66	75
1959	72	67	87	81	79	73	82	69	76
1960	77	82	90	86	84	77	85	77	81
1961	79	83	89	88	87	81	90	82	84
1962	92	94	95	90	97	85	97	91	91
1963	97	100	99	98	100	94	99	98	97
1964	100	100	100	100	100	100	100	100	100
1965	109	108	103	103	109	106	110	110	107
1966	119	113	107	106	113	112	118	120	113
1967	126	116	111	109	115	116	122	132	118
1968	130	122	108	109	113	120	119	135	120
1969	134	135	111	115	115	123	126	141	125
1970	146	137	114	123	123	133	135	155	133
1971	151	153	127	135	144	143	146	173	145
1972	172	164	136	142	167	158	158	189	160

Expenditures by commodity (Current prices; Mill. Skr)								Total food consumption	Population (thousands)
P_1Q_1	P_2Q_2	P_3Q_3	P_4Q_4	P_5Q_5	P_6Q_6	P_7Q_7	P_8Q_8	Y	
875	246	904	1,144	637	1,391	271	729	6,197	7,042
1,066	257	846	1,348	674	1,638	323	843	6,995	7,099
1,243	312	1,027	1,461	713	1,929	361	951	7,997	7,151
1,265	312	1,066	1,468	713	1,911	382	996	8,113	7,192
1,297	299	1,194	1,440	711	2,008	384	1,023	8,356	7,235
1,392	352	1,227	1,432	780	2,146	451	1,073	8,853	7,290
1,510	365	1,318	1,474	879	2,365	468	1,123	9,502	7,341
1,514	323	1,393	1,379	912	2,393	464	1,175	9,553	7,393
1,678	344	1,409	1,432	1,001	2,492	524	1,208	10,088	7,436
1,782	333	1,438	1,434	1,019	2,528	533	1,230	10,297	7,471
1,870	396	1,515	1,464	1,047	2,667	546	1,340	10,845	7,498
1,939	429	1,569	1,487	1,089	2,911	601	1,444	11,469	7,542
2,254	504	1,794	1,533	1,200	3,103	657	1,554	12,599	7,581
2,385	569	1,856	1,648	1,239	3,437	661	1,723	13,518	7,628
2,462	603	2,015	1,681	1,265	3,794	723	1,743	14,286	7,695
2,708	691	2,217	1,674	1,381	3,993	812	1,977	15,453	7,773
3,054	775	2,373	1,683	1,446	4,271	852	2,187	16,641	7,843
3,294	837	2,541	1,728	1,485	4,589	903	2,465	17,842	7,893
3,414	854	2,573	1,701	1,489	4,822	916	2,658	18,427	7,932
3,589	931	2,614	1,756	1,508	4,978	1,018	2,935	19,329	8,004
3,911	1,006	2,703	1,875	1,637	5,402	1,114	3,188	20,836	8,081
4,242	1,165	2,942	2,081	1,951	5,637	1,210	3,230	22,458	8,115
4,752	1,287	3.213	2,162	2,228	6,084	1,331	3,689	24,746	8,129

Table A.4. *Parameter estimates; all goods grouped into 4 commodities, 1950–1970*

Model/Parameter	Commodity			
	I	II	III	IV
Trend $-w$				
α_i	6.6144	1.1812	2.5972	−9.3928
	(0.5048)	(0.6217)	(0.3662)	(0.3523)
β_i	−0.0032	−0.0005	−0.0013	0.0050
	(0.0003)	(0.0003)	(0.0002)	(0.0002)
R^2	0.8899	0.1202	0.7098	0.9757
Auto $-w$				
α_i	0.0108	0.0290	0.0204	0.0063
	(0.0292)	(0.0243)	(0.0128)	(0.0153)
β_i	0.9632	0.8438	0.7844	0.9954
	(0.0802)	(0.1290)	(0.1245)	(0.0441)
R^2	0.8891	0.7041	0.6882	0.9659
CEDS $-\ln q$				
α_i	−4.2063	−5.2303	−5.6305	−4.9927
	(0.1006)	(0.0296)	(0.1027)	(0.0822)
e_i	0.2672	0.4371	0.3423	0.6823
	(0.0476)	(0.0136)	(0.0547)	(0.0417)
E_{ii}	−0.9430	0.0542	−0.3405	−0.2212
	(0.3266)	(0.1206)	(0.2422)	(0.6985)
R^2	0.8609	0.9889	0.9171	0.9905
CEDS $-\ln q-\varrho$				
α_i	3.4091	−4.5560	−3.8756	−2.3472
	(1.3807)	(0.5068)	(1.1390)	(2.9943)
e_i	0.2163	0.2275	0.3671	0.6830
	(0.0474)	(0.0863)	(0.0553)	(0.0372)
E_{ii}	−0.1484	−0.0264	−0.3922	−0.5748
	(0.3147)	(0.0925)	(0.2286)	(0.6357)
ϱ	0.7422	0.9328	0.2209	0.1726
R^2	0.9485	0.9956	0.9457	0.9937
CEDS $-\ln w$				
α_i	−0.7518	−1.6456	−1.7601	−1.4504
	(0.0958)	(0.0699)	(0.1799)	(0.0812)
e_i	−0.1414	−0.0392	−0.2777	0.2237
	(0.0463)	(0.0338)	(0.0871)	(0.0393)
E_{i1}	−0.7313	−6.7822	10.2092	0.9133
	(2.1014)	(1.5330)	(3.9480)	(1.7811)
E_{i2}	−0.7662	−2.6351	5.0297	0.4250
	(1.0460)	(0.7630)	(1.9651)	(0.8864)
E_{i3}	−0.3718	−1.8812	3.3453	0.2696
	(0.5431)	(0.3962)	(1.0204)	(0.4603)
E_{i4}	0.3186	−7.2045	6.5003	0.7895
	(1.7483)	(1.2754)	(3.2846)	(1.4818)
R^2	0.9240	0.9424	0.8910	0.9776

Model/Parameter	Commodity			
	I	II	III	IV
LESH$-pq$				
α_i	0.9798	1.0160	0.9221	0.9590
	(0.0207)	(0.0092)	(0.0565)	(0.0461)
β_i	0.2122	0.0338	0.1803	0.5736
	(0.0619)	(0.0264)	(0.0571)	(0.0591)
R^2	0.9987	0.9993	0.9839	0.9995
LESH$-w$				
α_i	0.9670	1.0232	0.8985	0.9415
	(0.0225)	(0.0116)	(0.0555)	(0.0544)
β_i	0.2370	0.0148	0.1935	0.5547
	(0.0636)	(0.0330)	(0.0612)	(0.0727)
R^2	0.9480	0.9509	0.6662	0.9814
LES$-w$				
c_i	0.0091	0.0233	0.0051	0.0148
	(0.0014)	(0.0012)	(0.0007)	(0.0047)
β_i	0.1930	0.1253	0.0965	0.5853
	(0.0084)	(0.0144)	(0.0071)	(0.0053)
R^2	0.5914	0.8385	0.7485	0.9888
LES$-w-\varrho$				
c_i	2.6513	1.4091	0.7916	3.1192
	(0.0963)	(0.1550)	(0.1067)	(0.5222)
β_i	0.1053	0.1866	0.1157	0.5924
	(0.0165)	(0.0676)	(0.0139)	–
R^2	0.8851	0.9108	0.6466	–
$\hat{\varrho}$	0.3594			
	(0.0880)			
RD$-w^*Dq$				
μ_i	0.1911	0.1613	0.0792	0.5684
	(0.0436)	(0.0216)	(0.0383)	–
π_{i1}	-0.0206			
	(0.0625)			
π_{i2}	0.0579	-0.0226		
	(0.0229)	(0.0153)		
π_{i3}	0.0210	0.0084	-0.0647	
	(0.0292)	(0.0152)	(0.0286)	
π_{i4}	-0.0583	-0.0437	0.0353	0.0667
R^2	0.1577	-0.4853	0.3244	0.7590
RDI$-w^*Dq$				
$\varkappa_i$	-0.0019	0.0057	-0.0045	0.0007
	(0.0035)	(0.0013)	(0.0030)	–
μ_i	0.2362	0.0156	0.2005	0.5477
	(0.0949)	(0.0363)	(0.0864)	–

Model/Parameter	Commodity			
	I	II	III	IV
π_{i1}	0.0023 (0.0706)			
π_{i2}	0.0331 (0.0208)	−0.0138 (0.0115)		
π_{i3}	0.0210 (0.0299)	0.0082 (0.0116)	−0.0666 (0.0281)	
π_{i4}	−0.0564	−0.0275	0.0374	0.0465
R^2	0.2741	0.1723	0.4116	–
ITRL$-w$				
α_i	−0.3670 (0.0035)	−0.1986 (0.0012)	−0.0975 (0.0018)	−0.3368 (0.0029)
β_{i1}	−0.0783 (0.1004)			
β_{i2}	−0.1077 (0.0373)	−0.1918 (0.0206)		
β_{i3}	−0.0747 (0.1356)	0.1141 (0.0372)	−0.0889 (0.0173)	
β_{i4}	−0.0491 (0.0333)	0.0207 (0.0141)	0.0278 (0.0341)	−0.1337 (0.0479)
R^2	0.8264	0.9062	0.9661	0.7622
ITRL$-w-\varrho$				
α_i	−0.3289 (0.0363)	−0.2488 (0.0326)	−0.0549 (0.0399)	−0.3674 (0.0321)
β_{i1}	−0.3381 (0.0725)			
β_{i2}	−0.0260 (0.0425)	−0.1953 (0.0280)		
β_{i3}	0.0225 (0.0305)	0.0194 (0.0174)	−0.0233 (0.0234)	
β_{i4}	0.1934 (0.0800)	0.0740 (0.0501)	0.0429 (0.0344)	−0.1234 (0.1127)
R^2	0.9446	0.9614	0.6928	0.9864
$\hat{\varrho}$	0.9339 (0.0300)			

Note:
R^2 is defined as

$$R_i^2 = 1-\sum_t e_{it}^2 \Big/ \sum_t (y_{it}-\bar{y}_i)^2;$$

where e_{it} is the residual and y_{it} the dependent variable for commodity i and year t. Since the residuals for a given commodity do not necessarily sum to zero, it is theoretically possible that $R_i^2<0$. Since the dependent variables are not the same in alla models, R^2:s are not necessarily comparable between models.

Table A.5. *Parameter estimates; food items grouped into 4 commodities, 1950–1970*

Model/Parameter	Commodity			
	A	B	C	D
Trend				
α_i	8.2159	−1.3383	−3.2661	−2.6115
	(0.4339)	(0.4451)	(0.3666)	(0.4447)
β_i	−0.0040	0.0008	0.0018	0.0014
	(0.0002)	(0.0003)	(0.0002)	(0.0002)
R^2	0.9398	0.3802	0.8328	0.6665
Auto−w				
α_i	0.0043	0.0904	0.0736	−0.0114
	(0.0266)	(0.0356)	(0.0275)	(0.0120)
β_i	0.9802	0.4917	0.7606	1.1043
	(0.0668)	(0.1994)	(0.0921)	(0.0953)
R^2	0.9228	0.2536	0.7913	0.8819
CEDS− ln q				
α_i	0.1444	−2.4022	−1.3463	2.8668
	(0.0938)	(0.2946)	(0.0886)	(0.3621)
e_i	−0.7874	2.1148	1.2728	2.3484
	(0.1542)	(0.4734)	(0.1426)	(0.5851)
E_{ii}	1.1412	−0.9464	−0.3691	−0.7819
	(0.5009)	(0.1493)	(0.0832)	(0.1337)
R^2	0.6609	0.9569	0.8169	0.6836
CEDS− ln $q-\varrho$				
α_i	4.4839	4.1620	0.0197	−5.2939
	(0.7892)	(1.9067)	(0.4919)	(1.3663)
e_i	0.1123	2.4757	1.3003	0.1671
	(0.1976)	(0.4787)	(0.1234)	(0.3418)
E_{ii}	0.3159	−0.7591	−0.3935	−0.2964
	(0.2650)	(0.1588)	(0.0687)	(0.2015)
ϱ	0.8680	0.4343	−0.0193	0.9900
R^2	0.9988	0.9955	0.9129	0.9809
CEDS− ln w				
α_i	−0.4576	−2.3866	−1.0823	−2.6768
	(0.0915)	(0.3225)	(0.1243)	(0.3254)
e_i	0.1746	2.0937	0.8475	2.0338
	(0.1483)	(0.5229)	(0.2016)	(0.5275)
E_{i1}	0.4710	−0.6318	−0.6103	−2.2024
	(0.2532)	(0.8926)	(0.3441)	(0.9004)
E_{i2}	0.0919	−0.5892	−0.2233	−0.3090
	(0.0805)	(0.2837)	(0.1094)	(0.2862)
E_{i3}	−0.1307	0.1944	−0.5888	−0.6377
	(0.0720)	(0.2538)	(0.0978)	(0.2560)

Model/Parameter	Commodity			
	A	B	C	D
E_{i4}	−0.1614 (0.0663)	0.2029 (0.2336)	0.0028 (0.0901)	−0.7789 (0.2357)
R^2	0.9837	0.5673	0.9281	0.8609
LESH−*pq*				
α_i	0.9380 (0.0186)	0.4209 (0.1684)	0.6259 (0.1117)	0.9057 (0.0763)
β_i	0.0851 (0.0331)	0.4149 (0.0475)	0.4483 (0.0486)	0.0516 –
R^2	0.9999	0.9996	0.9999	0.9992
LESH−*w*				
α_i	0.9427 (0.0190)	0.1094 (0.1383)	0.4296 (0.1072)	0.9589 (0.0899)
β_i	0.0574 (0.0224)	0.4469 (0.0371)	0.4816 (0.0436)	0.0141 –
R^2	0.9638	0.6756	0.9215	0.8882
LES−*w*				
c_i	8.8421 (0.2657)	0.8222 (0.1584)	3.4610 (0.2561)	−0.9037 (0.4735)
β_i	−0.2676 (0.0570)	0.3918 (0.0432)	0.3543 (0.0389)	0.5215 –
R^2	0.8714	0.2903	0.9184	0.6527
LES−*w*−ϱ				
c_i	0.8895 (0.0442)	0.0785 (0.0327)	0.3403 (0.0573)	0.2652 (0.0730)
β_i	−0.3507 (0.1162)	0.4815 (0.1120)	0.4487 (0.0863)	0.4205 –
R^2	0.9460	0.2382	0.8645	–
$\hat{\varrho}$	0.4631 (0.1054)			
RD−*w***Dq*				
μ_i	0.1614 (0.0579)	0.4088 (0.0946)	0.3498 (0.0718)	0.0800 –
π_{i1}	0.0802 (0.0445)			
π_{i2}	0.0004 (0.0244)	−0.0607 (0.0391)		
π_{i3}	−0.0166 (0.0278)	0.0307 (0.0273)	−0.0514 (0.0335)	
π_{i4}	−0.0640 (0.0256)	0.0296 (0.0198)	0.0373 (0.0214)	−0.0029 –
R^2	0.4163	0.5794	0.6706	–

Model/Parameter	Commodity			
	A	B	C	D
RDI$-w^*Dq$				
$\varkappa_i$	-0.0026	0.0013	0.0011	0.0002
	(0.0014)	(0.0021)	(0.0015)	–
μ_i	0.1663	0.4258	0.3424	0.0655
	(0.0565)	(0.0955)	(0.0722)	–
π_{i1}	0.0407			
	(0.0494)			
π_{i2}	-0.0280	-0.0336		
	(0.0287)	(0.0511)		
π_{i3}	0.0110	0.0389	-0.0686	
	(0.0306)	(0.0344)	(0.0367)	
π_{i4}	-0.0237	0.0226	0.0186	-0.0175
	(0.0359)	(0.0277)	(0.0256)	–
R^2	0.4813	0.6040	0.6884	–
ITRL$-w$				
α_i	-0.4005	-0.1773	-0.2999	-0.1222
	(0.0024)	(0.0025)	(0.0016)	(0.0015)
β_{i1}	-1.0786			
	(0.1486)			
β_{i2}	-0.3639	-0.1519		
	(0.1192)	(0.0445)		
β_{i3}	-0.5540	-0.2477	-0.5661	
	(0.1805)	(0.0768)	(0.1204)	
β_{i4}	-0.1328	-0.1096	-0.1651	-0.0885
	(0.0934)	(0.0339)	(0.0590)	(0.0273)
R^2	0.9501	0.3222	0.9208	0.8969
ITRL$-w-\varrho$				
α_i	-0.3891	-0.1873	-0.3019	-0.1217
	(0.0134)	(0.0143)	(0.0093)	(0.0068)
β_{i1}	-1.1747			
	(0.2082)			
β_{i2}	-0.2422	-0.2081		
	(0.1424)	(0.0836)		
β_{i3}	-0.5480	-0.2048	-0.5360	
	(0.2199)	(0.1204)	(0.1611)	
β_{i4}	-0.1954	-0.0901	-0.1708	-0.1688
	(0.1003)	(0.0526)	(0.0596)	(0.0202)
R^2	0.9747	0.5820	0.8736	0.9447
$\hat{\varrho}$	0.8674			
	(0.0574)			

Note: See note to Table A.4.

Table A.6. *Parameter estimates; food items grouped into 8 commodities, 1950–1970*

Model/Parameter	Commodity							
	1	2	3	4	5	6	7	8
Trend $-w$								
α_i	−3.885	−1.113	−0.2257	10.660	1.436	−2.644	−0.622	−2.611
	(0.249)	(0.237)	(0.4390)	(0.317)	(0.392)	(0.332)	(0.126)	(0.445)
β_i	0.0021	0.0006	0.0002	−0.0054	−0.0007	0.0015	0.0003	0.0014
	(0.0001)	(0.0001)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0001)	(0.0002)
R^2	0.9332	0.5544	0.0349	0.9831	0.3828	0.8002	0.5982	0.6663
Auto $-w$								
α_i	0.0231	0.0022	0.1045	−0.0027	0.0103	0.0719	0.0198	−0.0114
	(0.0124)	(0.0047)	(0.0321)	(0.0046)	(0.0101)	(0.0279)	(0.0079)	(0.0120)
β_i	0.8760	0.9555	0.2403	0.9853	0.8740	0.7175	0.6098	1.1043
	(0.0735)	(0.1168)	(0.2312)	(0.0324)	(0.1109)	(0.1121)	(0.1587)	(0.0953)
R^2	0.8875	0.7882	0.0566	0.9809	0.7753	0.6946	0.4508	0.8819
CEDS $-\ln q$								
α_i	−5.581	10.20	−0.5107	−10.38	−10.45	−0.2047	−0.2238	3.347
	(1.680)	(2.63)	(2.63)	(1.60)	(1.27)	(0.6050)	(1.1200)	(2.330)
e_i	0.0389	4.3555	1.3679	−1.0625	−1.0181	1.2893	1.6974	2.3496
	(0.4211)	(0.6599)	(0.6612)	(0.4009)	(0.3173)	(0.1520)	(0.2799)	(0.5841)
E_{ii}	0.3423	−1.0459	−0.9807	0.4598	−0.5587	−0.4633	−0.0868	−0.7814
	(0.1738)	(0.1589)	(0.1343)	(0.0935)	(0.1100)	(0.0783)	(0.2348)	(0.1334)
R^2	0.5296	0.7471	0.9455	0.8944	0.7224	0.8032	0.6743	0.6837
CEDS $-\ln w$								
α_i	−4.6391	−1.1118	1.2394	−7.2162	−1.4660	−3.2491	−1.2170	0.7292
	(1.4993)	(4.116)	(3.3998)	(1.3655)	(1.3390)	(0.8442)	(2.1872)	(2.0107)
e_i	0.2738	1.5138	1.8067	−0.2682	1.2416	0.5210	1.4417	1.7004
	(0.3766)	(1.0338)	(0.8539)	(0.3430)	(0.3363)	(0.2120)	(0.5493)	(0.5050)

E_{i1}	−0.2145	1.1317	0.1410	0.5719	−0.1170	−0.6628	−0.8423	−0.4058
	(0.3866)	(1.0614)	(0.8767)	(0.3521)	(0.3453)	(0.2177)	(0.5640)	(0.5185)
E_{i2}	0.0788	−0.5856	−0.0436	−0.2482	0.0010	−0.2203	0.2065	0.2884
	(0.1336)	(0.3667)	(0.3029)	(0.1216)	(0.1193)	(0.0752)	(0.1949)	(0.1791)
E_{i3}	−0.0487	−0.5364	0.1336	0.1408	0.3385	−0.3632	−1.1231	−0.1706
	(0.2675)	(0.7345)	(0.6066)	(0.2437)	(0.2389)	(0.1506)	(0.3903)	(0.3588)
E_{i4}	−0.1244	−0.2583	−0.5887	0.4657	0.1829	−0.2917	−0.7684	−0.0708
	(0.2788)	(0.7654)	(0.6322)	(0.2539)	(0.2490)	(0.1570)	(0.4067)	(0.3739)
E_{i5}	−0.0911	−0.1676	0.0450	0.6168	−0.2019	0.1745	−0.3854	−0.9250
	(0.1797)	(0.4933)	(0.4074)	(0.1636)	(0.1605)	(0.1012)	(0.2621)	(0.2410)
E_{i6}	0.1927	−1.0274	0.5537	−0.2112	−0.2931	−0.8301	−0.2483	−0.0128
	(0.2052)	(0.5633)	(0.4653)	(0.1868)	(0.1833)	(0.1155)	(0.2993)	(0.2752)
E_{i7}	0.3429	−1.2471	0.0151	−0.1921	0.1660	0.7830	0.0194	0.8089
	(0.3435)	(0.9429)	(0.7788)	(0.3128)	(0.3067)	(0.1934)	(0.5011)	(0.4606)
E_{i8}	−0.0272	−0.6400	0.3587	−0.2412	0.0255	0.1803	−1.4062	−0.7029
	(0.3379)	(0.9275)	(0.7661)	(0.3077)	(0.3017)	(0.1902)	(0.4929)	(0.4531)
R^2	0.9706	0.9038	0.5481	0.9975	0.9749	0.9658	0.8765	0.9464
LESH−pq								
α_i	0.7604	0.2732	0.2332	0.9708	0.7989	0.4312	0.6942	0.7426
	(0.0375)	(0.0836)	(0.1130)	(0.0117)	(0.0074)	(0.0575)	(0.0863)	(0.0401)
β_i	0.1084	0.0795	0.2698	0.0037	0.0431	0.3667	0.0406	0.0882
	(0.0179)	(0.0075)	(0.0269)	(0.0037)	(0.0026)	(0.0293)	(0.0106)	–
R^2	0.9999	0.9949	0.9985	0.9995	0.9997	0.9999	0.9994	0.9989
LESH−w								
α_i	0.8107	0.4530	−0.0096	0.9856	0.7852	0.3663	0.5644	0.8433
	(0.0423)	(0.0730)	(0.1039)	(0.0141)	(0.0135)	(0.0750)	(0.0533)	(0.0490)
β_i	0.0799	0.0537	0.3367	−0.0002	0.0445	0.3839	0.0536	0.0479
	(0.0171)	(0.0059)	(0.0246)	(0.0050)	(0.0035)	(0.0319)	(0.0109)	–
R^2	0.963	0.989	0.938	0.594	0.025	0.915	0.727	0.829

Model/Parameter	Commodity 1	2	3	4	5	6	7	8
LES−*w*								
c_i	2.2370	−0.0685	0.9806	2.9561	2.0102	3.3373	0.5369	1.6129
	(0.1529)	(0.0825)	(0.0938)	(0.0267)	(0.0395)	(0.1994)	(0.0418)	(0.0747)
β_i	0.1954	0.1691	0.3025	−0.1550	−0.0548	0.2947	0.0775	0.1706
	(0.0217)	(0.0133)	(0.0262)	(0.0194)	(0.0119)	(0.0223)	(0.0100)	–
R^2	0.790	0.976	−0.145	0.760	−1.425	0.874	0.486	0.668
RD−w^*Dq								
μ_i	0.0340	0.0585	0.3487	0.0820	0.0630	0.3031	0.0558	0.0549
	(0.0614)	(0.0446)	(0.1164)	(0.0319)	(0.0420)	(0.0866)	(0.0363)	–
π_{i1}	0.0093							
	(0.0250)							
π_{i2}	0.0231	−0.0203						
	(0.0115)	(0.0096)						
π_{i3}	−0.0309	0.0152	−0.0735					
	(0.0186)	(0.0129)	(0.0386)					
π_{i4}	0.0174	−0.0300	0.0081	0.0264				
	(0.0145)	(0.0080)	(0.0143)	(0.0172)				
π_{i5}	−0.0082	−0.0046	0.0038	0.0437	−0.0094			
	(0.0129)	(0.0077)	(0.0138)	(0.0107)	(0.0133)			
π_{i6}	−0.0078	0.0119	0.0337	−0.0285	−0.0142	−0.0742		
	(0.0196)	(0.0126)	(0.0261)	(0.0152)	(0.0144)	(0.0323)		
π_{i7}	0.0160	0.0106	−0.0007	−0.0249	0.0017	0.0144	0.0187	
	(0.0137)	(0.0078)	(0.0127)	(0.0110)	(0.0109)	(0.0122)	(0.0178)	
π_{i8}	−0.0189	−0.0058	0.0444	−0.0378	−0.0129	0.0391	−0.0357	0.0276
	(0.0201)	(0.0124)	(0.0229)	(0.0151)	(0.0142)	(0.0238)	(0.0148)	–
R^2	−0.0948	0.5202	0.4784	0.6934	−0.1304	0.6409	0.1717	–

RDI$-w^*Dq$								
$\varkappa_i$	0.0012	0.0002	−0.0002	−0.0038	−0.0002	0.0015	0.0017	−0.0004
	(0.0014)	(0.0009)	(0.0028)	(0.0011)	(0.0009)	(0.0018)	(0.0008)	–
μ_i	0.0364	0.0585	0.3347	0.0856	0.0624	0.3063	0.0599	0.0562
	(0.0634)	(0.0442)	(0.1153)	(0.0386)	(0.0353)	(0.0825)	(0.0289)	–
π_{i1}	0.0022							
	(0.0269)							
π_{i2}	0.0214	−0.0232						
	(0.0120)	(0.0100)						
π_{i3}	−0.0154	0.0125	−0.0522					
	(0.0277)	(0.0180)	(0.0622)					
π_{i4}	0.0268	−0.0135	−0.0348	−0.0252				
	(0.0151)	(0.0082)	(0.0195)	(0.0182)				
π_{i5}	−0.0056	−0.0011	0.0152	0.0131	−0.0142			
	(0.0135)	(0.0078)	(0.0177)	(0.0119)	(0.0140)			
π_{i6}	−0.0149	0.0076	0.0400	0.0236	−0.0075	−0.0807		
	(0.0203)	(0.0129)	(0.0340)	(0.0155)	(0.0137)	(0.0322)		
π_{i7}	0.0063	0.0039	0.0113	0.0051	0.0131	0.0047	0.0080	
	(0.0129)	(0.0075)	(0.0151)	(0.0113)	(0.0111)	(0.0113)	(0.0164)	
π_{i8}	−0.0208	−0.0077	0.0234	0.0048	−0.0130	0.0272	−0.0524	0.0385
	(0.0237)	(0.0139)	(0.0361)	(0.0199)	(0.0173)	(0.0262)	(0.0165)	–
R^2	−0.0421	0.5798	0.5608	0.7264	0.3149	0.7090	0.5650	–
ITRL$-w$, Ω								
α_i	−0.1726	−0.0367	−0.1391	−0.1349	−0.0957	−0.2489	−0.0500	−0.1220
	(0.0138)	(0.0012)	(0.0026)	(0.0016)	(0.0008)	(0.0021)	(0.0008)	–
β_{i1}	−0.2208							
	(0.0346)							
β_{i2}	−0.0243	−0.0008						
	(0.0126)	(0.0072)						
β_{i3}	−0.0169	−0.0096	−0.0436					
	(0.0252)	(0.0089)	(0.0265)					

Model/Parameter	Commodity 1950–1970							
	1	2	3	4	5	6	7	8
β_{i4}	−0.0485	0.0329	−0.0147	−0.2429				
	(0.0273)	(0.0126)	(0.0253)	(0.0330)				
β_{i5}	0.0011	0.0077	−0.0134	−0.0312	−0.8391			
	(0.0181)	(0.0068)	(0.0154)	(0.0140)	(0.0098)			
β_{i6}	−0.0840	0.0087	−0.0649	−0.0148	0.0068	−0.2121		
	(0.0308)	(0.0133)	(0.0258)	(0.0324)	(0.0197)	(0.0467)		
β_{i7}	−0.0395	−0.0010	0.0035	0.0163	−0.0043	−0.0258	−0.0489	
	(0.0128)	(0.0061)	(0.0110)	(0.0121)	(0.0088)	(0.0125)	(0.0136)	
β_{i8}	−0.0248	0.0227	−0.0221	−0.0360	0.0075	−0.0487	0.0485	−0.1107
	–	–	–	–	–	–	–	–
β_{iM}	−0.4576	0.0364	−0.1816	−0.3388	−0.1096	−0.4347	−0.0511	−0.1636
	(0.1233)	(0.0444)	(0.1038)	(0.0961)	(0.0680)	(0.1734)	(0.0416)	(0.1073)
R^2	0.9639	0.7654	0.2746	0.9931	0.9612	0.8657	0.6962	–
ITRL$-w$, Ω_0								
α_i	−0.1716	−0.0369	−0.1394	−0.1354	−0.0954	−0.2488	−0.0504	−0.1221
	(0.0343)	(0.0283)	(0.0444)	(0.0457)	(0.0402)	(0.0625)	(0.0326)	–
β_{i1}	−0.1872							
	(1.1155)							
β_{i2}	−0.0041	−0.0070						
	(0.3822)	(0.2193)						
β_{i3}	−0.0111	0.0058	−0.0802					
	(0.9053)	(0.3326)	(0.9253)					
β_{i4}	0.0016	0.0158	0.0434	−0.2703				
	(0.9224)	(0.3630)	(0.8725)	(0.7563)				
β_{i5}	−0.0116	0.0188	−0.0154	−0.0115	−0.0861			
	(0.6672)	(0.3077)	(0.5951)	(0.4556)	(0.3886)			

β_{i6}	−0.0542	0.0131	−0.0523	0.1730	0.0255	−0.1793		
	(1.3223)	(0.4308)	(1.0826)	(1.2615)	(0.8872)	(1.9171)		
β_{i7}	−0.0266	−0.0014	0.0313	−0.0190	−0.0029	−0.0181	−0.0435	
	(0.6171)	(0.3149)	(0.6059)	(0.5257)	(0.5185)	(0.5116)	(0.8624)	
β_{i8}	0.0139	0.0133	−0.0170	0.0019	0.0267	−0.0199	0.0345	−0.0826
	–	–	–	–	–	–	–	–
β_{iM}	−0.2794	0.0544	−0.0975	−0.2208	−0.0564	−0.2681	−0.0457	−0.0293
	(5.4818)	(1.6266)	(4.4667)	(4.1584)	(3.0049)	(7.8920)	(1.8738)	(5.1866)
R^2	0.9713	0.8344	0.4281	0.9890	0.9342	0.8665	0.7850	–

Note: See note to Table A.4.

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Complete systems of demand functions form an important part of many econometric macro models and they are also a useful tool for demand analysis in its own right. Econometric aspects of the application of these models are the subject of this volume. It includes discussions of data quality, model specification, choice of model, estimation and prediction. All applications are based on Swedish data. The results suggest that the most important issue in demand analysis is not that of finding the most general utility function but rather to find means of including constraints imposed on consumption by particular properties of commodities and markets and also taking into account sometimes sizable errors in data.

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