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ECONOMIC GROWTH IN THE VERY LONG RUN. On the Multiple-phase Interaction of Population, Technology, and Social Infrastructure

by

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Abstract

Economic growth in the very long run is described by a multiple– phase, dynamic process with potentially complex dynamics during transitions between regimes. Technology is assumed to rest on a managerial–administrative infrastructure which influences natality, mortality and the productivity of work. A given population adopts a temporarily efficient techno–infrastructure and determines the population of its heirs. Growth can occur within a regime by the reorganization of population into new groups. But this process cannot continue forever because of externalities. A way out exists in the adoption of an entirely new regime. Evolution is possible but the probability of escape from an old regime need not be unity. Fluctuations can occur with or without re–switching and under certain conditions a population can be trapped in a complex pattern of growth, fluctuation, re–switching and collapse. It is shown that realistic scenarios can be generated by the model. The paper concludes with a formal analysis of the possible events and the construction of probabilities that describe the chance that given phases will switch and that various kinds of qualitative histories can unfold.

ECONOMIC GROWTH IN THE VERY LONG RUN*

On the Multiple–Phase Interaction of Population, Technology, and Social Infrastructure

RICHARD H. DAY AND JEAN-LUC WALTER

> Our ultimate goal is to form laws of cultural dynamics.
>
> Paul Martin

> The archaeological data suggest that we must ... break away from the assumption that human cultures are inherently stable ....
>
> Mark Nathan Cohen

The theory of economic growth that flourished after mid–century was motivated, at least in part, by the exponential trend in aggregate output exhibited by the industrialized countries during the preceding century or two. Indeed, Tinbergen referred to his contribution as "a theory of the trend," and Solow's seminal work reflected the same stylized fact. That stylized fact, however, is only a relatively short run movement from the perspective of archaeologists for whom a generation or even a century is a short period, and who think in terms of millennia when contemplating the progress of human culture. Although the details become increasingly obscure in the more remote reaches of time, scientists in this field have firmly established a reasonably clear picture of human socio–economic development in its broadest terms. Human evolution over the 50 thousand years or so of modern homo sapiens' existence, devolves into the epoch of hunting and food collecting, that of horticultural settlement, and complex, nonliterate societies, the historical epoch of urban civilization, household agriculture and trading empires, and very recently the industrialized economy. Some pundits believe we are already in the midst of a transition to a new epoch of post- industrialization.

At some places and times the transition among epoches seems to have been smooth; at others crises seem to have occurred with "jumping" and "re-switching" prior to successful adoption of a new regime. Moreover, examples exist when a rapid collapse and reversion to a preceding regime seems to have occurred. Population fluctuations within a given way of life appear in the archaeological and historical record, and in some well-known cases of relatively isolated cultures, the socio-economic process seems to have been stuck in a more or less stationary or fluctuating state for very long periods of time.

The present contribution provides a model of this complex picture of socio-economic evolution. Economic growth is described by a multiple-phase, dynamic process. The effective use of a given technology (or set of technologies) is assumed to rest on an associated social system that incorporates managerial- administrative practice and measures for public health, welfare and defense. Natality, mortality and the productivity of work are all assumed to depend on this social system. A given "regime" consists of more or less independent "groups", each organized within a given technology-social system pair which I call a techno-infrastructure. A given population adopts a temporarily efficient techno-infrastructure and determines the population of its heirs. Although expansion within a group is limited by internal diseconomies of group size, growth can occur within such a regime by the reorganization of population into new groups. This process cannot continue forever because of external diseconomies associated with the total population. A way out exists in the adoption of a new technology and its associated infrastructure. The probability of escape from an old regime need not be unity, however. Fluctuations can occur with or without re-switching. Under certain conditions a population can be trapped in a complex pattern of growth, fluctuation, collapse, reversion to an old regime and renewed growth.

Although the picture that emerges is very different in some ways from the models of growth that have dominated economic thinking until now, it is built up from classical ingredients: the interaction of population, economic productivity and reproductive behavior. But there are crucial new elements, that, added to the classical assumptions, lead away from stationary states or steady, balanced growth.

The present introduction to this theory is divided into four parts: (1) a background survey of the major epoches and the hypotheses of social infrastructure, productivity, and demoeconomic behavior upon which the analysis rests; (2) the statement of a formal model that expresses these hypotheses; (3) an illustration of the kinds of histories that can be generated by the model and an explanation of how a variety of scenarios that seem like "real world" developments can occur; and (4) an introduction to the mathematical analysis of processes of this kind.

1. BACKGROUND

1.1 The Epoches.

Some fifty millennia ago, when modern people replaced their Neanderthal predecessors, the change was associated with an improvement in social organization and technology. According to Butzer [1977], the new bands were probably twice the size of the earlier groups. Their hunting–gathering technology involved an expanded ensemble of specialized stone implements of consummately skilled manufacture for various tasks of killing game, processing food and fabricating clothing and habitations. This great advance was apparently made possible by an improved brain and vocal organs that yielded a distinct advantage in communication, hence in social interaction. The new linguistic capacity may also have been intimately related to a superior creative capacity that made possible the striking improvement in technology and adaptation to virtually every nook and cranny of the globe; a process that came to an end some 10,000 years ago when the world (both old and new) was essentially filled with representatives of the hunting and gathering culture.

Binford [1968] and in an especially comprehensive treatise, Cohen [1977] demonstrate that during the closing of the global frontier, three roughly coincident developments occurred: the disappearance of the megafaunal species, the appearance of villages, and the domestication of plants and animals. The subsequent agricultural and herding societies based on horticulture and animal husbandry, marked the beginning of a transition to a new epoch, one that spread throughout a very large part of the world. The new culture displayed considerably more variety of both technology and social style. And it supported a more dense population. As this new culture spread, and it seems to have done so quite steadily, human numbers exhibited a worldwide surge. "Earlier" people were displaced, settled themselves, or fused into larger, more productive groups than before. Hunting cultures gradually faded into remote areas relatively unsuited for agricultural activity.

About 3000–5000 years ago urban–agricultural societies began to organize into centrally controlled, bureaucratically administered city states that used writing and accounting to monitor and control economic transactions. Sagan [1985] suggests that these early civilizations were preceded by intermediate, complex societies that possessed roads, schools, police, standing armies and bureaucracies, but not written languages, prominent examples of which persisted in Africa and Polynesia, until the European commercial expansion. In any event, the emergence of empires about 1500 B.C. based on widespread trading networks made possible a great increase in specialization and a pronounced expansion again in productivity. Another surge in population followed. Social, cultural and scientific progress of various kinds occurred throughout this age, leading after the Renaissance to a breakthrough to a new commercial age when nation states and trading empires spread civilization throughout much of the world. Then came the Industrial Revolution, based on power technology and large scale capital. It led to still another surge in productivity and population, and, compared to previous rates of increase, a truly explosive one, due in large measure to the decrease in mortality rates that accompanied this regime.

This in barest outline ~~is the~~ grand dynamics of *homo sapiens sapiens*! The actual number of epoches used in describing it is somewhat arbitrary.[1] More importantly, within the major epoches that mark these vast rearrangements of human activity and numbers, various goods, techniques, rules of behavior and institutional forms were invented, innovated, diffused and abandoned in over–lapping waves of activity and organization, a process that has accelerated with growing amplitude and shortening period. Economic evolution is, thus, much richer and more varied than this brief sketch portrays. What is crucial to the present analysis, however, is the practical existence of distinct socio–economic epoches.[2]

1.2 The Techno–infrastructure.

The key to understanding their significance for the theory of economic growth, is the explicit recognition that each is based on a distinct managerial–administrative infrastructure. Butzer already emphasized the point that it was more efficient organization that enabled humans to specialize in the harvest of dominant species (mammoth, horse, etc.), and to adopt quasi– permanent settlements, religion and specialization in the production of weapons and other implements.

When agriculture emerged, higher levels of organization and more complex societies evolved with it. People assembled into villages; classes and political organizations emerged. The increasingly intensive systems of cultivation required them. A literate elite, large scale public architecture, a standing military and a permanent bureaucracy characterized the regime of civilized urban centers based on irrigated agriculture, all stemming "from the need to organize local production and long distance trade." And when the modern industrial economy began to emerge, its growing armies of white and blue collar workers, depended on their abilities to function appropriately on vast systems of transportation, education, police, justice and public health. These in turn require huge forces of workers and administrators.[3]

Evidently, a salient feature of socio-economic life in all the great epoches is the division of effort between "managerial and organizational infrastructure" and "work." The former produces the social cohesion, coordination and knowledge upon which the productivity of labor is based. Given that effort, the work force can effectively process materials and fabricate goods. A given technology defines the possibilities for specialization and co-operation. Its effective implementation depends on the existence of the infrastructure and its managerial "know-how", a prerequisite that Boserup [1981] calls the "administrative technology."

In the simplest social groups, this infrastructure may be created by many or even all individuals part of the time. Even in the paleolithic hunting bands, individuals played distinct, specialized roles of social and religious leadership. Although the term "infrastructure" possibly exaggerates these functions in such simple societies, the presence of such a division of labor is obviously crucial to cohesiveness in the very large, nonliterate, complex societies, and in the huge agglomerations of early civilization and of our own industrial age. It is clear on the basis of these observations that for society to switch from one epoch to another, it is essential for it to possess a great enough population to support the new technology by providing an appropriate infrastructure. The infrastructure requirements constitute a threshold of population *that must be surpassed before a transition is possible.*

In addition to this lower threshold, is an upper bound on population beyond which the effective operation of technology with a given infrastructure cannot be maintained. This is because an excessive population cannot be coordinated: the planning, organization and control of public goods and services will not be effectively managed. Technology and infrastructure are, therefore, characterized by both lower and upper thresholds which define its population *domain of viability*. This combination of production technology and administrative technology with its division of effort between work and management (where these terms are broadly conceived), and with its population range of viability, I shall call a *techno- infrastructure*.

In addition to the internal diseconomy caused by expanding group size, due to problems of information, communication, and coordination within a given group possessing a fixed infrastructure, there exists also an *external diseconomy*, also determined by the technology, which derives from aggregate population size. It is induced by the absorbing capacity of the environment. The earth's absorbing capacity for peoples possessing a given techno–infrastructure can be stated in terms of the space available, which depends on the technology, and which can be expressed in terms of the average population density. The supply of nonhuman resources is diminished when human densities become too large: the productivity of agriculture is reduced due to the scarcity of land, water and other resources, and the waste absorbing capacity of the environment is increasingly exhausted.

The externality factor is a characteristic of the techno– infrastructure in that the absorbing capacity of the earth depends upon the implied way of life. For example, hunter–gathering societies are limited by the available game; horticultural societies by the supply of arable land; and industrial society by the supplies of water and oxygen and by the pollution absorbing capacity of the environment. A change in regime may overcome this constraint. Once the process of fission and diffusion of groups within a given culture has run its course, it is the only avenue for further development. It is the only avenue, that is, given the absence of technological innovation and diffusion that is "neutral" to the techno–infrastructure in the sense of being compatible with the given social system. This latter type of technological change, of course, plays an important role and occurs more or less continually. Its effect, however, is to accelerate growth within a given regime, which, as shall become evident below, hastens the process of switching among alternative regimes.

Consequently, little is lost if in this study we abstract from it *in order to focus on the process of epochal development in the sense defined here.*

1.3 Population.

Evidently, population size plays an essential role in determining which techno-infra-structures are viable. A sufficient population size is necessary for any productive activity at all, and once a given technology has been adapted, productivity is influenced by growth in numbers. Eventually, this productivity must decline because of the internal and external diseconomies. Any idea of well-being must depend on productivity, and this initial formalization of the theory will follow Cohen in using average product as the key variable. If average product is adequately high, population can expand. If it is too low, the attendant adversity will motivate a reorganization of society in search of a means to insure survival and to improve welfare. This process of switching is the heart of the matter, but the relation of population to welfare is its crucial antecedent.

Within the limits of survivability, the "demand for children" can be expressed like that for any other costly good, but the correlation of net population growth rates with welfare need not be thought of in literally rational terms. When welfare is low enough, obviously no children will survive. When some threshold of material well-being is surpassed, some children will survive, and this surviving number will increase with rising well-being until the choice of individuals, social custom or biological constraints introduces sufficient pressure to place an upper bound on further expansion.

The connection between productivity and population growth rates was the essence of the classical theory of development, and it has been incorporated in toto by modern anthropological - economic growth theorists like Cohen and Boserup. Obviously, the connection is truly subtle and highly variable, but for purposes of analyzing economic development in the very long run it would be inadmissible to omit it. For purposes of developing a formal model, the connection must be made precise, and this will be done using the standard form, long incorporated by Nelson [1956], Solow [1956], or Haavelmo [1958], or more recently by Day [1983] or by Day, Kim and Macunovich [1987].

What we have then is the interaction of population, productivity, welfare and population growth rates that forms the basis of the classical theory of economic growth. What has been added to the latter is the concept of the techno–infrastructure, and the explicit incorporation of internal and external diseconomies associated with excessive population numbers within a given technology and administrative framework.

1.4 Fission, Fusion and the Switch in Regime.

The key hypothesis originated by Binford, developed in Boserup, and buttressed in a comprehensive survey of the evidence by Cohen, is that population growth brings about a need to switch to progressively more intensive techniques in order to avoid an excessive decline in well–being, and that in order to switch an appropriate infrastructure is required. According to this theory, population growth is necessary to bring about major reorganizations of society and is sufficient in creating the economic pressures that motivate a social transformation.

This process of socio–economic evolution can be most easily identified at the transition between the hunting and food collecting epoch and the succeeding epoch of settled agriculture. Under favorable conditions, a hunting and food collecting band grows through a normal increase in population. As it does it draws on a greater and greater area whose scope eventually taxes the energies of the group. Productivity, and hence, welfare begins to fall. At some point the band may split to form two separate groups, or in a more gradual way "shed" some of its members who will fuse with others who have separated from other bands to form a new productive entity. The new groups move apart, each occupying about the same space as did the original bands, and each following essentially the same life as before. In this way in a process originally described by Birdsell [1958], an originally small, insignificant population spreads itself throughout all those areas where hunting and food collecting is possible.

The process will be slow or fast depending on the yield of the environment, the quality and specialization of implements and the effectiveness of cooperation in the hunt. Also crucial are mores of reproduction, conditions of hygiene, and external environmental factors that determine mortality in the species. But it can take place at a very high speed, as shown in recent simulation studies by Martin [1980].

The new groups that emerge could choose a new technology, but this may not be possible until there is a big enough population for bands to be fused; or it may be that the given technology makes possible a superior well-being just by splitting the groups, each adopting the same infrastructure–technology pair as before. But once the world becomes "full", that is, once the external diseconomies of total population, using a given techno–infrastructure become prominent enough, well–being cannot be maintained through further group formation. It can be accomplished (given the caveat about neutral technological change above) only by a jump to a new techno–infrastructure altogether.

The process of fission that characterized the expansion throughout the epoch of the hunting band did not disappear at the end of the age. It is still an important phenomenon. Fusion became increasingly important as formerly disparate groups were conquered and assimilated, or combined among themselves to form new civilizations to oppose the others. Nonetheless, during the spread of civilization, great empires that had once been formed often broke up into smaller geographical units within which growth eventually resumed. The Roman Empire, for example, divided into a considerable number of smaller states. Although population declined in some places, particularly in Rome, population growth in Europe quickly resumed. Similar breakups and reunifications occurred in China. Although fusion and fission in advanced societies is usually, if not universally, accompanied by war, it seems likely that the underlying economic forces causing these changes includes those of population growth, productivity and the efficacy of administrative technology in a manner described in general and somewhat abstract terms set forth here.

1.5 Alternative Scenarios of Economic Development.

The transition to a new techno–infrastructure rests both on its sufficient productivity and on its "reachability". In the absence of such a regime, the expansion of population could converge to an equilibrium, or, what seems more likely, to a fluctuation in numbers as originally argued by Malthus and more recently in the archaeological literature by Zubrow [1971], who provides evidence of such dynamics in the data on prehistoric agriculture in what is now the southwestern United States. In either case, economic evolution would come to a halt, awaiting the discovery of new techniques and the requisite social forms.

A more extreme result of long run growth in the absence of a reachable regime is the overshoot of an equilibrium and the collapse of the culture, with an attendant reversion to a preceding, "less advanced" techno–infrastructure. Such a collapse, for which there are several notable examples in the archaeological record (ancient Egypt, Teotihuacan), could be followed by a new expansion and evolution, but it could also be followed by still another collapse. See Sablov and Renfrew.

Finally, it may be that a regime is reachable, but because of the previous development history an expansion within a regime is followed by fluctuations with switching and re–switching that delay an eventual permanent transition to a new epoch. Such a scenario seems to mimic events that have been played out at one place or another in former times. Sagan, (p. 235), for example, observes that prior to contact with Western culture some societies appear to have spent hundreds of years alternating between band organizations and primitive, kinship societies.

Broadly speaking, however, the general trend of growth has involved a progression from one regime to another, each marked by the increasing size and complexity of its managerial infrastructure, and each requiring a striking advance in administrative as well as production technology.

2. A FORMAL MODEL

The grand dynamics of our story can be portrayed by a formal model which illuminates the underlying interaction of population, productivity, welfare and social organization. The first step is to reconsider the aggregate production function; the second is to summarize the salient features of natality and mortality; the third is to combine these classical but now modified ingredients to obtain a more general theory of growth that applies, not just for a single epoch, but which describes both the change within an epoch and the switch from one techno–infrastructure to another.

2.1 Production in a Group.

For simplicity, consider a communal group made of heterosexual pairs, or "households" and their children. Each pair supplies one adult equivalent of effort to society, either as part of the work force or as part of the infrastructure; one adult equivalent of effort is utilized in household production, childrearing or leisure. The group possesses a technology that rests on a managerial and administrative infrastructure whose presence is necessary for effective production within the culture of the group. Given this infrastructure, effective work can be undertaken using the available technology. With this setup, two distinct inputs must be distinguished: administrative or managerial effort, M, and labor, L. If the group size is x (measured in numbers of "households"), then $x = M + L$.

According to the theory under consideration, planning, coordination and control of economic activity becomes increasingly difficult as population grows within a group. For simplicity, it may be assumed that for any given regime there is some maximum number compatible with any effective socio–economic order. Let this number be N, call it the "upper viability threshold". The term $S = N - x$ represents the "social space" or "social slack" within which the group functions. If S is relatively large, a group may increase for some time with little depressing effect on productivity. When S is relatively small, there is little "room" for expansion, and increases in group size begin to lower productivity. If $S \leq 0$ the group cannot function.

Suppose now that the *intra-group production function* can be represented by the product of three separate factors involving separately the managerial input M, the labor input L, and the social space S; that is, let $f(M, L, S) = g(M)h(L)k(S)$ where $g(\cdot), h(\cdot)$, and $k(\cdot)$ are strictly increasing concave functions on $\mathcal{R}^+$ with $g(M) = h(L) = k(S) = 0$ for $M, L, S \leq 0$. Suppose in addition that M is fixed so that it is a parameter for a given group. Then, the production function for the group can be re-expressed as

$$Y = f(x; M, N) := \begin{cases} g(M)\, h(x - M)\, k(N - x), & M \leq x \leq N \\ 0, & x \leq M \text{ or } x \geq N. \end{cases} \tag{1}$$

The separate factors in the production function all have positive marginal productivity. When the constraints implied by the lower and upper thresholds M and N are taken into account, however, group effort as a whole first has increasing, then diminishing average productivity and, after a maximum output is reached, declining absolute productivity. The interval (M, N) is the group's *viability domain* given its fixed techno-infrastructure.[4] Group size must exceed M but not N.

2.2 Fission, Fusion and the Social Production Function.

Given a fixed techno-infrastructure, a population could expand beyond the feasibility domain for a single group through "fission", the splitting of groups in two, or "shedding and fusion", the formation of a new group from individuals splitting off from existing groups and combining to form a new entity.

Let x be the total population organized into n groups of average size x/n. For n

groups production is $nf(x/n) = Kh(max\{0, x/n - M\}) \cdot k(max\{0, N - x/n\})$. Clearly, $nf(x/n) > 0$ on the open interval $V_n := (nM, nN)$. According to the theory, the process of fission, shedding and fusion occurs so as to maintain temporarily efficient production. Consequently, social production is $\max_{n \in \mathcal{N}} \{nf(x/n)\}$ where $\mathcal{N}$ is the set of positive integers. Thus, the production function as a whole is the efficiency frontier of a scalloped sequence of overlapping component functions, each member of which is an integer multiple of its predecessor: "n" times the range of viability and "n" times the maximum attainable output. It presumes that, as population expands, groups split or shed and fuse so as to maintain overall population productivity at as high a level as possible.

Eventually, the absorbing capacity of the environment must be exceeded and this absorbing capacity cannot be expanded by forming new groups. Denote this externality factor by the term $p(x; \bar{x})$, a decreasing function on $[0, \bar{x}]$ with $p(0, \bar{x}) = 1$ and $p(x, \bar{x}) = 0$, all $x \geq \bar{x}$. Incorporating this externality the *social production function* is

$$Y = F(x) := \max_{n \in \mathcal{N}} \{nf(x/n)\} p(x; \bar{x}). \tag{2}$$

Evidently, population is bounded above by $\bar{x}$. Define $\bar{n} := \max_n \{nM \leq \bar{x}\}$. Then $\bar{n}$ is the maximum number of groups compatible with $\bar{x}$ and the requirements of the techno--infrastructure.[5]

Figure 1 shows an example in which $\bar{n} = 3$. The dotted lines show the successive production functions for 1, 2, and 3 groups when the externality factor does not play a role. The solid lines show how these are modified by the externality factor. The social production function is the envelope of the solid lines.

-Figure 1 about here-

Average productivity, which plays a key role in the theory is

$$y = Y/x = F(x). \tag{3}$$

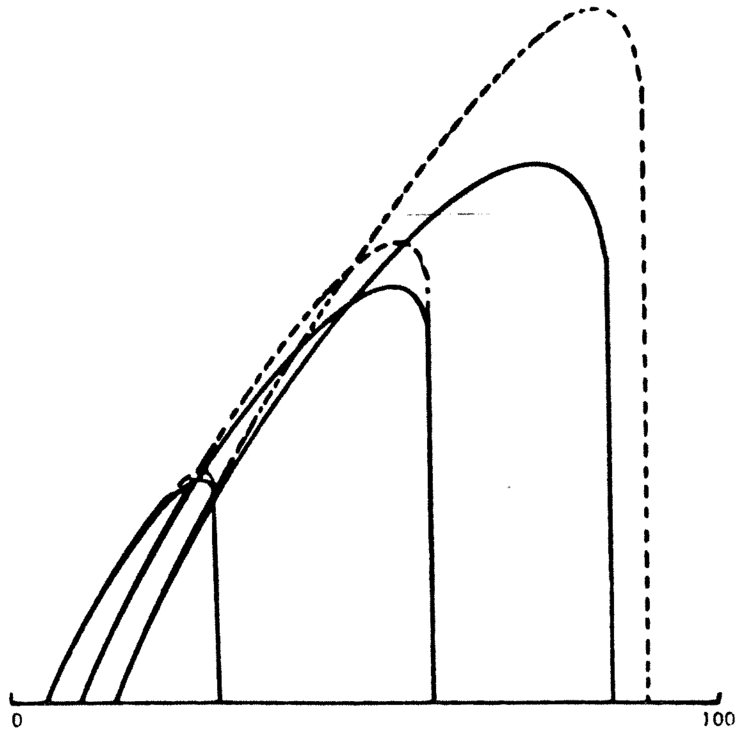


Figure 1: THE SOCIAL PRODUCTION FUNCTION
Equation (1') was used for 1, 2 and 3 groups
The parameters are $\beta = .9$, $\lambda = .1$, $\delta = .1$,
$M = 5$, $N = 30$, $\bar{x} = 85$.

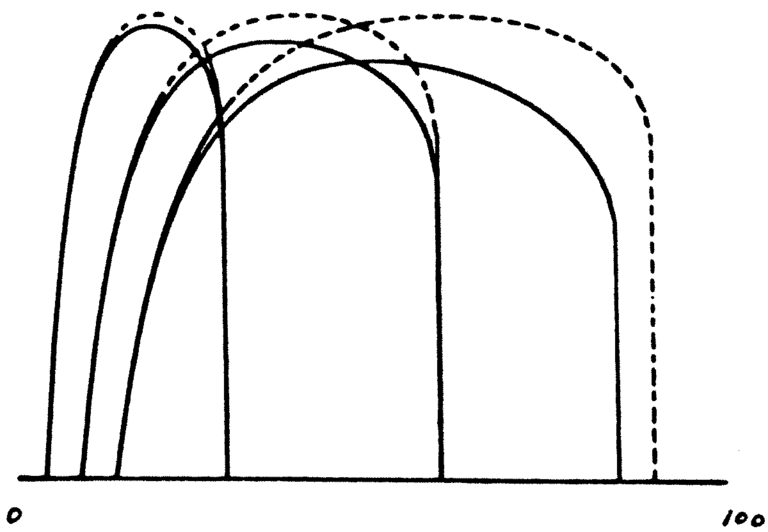


Figure 2: AVERAGE PRODUCTIVITY
Using the social peoduction function
of Figure 1.

Given a social production function like that shown in Figure 1, the graph of average productivity within a given regime would be like that shown in Figure 2.

-Figure 2 about here-

2.3 The Aggregate Social Production Function and Average Family Welfare.

Suppose now that we have a collection of alternative regimes waiting to be discovered or created in some kind of morphogenesis that occurs when productivity falls, each represented by a technology and by its characteristic threshold parameters. The population may now choose between expansion within a given regime by fission and diffusion *or* by a switch in regime.

Let us denote the sequence of alternative technologies by $\mathcal{T} = \{1, 2, 3 \ldots\} \subset \mathcal{N}$. Assuming that society uses an efficient technology the *aggregate production function* is

$$G(x) = \max_{i \in \mathcal{T}} \{F^i(x)\} \tag{4}$$

where each F^i is defined by (2) and where each component group production function f^i is defined by (1). The aggregate production function is the efficiency frontier of a scalloped sequence of overlapping, scalloped social production functions, each member of which is made up from the basic production function for a single group for a given regime. The aggregate technology $\{(x, G(x); x \in \mathcal{R}^+\}$ is, of course, nonconvex.[6]

The efficiency criterion underlying the aggregate production function implies that the average aggregate product is maximized temporarily and locally over alternative techno-infrastructures so that

$$y = G(x)/x = \max_{i \in \mathcal{T}} \{F^i(x)\}/x = \max_{i \in \mathcal{T}} \{F^i(x)/x\}. \tag{5}$$

2.4 Demoeconomic Behavior.

Assume that the time period is a generation. The number of families being x_t, the total population of adults and children is $P_t = (2 + b_t)x_t$ where b_t is the number of children per female. The maximum number of surviving children per female (Ricardo's "natural rate of growth") depends in general on the average well-being. The actual surviving number may be, and under some conditions actually is, smaller than this "natural rate". The actual rate may depend on preferences and social mores. Likewise, the number of children surviving to maturity depends on economic circumstances. Below some starvation level of income, say c_i, naturally, survival is impossible. Above this level the survival rate increases sharply. It approaches unity and possibly dips somewhat at very high income levels.[7]

For the sake of the theory, the net result of these considerations is the specification of a *demoeconomic function* that defines the average number of adult females that emerge in a given period per female existing in the preceding period. Formally, we suppose that $\pi(\cdot)$ is a function of average well-being that is fixed for a given infrastructure but may change when a transition occurs. Thus we index $\pi_i(y)$, $i \in \mathcal{T}$. We shall assume that $\pi_i(y)$ is quasi-concave for $y \geq c_i \geq 0$ with $\pi_i(y) = 0, 0 \leq y \leq c_i$, $i \in \mathcal{T}$. The parameter c_i will be called the *net birth income threshold* for the ith regime. Let $\lambda_i := \sup_{y \geq 0} \pi_i(y)$. Then λ_i is the maximum net rate of population growth in regime i.

2.5 Phase Structures and Regimes.

Given the assumptions made so far, the number of families that emerge in time period $t+1$ from the population of period t when the latter is organized into groups with the ith techno- infrastructure is $x_{t+1} = \pi_i(y_t)x_t$. Recalling that the average well-being is assumed to be the average product $y = G(x)/x$ then

$$x_{t+1} = \theta_i(x_t) := \pi_i[F^i(x_t)/x_t]x_t \tag{6}$$

when the population is organized in the ith *techno-infrastructure*. The map $\theta_i(\cdot)$ is called the ith *phase structure*.

It could be that for some n, and some i there exist feasible $x \in (nM^i, nN^i)$ such that $\theta_i(x) = 0$ because $F^i(x)/x < c_i$. If in the course of development from an initial population

x_o such a population is generated, then it is the last because $x = 0$ is a fixpoint for any phasestructure (6). This motivates the following:

DEFINITION 1: *Regimes and the viability domain.* The set $\mathcal{A}^* := \{x \mid \theta_i(x) > 0 \text{ for some } i\}$ is called the *viability domain*. Define

$$I(x) := \min\{\arg\max_{i \in T}\{F^i(x)\}\} \text{ for all } x \in \mathcal{A}^* \tag{6a}$$

and

$$I(x) := 0 \text{ all } x \in \mathcal{A}_o := \mathcal{R} \setminus \mathcal{A}^*. \tag{6b}$$

Now let

$$\mathcal{A}_i := \{x \mid I(x) = i\}. \tag{7}$$

It is the set of populations for which the ith phase structure governs development. We shall call it the ith *regime*. Thus when $x \in \mathcal{A}_i$, the ith phase structure determines the succeeding population. The set $\mathcal{A}_o$ is called the *null* regime. For any $x \in \mathcal{A}_o$ the succeeding population is zero, so we define $\theta_o(x) := 0$ all $x \in \mathcal{A}_o$.■

Obviously, $\mathcal{A}^* := \cup_{i \in T}\{x \mid F^i(x)/x > c_i\}$.

2.6 The Multiple–Phase Dynamic Process.

The grand dynamics of demoeconomic development, involving the interaction of population, productivity, technology and social infrastructure can now be represented as the *multiple-phase dynamic process*

$$x_{t+1} = \theta(x_t) := \theta_{I(x_t)}(x_t) = \theta_i(x_t), x_t \in \mathcal{A}_i. \tag{8}$$

Nothing guarantees that every phase zone is nonempty. An empty phase zone for a given regime means that it is dominated by other uniformly more productive regimes. Moreover, not all techno– infrastructures may be reachable from an initial population.

But some history of phases unfolds for any initial population. The *phase progression* $I(x_t), t = 0, 1 \ldots$ represents this history. *It describes economic development as an epochal evolution.*

For change to occur within a given regime, there must be a number n of groups into which the population of families are divided which are compatible with the population viability thresholds M_i and N_i. If regime A_i governs growth during period t, then there exists a number of groups, say $n(t)$, such that $n(t)M^i \leq x_t \leq n(t)N^i$.

Begin with an initial population of families x_o in some base period. Suppose $I(x_t) = 1$ for $t = 0, \ldots, s_1$ but that the regimes switch and $I(x_t) = i_2$ when $t = s_1 + 1$. The first epoch lasted for s_1 generations. Suppose that $I(x_t) = i_2$ for $t = s_1 + 1, \ldots, s_2$ but not for $t = s_2 + 1$. Then again the regime switches, let us say to i_3. The second epoch lasted $(s_2 - s_1)$ generations. The third regime is i_3 with duration $s_3 - s_1 - s_2$ and so on. Within each epoch the number of groups $n_i(t)$ will change when fission or shedding and fusion occurs, so the number of groups in the ith regime forms a sequence $n_{ij}, j = 1, \ldots, g_i$, where n_{i1} is the initial number of groups formed at the switch into regime i. This process can continue so long as there are productive techno- -infrastructures to be adopted when population gets large enough.

3. "REAL WORLD" HISTORIES

The model specified above is probably the minimal variation on the classical–neoclassical growth theory that incorporates the new theory of socioeconomic growth in the very long run. Some examples will illustrate how the theory can "explain" some of the more complex patterns of development found in the archaeological–anthropological–economic historical literature.

3.1 Specific Functional Forms.

First, we have to adopt specific functional forms for the components of the theory.

Consider the group production function for techno-infrastructure i.

$$y = f^i(G) = K_i(G - M_i)^{\beta_i}(N_i - G)^{\gamma_i},\ M_i < G < N_i, \tag{1'}$$

Let the externality factor $p(x;\bar{x}_i) := (1 - x/\bar{x}_i)^{\delta_i}$ for $x \in [0, \bar{x}_i]$.[8] The social production function for a given techno-infrastructure can be shown to be

$$y = F^i(x) = K_i \max_{n \in \mathcal{N}}\{n^{1-\beta_i-\gamma_i}(x - nM_i)^{\beta_i}(nN_i - x)^{\gamma_i}(1 - x/\bar{x}_i)^{\delta_i}\}. \tag{2'}$$

Next, suppose the demoeconomic function is $\pi_i(y) := \max\{0, \min\{\alpha_i(y - c_i), \lambda_i\}\}$ which gives a positive linear function with positive slope α_i on the interval $[c_i, \lambda_i/\alpha_i - c_i]$.

From these, the ith phase equations must be

$$x_{t+1} = \theta_i(x_t) = \max\{0, \min\{\alpha_i(F^i(x_t) - c_i x_t), \lambda_i x_t\}\}.^9 \tag{5'}$$

The production function (1') is concave on its feasibility domain $v^i = (M^i, N^i)$ when $0 < \beta_i, \gamma_i < 1$. . When fission occurs this function is "stretched", but because of the externality it may become quasi–concave near $\bar{x}_i$. The function $F^i(\cdot)$ is certainly piecewise quasi– concave and piecewise monotonic. Because $\alpha_i c_i x$ is linear, the term $\alpha_i F^i(x_t) - \alpha_i c_i x_t$ retains essentially the same profile as $F^i(\cdot)$.

Given all this, the ith regime is $\mathcal{A}_i = \{x; F^i(x)/x > c_i\}, i \in \mathcal{T}, \mathcal{A}_o := \{x; F^i(x)/x \leq c_i\}$. The multiple-phase dynamic process is given by

$$x_{t+1} = \begin{cases} \theta_i(x_t) = \min\{\alpha_i(F^i(x_t) - c_i x_t, \lambda_i x_t\}, & x_t \in \mathcal{A}_i, \\ 0, & x_t \in \mathcal{A}_o. \end{cases} \tag{8'}$$

3.2 Complex Dynamics at the Transition.

One of the most striking possibilities in this multiple–phase process, is one in which fluctuations occur between the regimes, with switching and re–switching occurring at irregular intervals, and then a permanent switch followed by growth within a succeeding regime. *Development in the long run is portrayed as a sequence of growth trends interspersed with* *fluctuations. Smooth transitions can also occur with monotonic growth continuing at some transitions but fluctuations and crises at others.*

In Figure 3a a phase diagram for Equation (8') using specific parameter values is displayed. In Figure 3b a trajectory beginning from a very small population is shown. After a period of growth, a smooth transition to a second regime occurs. This is followed by growth, then fluctuations with re–switching – a prolonged period of crises, if you will – followed by a successful jump, further growth, and then fluctuations again. Figure 4 presents another phase diagram and trajectory starting from a slightly different initial population *but with the same parameters*. In this example, fluctuations with re- switching occur even in the first regime, but eventually a permanent transition comes about. In order to bring out the potential instabilities inherent in such a process, the parameters have been adjusted so that the probability of switching regimes *without* fluctuations is very small. Nonetheless, the probability of jumping is 1, as is shown in Theorem 1 below.

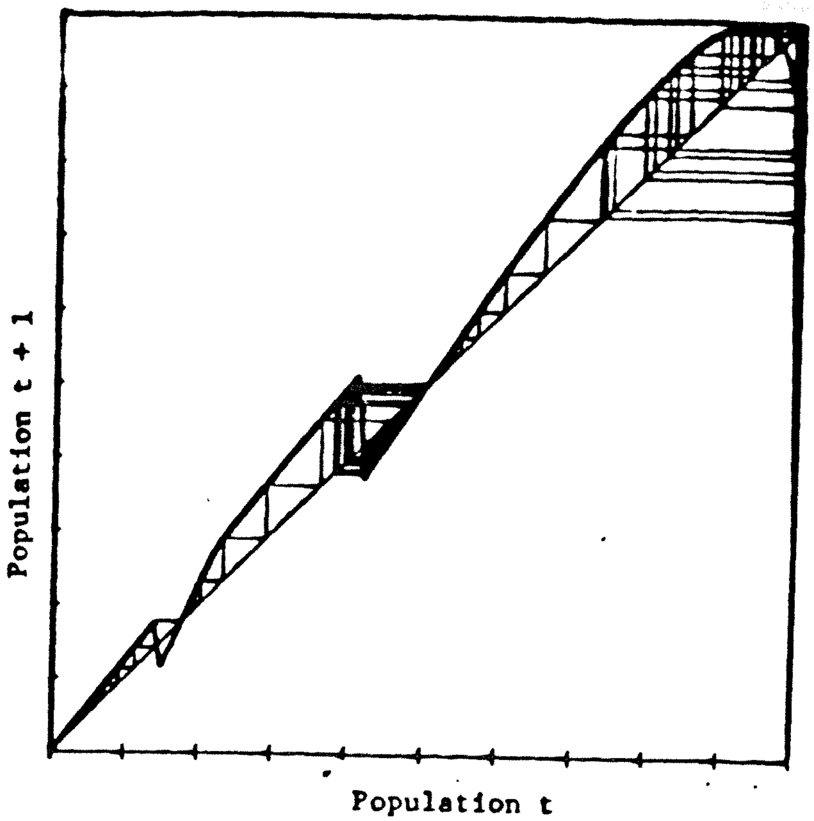
-Figures 3 and 4 about here-

3.3 The Very Long Run Growth Trend.

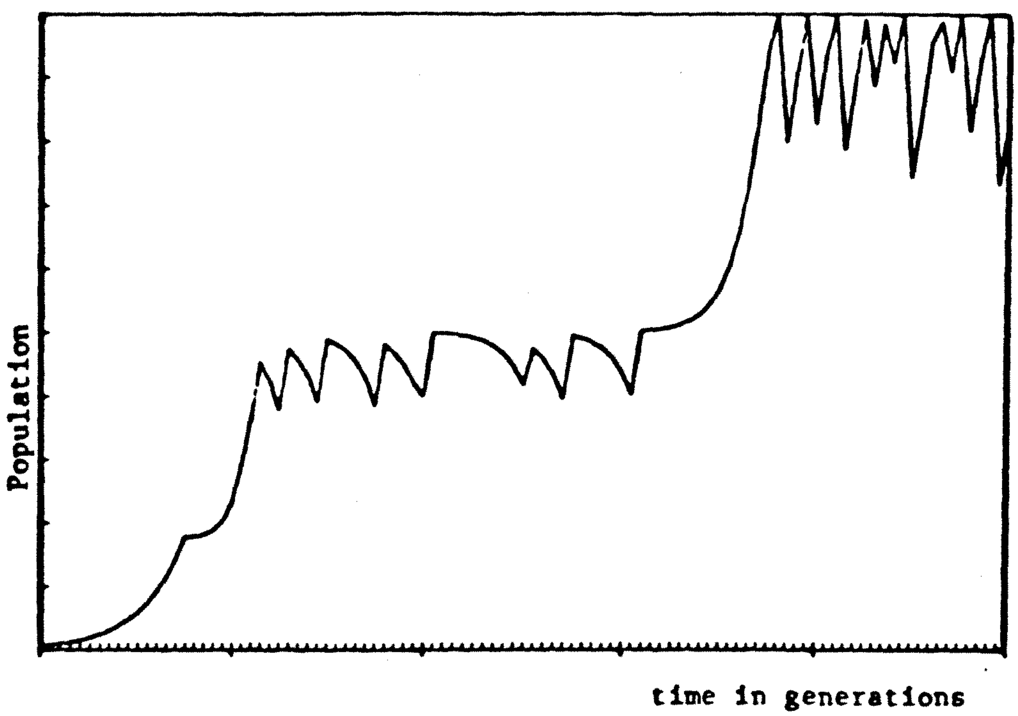
Consider now the following stylized epoches based on Deevey [1960].

TABLE 1: THE EPOCHS

Regime	Duration	Number of Generations before 1975	Epoch	Beginning Population (# of "families")
1	40,000–8,000 B.C.	1680	Hunting and Collecting	$.325K^2$
2	8,000–3,000 B.C.	400	Village Agriculture	$1.5K^2$
3	3,000–1750 B.C.	200	Civilization and Trading Empires	$25K^2$
4	1750 A.D.–1975 A.D.	9	Industrial Revolution	$250K^2$

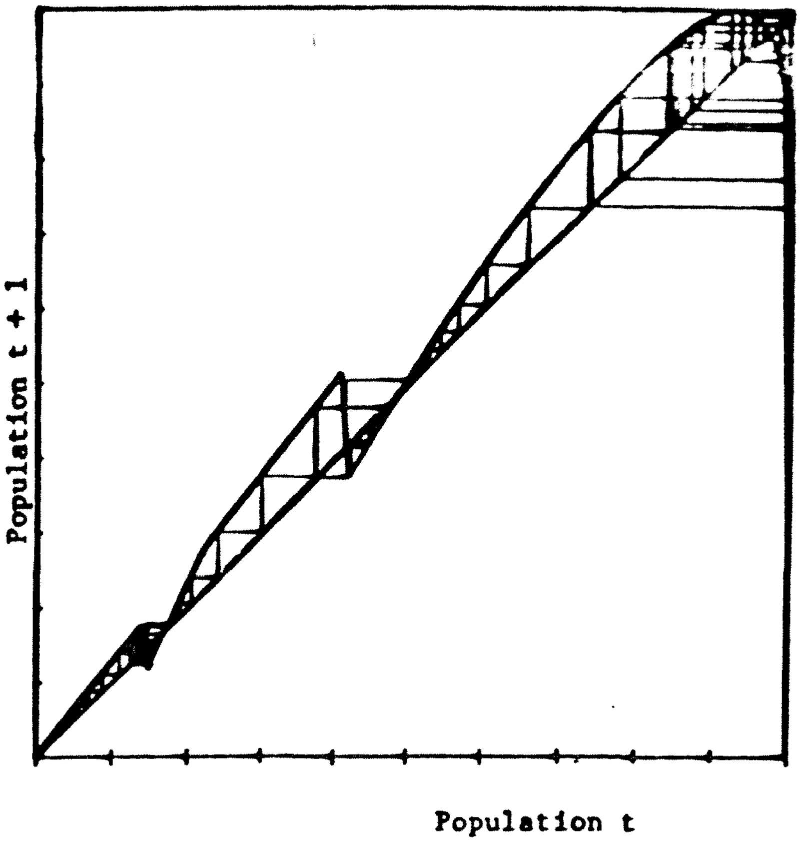


(a) Phase Diagram

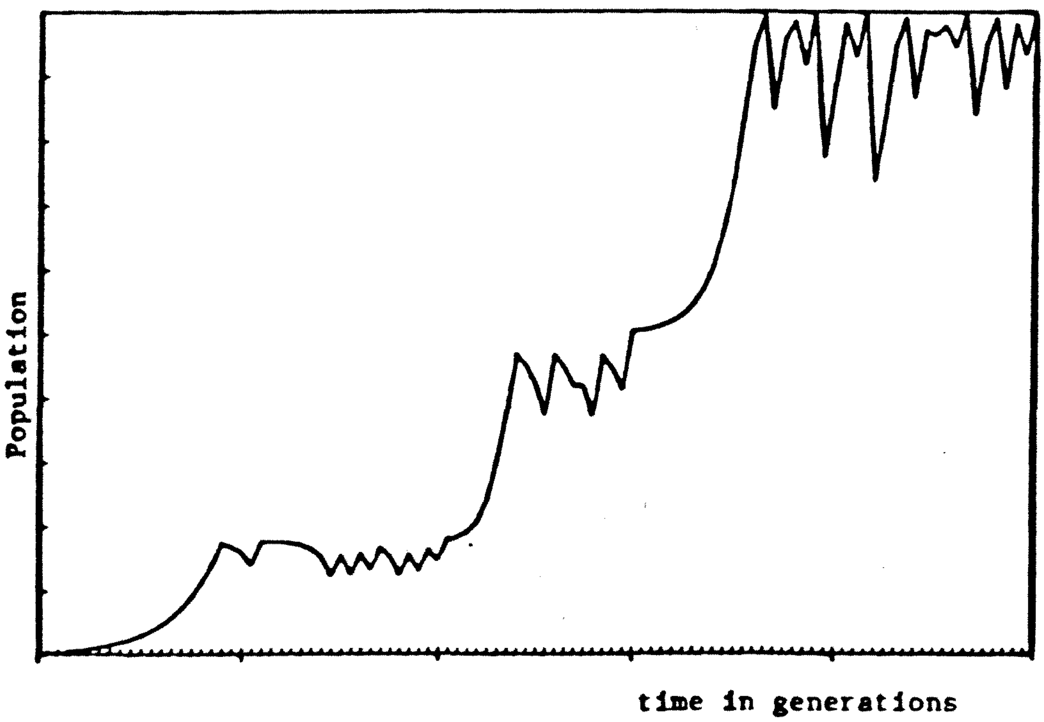


(b) Implied History

FIGURE 3: COMPLEX DYNAMICS WITH RESWITCHING.



(a) Phase Diagram



(b) Implied History

FIGURE 4: COMPLEX DYNAMICS WITH RESWITCHING.

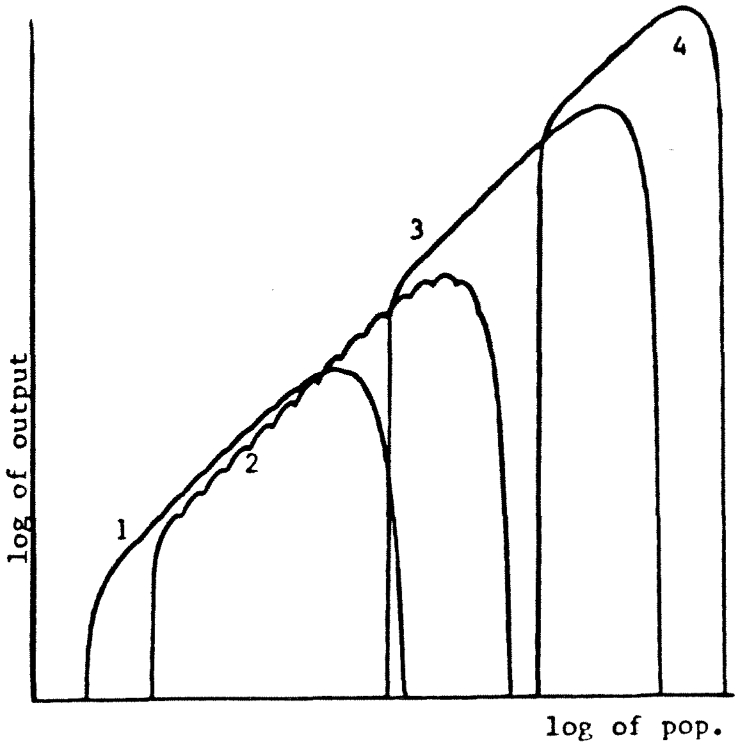
Our problem, given this aggregated set of epoches, is to estimate the parameters of equation (8'), so that the regime switchings occur more or less in the order and with growth within a regime occurring for roughly the duration presented in the table. A crude set of "guestimated" parameter values that will accomplish this is the following:[10]

TABLE 2: PARAMETER VALUES

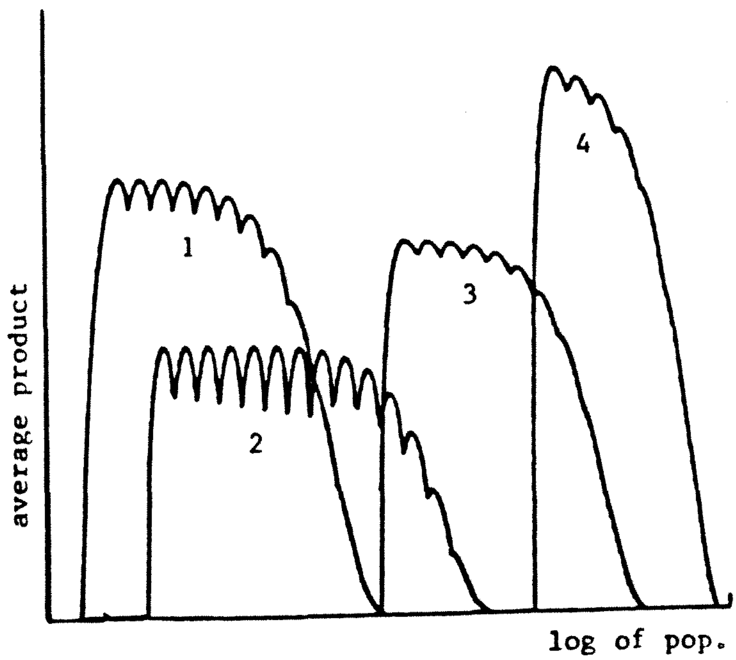
Regime	M	N	$\geq$	K	β	γ	δ	λ
1	5	30	104K	7	.9	.1	.1	1.001196
2	40	100	$2.2K^2$	20	.6	.1	.1	1.012167
3	50K	$2K^2$	$180K^2$	5	.6	.1	.1	1.014128
4	$5K^2$	$500K^2$	$1.3K^3$	800	.6	.1	.1	1.222845

In order to record the huge range in the data over so many millennia, log transformations have been used to plot population and output. This has the effect of giving the early epoches, which lasted a long time, a weight comparable with the more recent epoches which grew at accelerating rates for much shorter durations.

Figure 5a shows the aggregate production function. Figure 5b illustrates the implied average product. Because of the logarithmic scale we cannot see much of the detail in the former diagram. Contrastingly, the scalloped profile due to the fission–shedding–fusion process shows up boldly in latter chart. Figure 6 gives the history of population. Evidently, the model presents a story of socio–economic evolution more–or– less like that described



(a) Aggregate Production



(b) Average Product

FIGURE 5: THE AGGREGATE PRODUCTION FUNCTION AND AVERAGE PRODUCT WITH 4 REGIMES.

See Table 1.

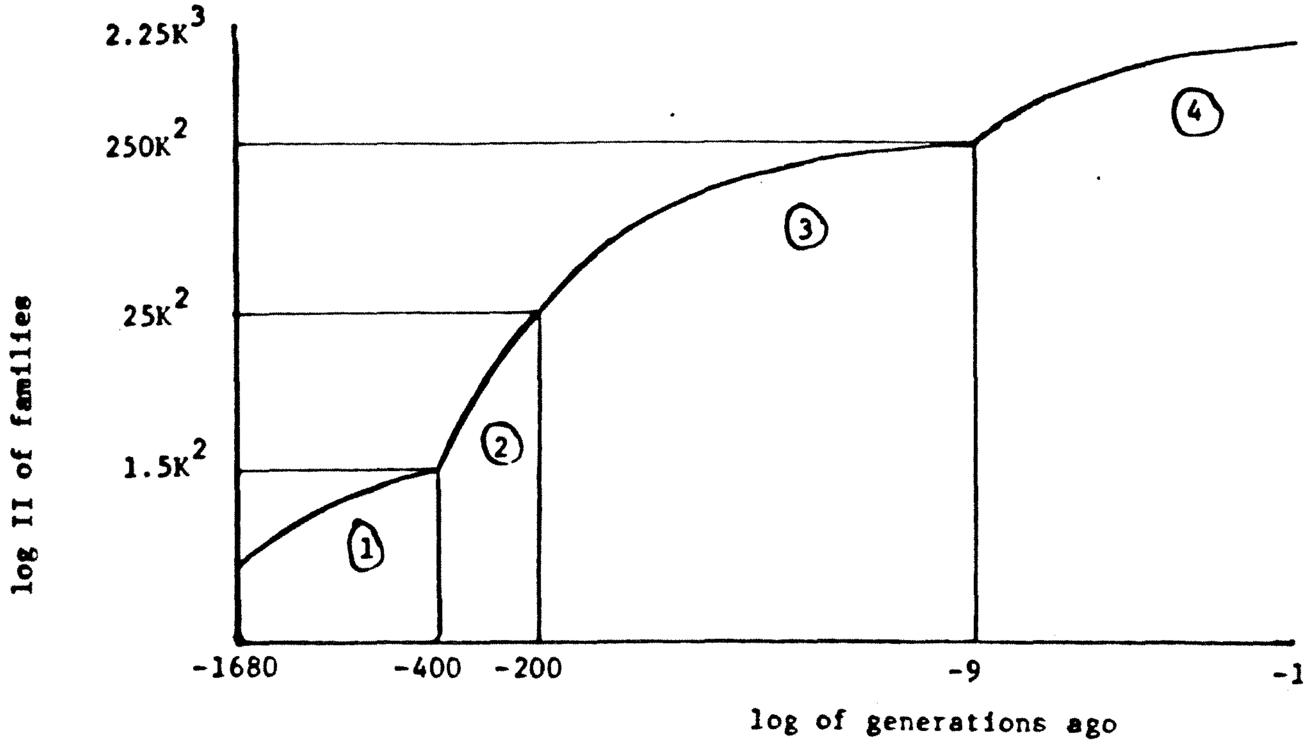


FIGURE 6: THE PROGRESS OF AGGREGATE POPULATION THROUGH THE FOUR MAJOR EPOCHS.

by Deevey's population data.

-Figures 5 and 6 about here-

4. MATHEMATICAL ANALYSIS

We have shown that it is possible to construct models using specific functional forms that generate patterns of development reminiscent of those in the record. It is the purpose of this concluding section to derive precise "general" conditions under which these results occur and to give a formal characterization of scenarios that are possible within this framework. For this purpose, history is described by sequences of qualitative events whose conditional probabilities of occurrence can be derived in principle from the underlying parameters of the techno– infrastructure and demoeconomic behavior. The necessary concepts are developed and the central results presented. Proofs of the latter will be found in the Appendix.

4.1 Trajectories and Orbits.

A *trajectory* of (8) with initial condition $x_o = x$ is a sequence $\tau(x) := (x_n)_{n=0}^{\infty}$ such that x_{n+1}, x_n satisfy (8) for all n. If $\theta^n(x)$ is the nth iterated map generated from θ then $\tau(x) = \left(\theta^n(x)\right)_{n=0}^{\infty}$. A trajectory is a model generated history or scenario and in what follows shall be referred to synonymously as such.

4.2 Viability.

In order to keep the number of evolutionary possibilities within reasonable bounds and to simplify the analysis, some regularity conditions will be adopted.

First, assume that the number of distinct infrastructures is finite, i.e., that $\mathcal{T} = \{1, \ldots, r < \infty\}$. Next, in order to insure that $\mathcal{A}^*$ is a connected set, assume that the infrastructure M_i is less than half the upper feasibility bound on group size, that is, that $M_i < \frac{1}{2}N_i$ all $i \in \mathcal{T}$. Also assume that $M_i < M_{i+1} < \bar{x}_i < \bar{x}_{i+1}$ so that neighboring feasibility domains overlap and are well ordered. If it is also assumed that $\max_x \theta(x) \leq \bar{x}_r$, then it is easily seen that $\theta(\cdot)$ is continuous and maps into $[0, \bar{x}_r]$ but it need not be that $\theta(\mathcal{A}^*) \subset \mathcal{A}^*$. Indeed, if M_1 or c_1 is positive, then $\theta(x) = 0$ for any $i \in \mathcal{T}$ and any x such that $G^i(x)/x < c_i$. Such x will exist sufficiently close to M_1 and $\bar{x}_r$ and perhaps at other populations as well. Hence, $\mathcal{A}_o \cap (0, \bar{x}_r) \neq \emptyset$ and evolution comes to an end for any trajectory that enters $\mathcal{A}_o$. Such a possibility is worth thinking about because extinction is such a common occurrence in the biological world of which we are a part. Nonetheless, the insights of the present theory are of considerable interest in the absence of such catastrophes, so we shall assume M_1 and c_1 are zero. Then, $\theta(0) = 0 = \theta(\bar{x}_r)$ and, given the previous assumptions, $\theta(x) > 0$ all $x \in (0, \bar{x}_r)$. Thus, $\mathcal{A}^* = (0, \bar{x}_r)$ and $\theta(\mathcal{A}^*) \subset \mathcal{A}^*$; once the system "starts up" it can continue.[11]

4.3 Peaks and Tails.

It is evident from the preceding sections that local minima can occur for the map $\theta(\cdot)$ at population levels for which the number of groups in the population changes within a given regime, or at populations for which a regime switch occurs (and the preexisting groups are fused to form a smaller or decomposed to form a larger number of groups). These local minima, or *tails* are turning points where the slope of $\theta(\cdot)$ changes from negative to positive. In between these tails are local maxima, or *peaks*, which are associated with the maximum population possible for a given number of groups within a given regime. For our present purposes the two kinds of switch points, one kind due to a change in the number of groups and one kind due to a switch in regime, need not be distinguished.

Let $\beta_o = 0, \beta_1 < \beta_2 < \ldots < \beta_s = \bar{x}_r$ be the $s+1$ local minimizers and let $\alpha_i, i = 1, \ldots, s$ be local maximizers of θ. Of course $\beta_{i-1} < \alpha_i < \beta_i, = 1, \ldots, s$. By definition, the peak $\theta(\alpha_i)$ is the maximum emerging population that can occur from established populations in the neighborhood of α_i. Likewise, the tail $\theta(\beta_i)$ is the minimum population that can emerge in the neighborhood of β_i. Note that because of the splitting of socioeconomic groups, s will not be smaller than r.

Now let Z be an interval and let $\theta_Z(\cdot)$ be the restriction of $\theta(\cdot)$ to this interval. It shall be assumed that for all $x \in [\beta_{i-1}, \alpha_i]$, θ is strictly increasing and $\theta(x) > x$. It shall further be assumed that θ is concave and strictly decreasing on $[\alpha_i, \beta_i], i = 1, \ldots, s$. Thus, θ is not constant in the neighborhood of a peak. These assumptions mean only that θ_i is concave on the relevant parts of its domain. They rule out feasible regimes in which only a contraction can occur. While models that violate these assumptions would be of considerable interest for describing some kinds of history, to exclude them reduces the number of cases to be explored which, as shall soon be seen, is still quite large. The instability or local expansiveness of the map θ plays a crucial role in the present theory. Specifically, we shall make use of

$$\text{Condition E: for all } x \in (\alpha_i, \beta_i), \theta'(x) < -1.$$

This condition rules out convergence to a stationary state almost surely. When it prevails we can get very strong results.

4.4 Events.

Now consider the set of trajectories $S := \{\tau(x) | x \in (0, \alpha_1)\}$. Our objective is to give a characterization for all trajectories in S. To proceed we decompose $[0, \bar{x}_r]$ into intervals according to

DEFINITION 2: *Event thresholds and event zones.* Set $\gamma_1 = \beta_o = 0, \gamma_{s+1} = \beta_s = \bar{x}_r$. For $i = 2, \ldots, s$ define $\gamma_i \in [\beta_{i-1}, \alpha_i]$ by

$$\begin{array}{lll} \gamma_i & = \theta(\gamma_i) & \text{if } \theta(\beta_i) \leq \beta_i \\ \gamma_i & = \beta_i & \text{if } \theta(\beta_i) > \beta_i \end{array}$$

The interval $Z_i := [\gamma_i, \gamma_{i+1}], i = 1, \ldots, s$ will be called the ith *event zone* and the parameter γ_i the ith *event threshold.* ■

The types of trajectories that occur can now be characterized in terms of these event zones. For this purpose, the following definitions will be used.

DEFINITION 3: *Events*. An event S_i is a subset of $S := \{\tau(x)|x \in [0, \alpha_1]\}$ defined with reference to the ith event zone. The *null event*, N_i, contains trajectories that never reach the ith event zone; the *reaching event*, Γ_i, contains all trajectories that surpass the ith event threshold. The *touching event*, Γ_i^*, contains all reaching trajectories whose first element past the ith threshold belongs to the ith event zone. The *skipping event*, J_i^k, contains trajectories that skip the ith event zone the first time they enter a higher zone, *i.e.*, $J_i^k = \Gamma_i \setminus \Gamma_i^*$; the *growth event*, J_i^g, contains trajectories that grow monotonically in Z_i after initially surpassing the ith event threshold and then jump to a higher zone; they may return to Z_i or to some lower event zone after this first escape; the *fluctuation and jumping, or local chaos event*, J_i^{lc}, contains trajectories that oscillate a finite number of periods and jump to a higher zone after first entering Z_i; the *sticking event*, T_i^s, contains trajectories that do not escape Z_i; the *reversion event*, T_i^r contains trajectories that touch Z_i, revert to an earlier event zone and never exceed γ_{i+1}. The *jumping event*, $J_i := J_i^g \cup J^{lc} \cup J_i^k$, contains all trajectories that skip Z_i or that enter Z_i and then jump to a higher zone $Z_j, j > i$. The *trapping event* $T_i := T_i^s \cup T_i^r$, contains all trajectories that enter Z_i but that never reach a higher zone. ▪

Figure 7 shows a map $\theta(\cdot)$ that satisfies the assumptions made so far. Generally speaking, any trajectories that enter the sets G_i grow; those that enter the sets F_i fluctuate, and those that enter the sets E_i escape to a higher zone.

Consider an initial condition $x_o \in Z_1$. Note that $Z_1 = (0, \beta_1) = A_1$. Growth in G_1 and a smooth transition to the second regime occurs unless $\theta^s(x_o) = \beta_1$ for some

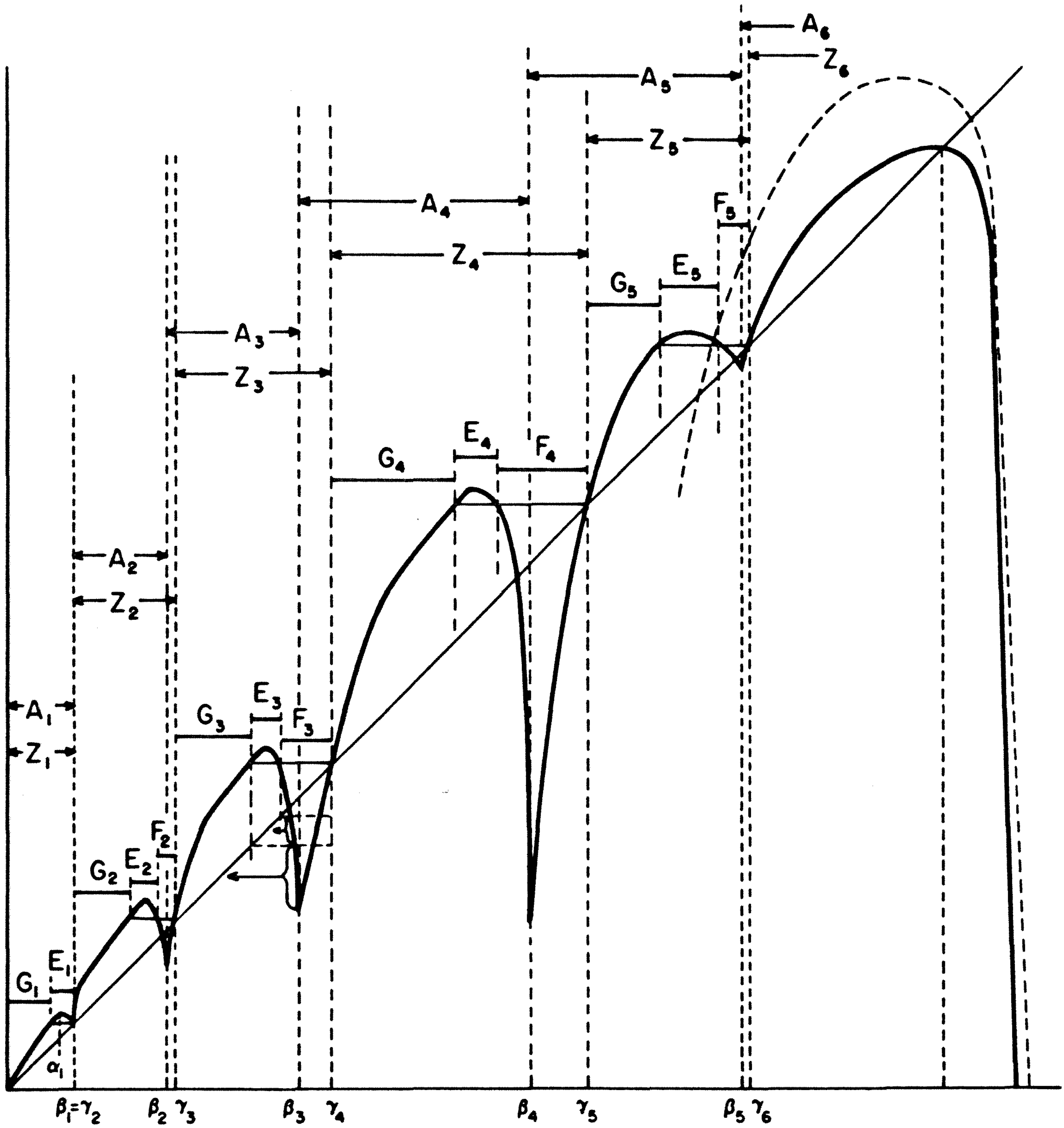


FIGURE 7: MULTIPLE PHASE DYNAMICS. There are 6 or 7 regimes, each exhibiting a different canonical type of transition possibilities.

s. Because there are only a countable number of points mapping into β_1, **almost all trajectories beginning in** $(0, \alpha_1)$ must enter the escape interval E_1. **Thus, we can see that** the monotonic growth and jump event, J_1^g, occurs almost surely for x chosen at random in $(0, \alpha_1)$.

-Figure 7 about here-

In Regime 2, growth continues. If the trajectory enters the escape interval E_2, a jump to Regime 3 occurs. If, instead, the trajectory enters the interval F_2, then fluctuations emerge. Note that a switch in Regime occurs if x_t enters the sub–interval (β_2, γ_3), but population declines and Regime 2 is re–adopted. As one can see, there is a small interval in F_2 which leads to E_2 and a successful jump to Regime 3. What is the probability of escape when the trajectory enters F_2? Evidently, it is positive. Is it one? That question is answered in the affirmative below when Condition E prevails.

Suppose, then, that the trajectory passes γ_3 and into event zone Z_3. Evidently, if the interval E_3 is entered, we have J_3^g. A jump to the Regime 4 with further growth would occur. If the interval F_3 were entered, fluctuations with re–switching could occur. Note that there are intervals in F_3 from which trajectories will enter E_3 or G_3. In the former case escape to the next regime occurs. In the latter case growth within Regime 3 resumes, followed by all the possibilities already noted. Must a jump to Regime 4 occur, or could "history" be trapped in Regime 3? It is shown below that if Condition E prevails a jump must occur. If that condition does not hold, then a trapping event could occur with positive probability.

Suppose an escape to Regime 4 does occur. As in the previous case, if x_t enters E_4, a transition occurs. But if it enters F_4, fluctuations emerge again with the possibility of switching and re–switching. The trajectory may enter E_4 and escape, emerging into Regime 5 and a continuation of growth in G_5. Or it may revert to G_4. If so, growth resumes and the story is repeated with the possibility of a successful jump or a crisis with fluctuations and re–switching. But there are also small intervals in F_4 which will lead to a reversion to Regime 3. If this happens then all the qualitative histories already described can unfold. Given this possibility of switching, re– switching and reversion, is the probability of escape one? Or, can society be trapped with positive probability into an endless pattern of growth and complex fluctuations among Regimes 3,4 and 5? If Condition E does not hold, then the latter is possible. If it does, growth will resume almost surely.

If a successful jump is made to Regime 5, then growth does resume and, as shown in the diagram, a smooth transition to the next regime, 6, is possible. Contrastingly, however, if the interval F_5 is entered, the economy is trapped. Inside this interval cycle, chaotic fluctuations or convergence can occur but not escape. Thus, there is a positive probability that trajectory beginning near a will trapped in F_5. If Condition E prevails fluctuations would continue. Otherwise, trajectories would converge to a classical stationary state with positive probability.

If the transition to Regime 6 does occur, then population converges to a stationary state. Suppose that the socioeconomic menu is augmented by Phase structure 7. Then growth would resume, fluctuations would re–emerge and then a reversion to some earlier techno–infrastructure.

4.4 Peaks, Tails and Types of Transitions.

Whether or not jumps, traps, reversions, *etc.*, occur depends essentially on the local peaks and tails of the θ_{Z_i} (the restriction of θ to the ith event zone). To give this observation precise meaning, we specify

DEFINITION 4:

(i) Let $\theta(\alpha_i) \in Z_{j_i}$. Then $h_i : j_i - i$ will be called the ***size of the ith peak.***

(ii) The size of the ith tail is

$$t_i = 0 \text{ if } \theta(\beta_i) \geq \gamma_{i+1}$$
$$t_i = 2 + k \text{ if } \gamma_{i-k} \leq \theta(\beta_i) < \gamma_{i-k+1}, k = 1, \ldots.$$

If $\theta(\beta_i) < \beta_i$ and $h_i \geq 1$, then γ_{i+1} has two pre–images in Z_{i1}, say $\phi_i < \psi_i$.

Then

$$\begin{aligned} t_i &= 1 \quad \text{if } \psi_i \leq \theta(\beta_i) < \beta_i \\ t_i &= 2 \quad \text{if } \gamma_i \leq \theta(\beta_i) < \psi_i. \end{aligned}$$

(iii) The interval $E_i := (\phi_i, \psi_i)$ will be called the ith *escape zone*.

(iv) The interval $F_i := (\psi_i, \gamma_{i+1})$ will be called the ith *fluctuation zone*. ∎

The size of the ith peak determines the highest regime reachable from trajectories entering Z_i. The size of the ith tail determines the lowest regime reachable from trajectories entering Z_i. Figure 8 illustrates the types of transitions from one event zone to another and shows how these are determined by the peaks and tails. Three distinct types are shown. In 8a, b and c the probability of escape from Z_i is zero. In 8a, convergence to a stationary state occurs; in 8b, fluctuations within a fixed regime take place; and in 8c, endless, possibly chaotic fluctuations occur with re–switching (in the number of groups and/or techno–infrastructures). When such sticking events occur, further development in the sense of transitions to regimes with larger managerial infrastructures and higher productivity cannot occur.

In Figure 8d, there is a positive probability for the monotonic growth and jumping event J_i^g, but also a positive probability of the sticking event T_i^s in which endless chaotic fluctuations are possible.

In Figures 8e–h, transition types occur in which the probability of jumping from event zone Z_i occurs is unity, but locally chaotic fluctuation with re–switching may occur

TYPE 1: PROBABILITY OF ESCAPE = 0. (STICKING EVENTS)

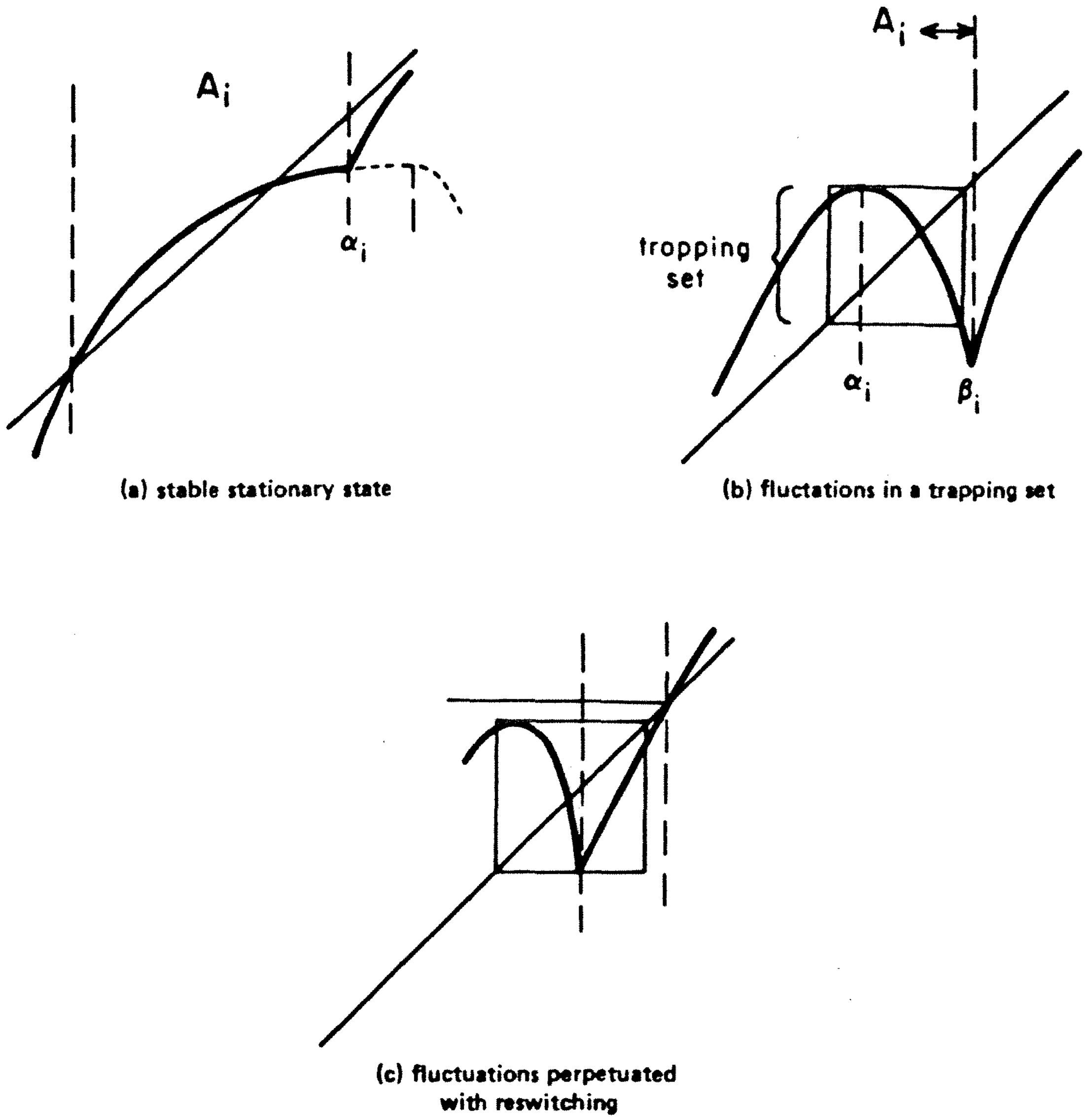


FIGURE 8: TRANSITIONS

TYPE 2: PROBABILITY OF ESCAPE AND PROBABILITY OF STICKING ARE POSITIVE

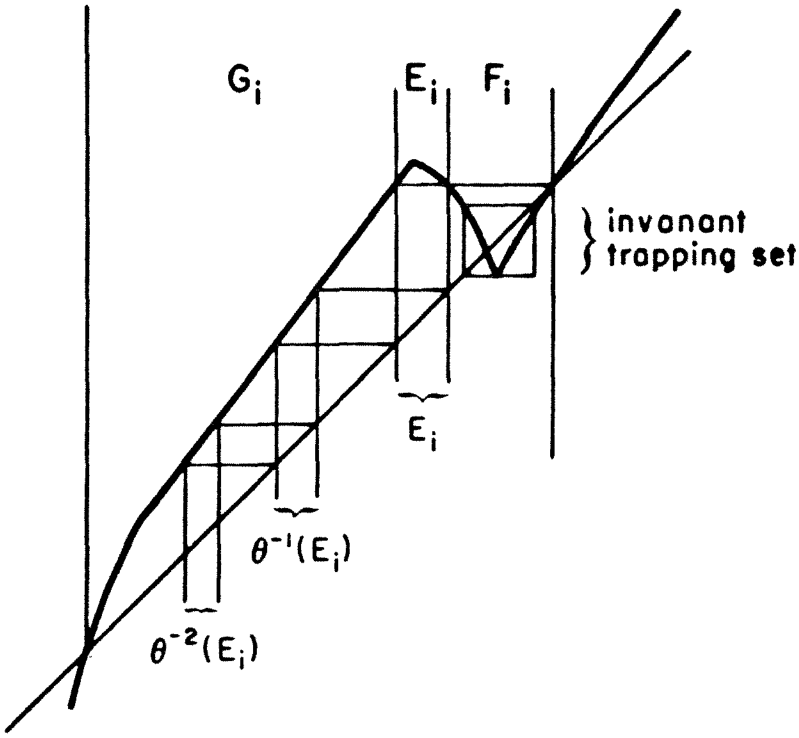
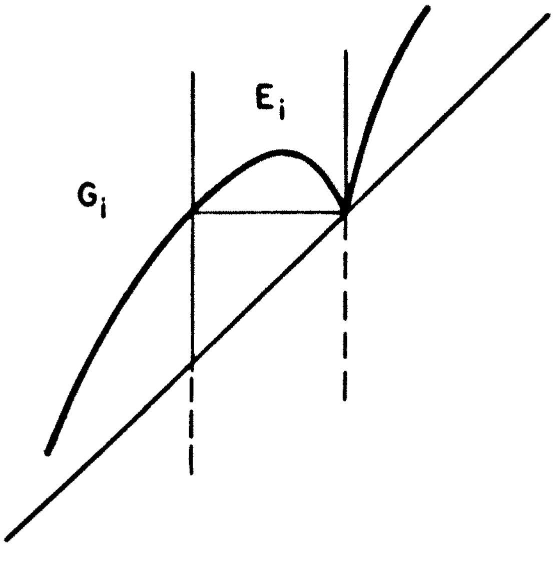
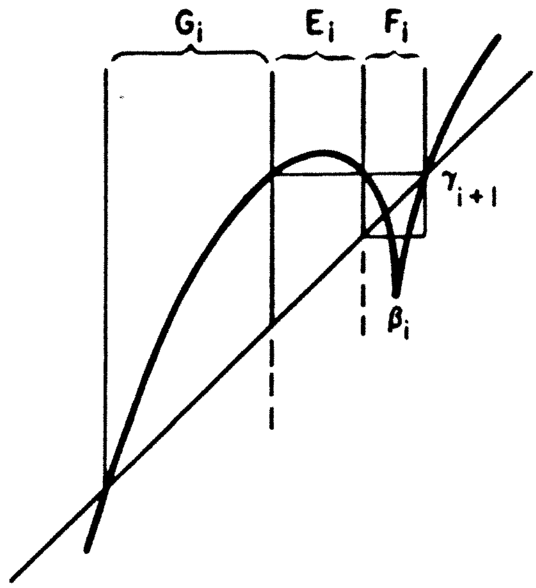


FIGURE 8: TRANSITIONS (continued)

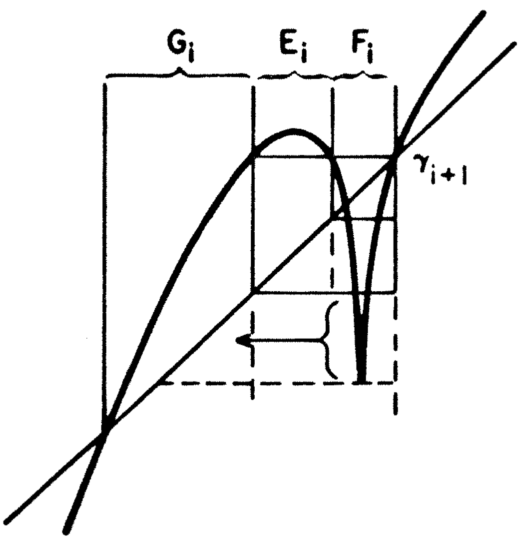
TYPE 3: PROBABILITY OF ESCAPE IS ONE.



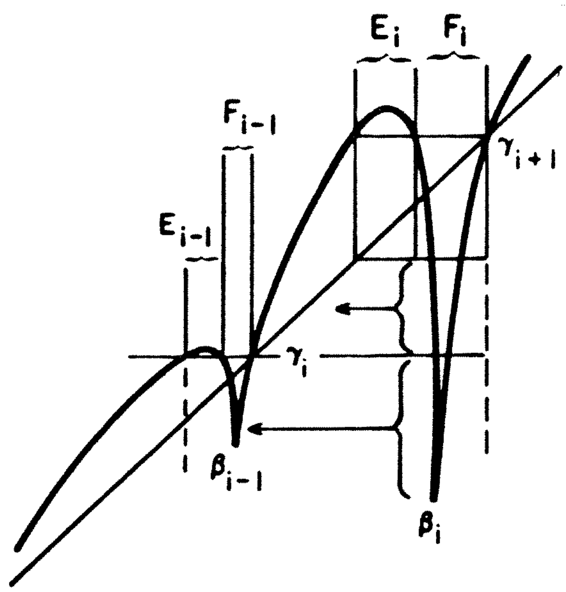
(e) growth and jump almost surely



(f) growth or fluctuations and jump almost surely



(g) growth or fluctuations and jump



(h) growth with fluctuations and reversions possible.

FIGURE 8: TRANSITIONS (continued)

between an initial period of growth and the jump to a higher regime or the reversion to a lower regime can occur.

-Figure 8 about here-

4.5 Qualitative History.

We can now think of history as a sequence of qualitative events. In order to determine the probability of given event sequences, we need to derive a conditional probability measure on the event zone Z_i. To do this, let p be a probability measure on $[0, \alpha_1]$ that is absolutely continuous with respect to Lebesgue measure, μ, so that the "density" of any point $x \in [0, \alpha_1]$ can be represented by a continuous function, say $\delta(\cdot)$, and the probability of "choosing" any $x \in [a, b] \subset [0, \alpha_1]$ is $\int_a^b \delta(x)dx$. The density $\delta(\cdot)$ can be used to represent a prior degree of reasonable belief (Jeffries probability) that population in the initial regime occurred within a certain interval. Because the trajectory $\tau(x)$ defines a one–one map $\tau : [0, \alpha_1] \rightarrow S := \{(\theta_n(x))_{n=0}^{\infty}\}$ the measure p can be considered to be a probability measure on S.

Let S_i be an event in Γ_i and suppose $p(\Gamma_i) > 0$. Then $p_i(S_i) := p(S_i)/p(\Gamma_i)$ is the conditional probability of the ith event given that the ith reaching event has occurred.

Using this concept, qualitative history can be given a rigorous treatment within the framework of our theory. Indeed, we have the following theorem whose proof is given in the Appendix and which characterizes the chance that given trajectories exhibit specific characteristics. We recall that "events" can be associated either with the fission of groups or the switch in techno–infrastructures.

THEOREM 1:

(i) If $\Gamma_i^s \neq \emptyset, h_i = 0$ *and* $t_i = 1$, *then the* i*th sticking event,* T_i^s, *has conditional probability one,* $p_i(T_i^s) = 1$.

(ii) If $\Gamma_i^s \neq \emptyset, h_i \geq 1$ *and* $t_i = 0$, *then the* i*th growth event,* J_i^g, *has conditional*

probability one, i.e., $p_i(J_i^g) = 1$.

(iii) If $\Gamma_i \neq \emptyset$ *and there exists* $j < i$ *such that* $j + h_j > i$, *then the* i*th skipping event* J_i^k *has conditional positive probability, i.e.,* $p_i(J_i^k) > 0$.

(iv) If $\Gamma_i^* \neq \emptyset, h_i \geq 1$ *and* $t_i = 1$, *then the* i*th growth event* J_i^g *and the* i*th sticking event* T_i^s *have positive probability, i.e.,* $p_i(J_i^g) > 0, p_i(T_i^s) > 0$ *and* $p_i(J_i^g) + p_i(T_i^s) = 1$.

(v) If $\Gamma_i^* \neq \emptyset, h_i \geq 1$ *and* $t_i = 2$ *or* 3, *then the* i*th growth event* J_i^g *has positive probability and the* i*th local chaos event* J_i^{lc} *have positive probability. Moreover, if Condition E is satisfied, then the probability of jumping is unity, i.e.,* $p(J_i) = p(J_i^g) + p(J_i^{lc}) = 1$. *If this condition is not satisfied, then the probability of sticking in* Z_i *is nonnegative, i.e.,* $p(T_i^s) \geq 0$ *and* $p(J_i^g) + p(J_i^{lc}) + p(T^s) = 1$.

(vi) If $i \geq 2, \Gamma_i^* \neq \emptyset, h_i = 0$ *and* $t_i < 3$, *then the* i*th trapping and reversion event* T_i^r *has positive probability. If in addition Condition E is satisfied, then* $p_i(T_i^r) = 1$.

(vii) If $i \geq 2, \Gamma_i^* \neq \emptyset, h_i \geq 1$ *and* $t_i < 2$, *then the* i*th growth event* J_i^g *and the* i*th reversion event have positive probability. If in addition Condition E is satisfied, then* $p(J_i^g) + p(T_i^r) = 1$. ▪

Consider now sequences of events $S_1, \ldots, S_q$ that constitute a qualitative history in terms of the event zones $Z_1, \ldots, Z_q$. Suppose, for example, that $p(T_1) > 0$. Then $p(N_i) > 0$, $i = 2, \ldots, q$. History could be trapped in the first event zone with positive probability, experiencing endless fluctuations, perhaps with re–switching between phase structures θ_1 and θ_2 in the interval (ψ_1, γ_2).

Constrastingly, suppose $h_j \geq 1, j = 1, \ldots, r-1$. Then, $p\big(\theta(x) \in \bigcap_{j=1}^{r-1} J_j\big) > 0$. If in addition, $t_j \neq 1, j = 1, \ldots, r-1$, then $p(\tau(x) \in \bigcap_{j=1}^{r-1} J_j) = 1$. If, however, $t_j = 1$ for some $1 \leq j < r$, then evolution would have a positive probability of being trapped in a set bounded by the $j+1$ event threshold γ_{j+1}.

In this way we can give an exact meaning to the probability of occurrence of any qualitative history. If we are dealing with an unstable system, trajectories that do not get trapped evolve. Those that are trapped fluctuate, and those that are trapped and have large tails revert almost surely. These facts are stated formally in the last two theorems which are simple corollaries of Theorem 1.

THEOREM 2: *Evolving Trajectories: Suppose*

(i) $\Gamma_i \neq \emptyset$, $h_i \geq 1$, *and for all* $Z_j, j = 1, \ldots, i$ *Condition E prevails.*

(ii) Let $G_i := \{\tau(x) \in \Gamma_i |$ *there exists no* t *such that* $\theta^t(x) \in [\psi_j, \gamma_{j+1}]$ *with* $t_j = 1, 0 \leq j \leq i, \}$.

(iii) For all $Z_j, j = 1, \ldots, i$ *Condition E prevails.*

Then for all $\tau(x) \in G_i$ *there exists* n *such that* $\theta^n(x) > \gamma_{i+1}$.∎

Recall that the interval $[\psi_j, \gamma_{j+1}]$ traps all trajectories that enter if $t_j = 1$. The theorem states that almost all trajectories that are not trapped will evolve if the humps are big enough ($h_i \geq 1$) and the map is expansive in the zone of fluctuation.

On the other hand, if the hump is small ($h_i = 0$) and the tail large, almost all trajectories will revert to a zone of lesser index:

THEOREM 3: *Reverting Trajectories. Suppose*

$p(\Gamma_i^*) > 0, h_i = 0$, $t_i > 2$, *and Condition* E_i *prevails,*

then for any $\tau(x) \in \Gamma_i^*$, $p_i(T_i^\tau) = 1$. ∎

5. SUMMARY

(1) Evolution in this theory is driven by an unstable, deterministic (intrinsic) process, not by a random shock (extrinsic) process. (2) Nonetheless, the probabilities of various possible historical scenarios can be derived in terms of sequences of qualitative "events." (3) If the socioeconomic menu is finite, then evolution in terms of continued progression to "higher" regimes eventually comes to an end. (4) If the map $\theta(\cdot)$ is unstable and closed,

then history must involve endless fluctuations eventually sticking within a given regime or cycling in a nonperiodic fashion through an endless reversion sequence of regimes. This case could be called the *Hindu paradigm*. (5) If, by way of contrast, there were one last reachable regime with a stable stationary state, world history could converge after possibly many periods of local chaos to a classical equilibrium.

It is easy to see that the probability of trapping events is increased if *ceteris paribus* successive infrastructures are "large." They must occur if at least one peak is "small" ($h_i = 0$). But a given hump h_i can be increased by *decreasing* the successive infrastructure M_{i+1} while maintaining productivity (increase $g(M_{i+1})$). *Thus, the key to continued evolution is the identification of new socioeconomic infrastructures that overcome the internal and external diseconomies of population.*

An improvement in production within a given techno– infrastructural regime is also a possible way, one that we have not incorporated in the present analysis so as to highlight a new point of view. But it should be clear that improvements in technological productivity alone that leave unchanged the techno- infrastructural thresholds M_i and N_i can only accelerate progress through the several epoches. The moral of the theory would seem to be that it is the creative human faculty focussed on the design for group living that is the ultimate resource in a finite world.

APPENDIX

PROOF OF THEOREM 1:

(i) *In this case $\theta(Z_i) \subset Z_i$ so all trajectories that enter Z_i remain there. Because the stationary state in Z_i is unstable, fluctuations persist. These may converge to cycles or they may be chaotic.*

(ii) *Here $\theta(\beta_i) \geq \beta_i = \gamma_{i+1}$. In this case γ_i and possibly β_i (if $\theta(\beta_i) = \beta_i$) are fix points, but by assumption they are repellent. By concavity $\theta(x) > x$ all $x \in (\gamma_i, \beta_i)$ so except for the countable sequence $\theta^{-t}(\beta_i)t = 1, \ldots$ (if β_i is a fix point) for all x there must exist an n such that $\theta^n(x) > \beta_i = \gamma_{i+1}$.*

(iii) *$\Gamma_i \neq \phi$ implies there exists $j, 1 \leq j < i$ with $j + h_j \geq i$ such that $p(J_h) > 1$ all $1 \leq h \leq j$. Consequently, $\Gamma_j^* \neq \emptyset$ and $p(J_j) = \Pi_{h=1}^{j} p(J_h) > 1$. Let E_j be the escape interval in j. It is nondegenerate because $h_j > i - j$. By continuity and concavity of $\theta(\cdot)$ there exists a nondegenerate interval $E^{h_j} - j \subset E - j$ such that $\theta(x) \in Z_{j+h_j}$ for all $x \in E_j^{h_j}$. Let $E_j^* := \{\tau(x) \in \Gamma^* - j | x_{n_j}(x) \in E^{h_j} - j\}$. Then $p_j(E^{h_j} - j) = p(E^* - j)/p(\Gamma^* - j) > 0$ but $p(J_i^k) \geq p_j(E_j^{h_j})$.*

(iv) *Here $\theta(\beta_i) < \beta_i$ and γ_{i+1} the event threshold for event $i+1$ has two preimages $\phi < \psi$. Let $\gamma = \gamma_{i+1}$. I shall call the open interval $E := (\phi, \psi)$ the **jump interval** and the set $F := (\psi, \gamma)$ the **fluctuation interval**. Any trajectories that enter E, jump to a higher zone, i.e., for any $x_n \in E, x_{n+1} = \theta(s_n) > \gamma$. The set $\gamma^{-n}(E)$, therefore, gives the set of points that jump into a higher zone after $n+1$ periods and the open set $J := \bigcup_{n \in \mathcal{N}} \theta^{-n}(E)$ gives the set of all initial conditions that eventually jump.*

The map θ_{z_i} is an increasing C_1 diffeomorphism so its inverse image $g := \theta_{z_i}^{-1}$ is likewise an increasing diffeomorphism from $[\gamma_i, \gamma]$ to $[\gamma_i, \phi]$. For all $x \in (\gamma_i, \phi), \theta (x) > x$ so $g(x) < x$. Let $(\phi^n)_{n \in \mathcal{N}}$ be two sequences defined by $\phi^{n+1} = g(\phi^n)$

and $\psi^{n+1} = g(\psi^n)$ *respectively. Because* $g(x) < x$ *all* $x \in (0, \phi)$

$$\phi^{n+1} < \psi^{n+1} < \phi^n < \psi^n$$

all $n \in \mathcal{N}$. *Because 0 is the only fixpoint on* $[\gamma_i, \phi]$ *we have*

$$\lim_n \phi^n = \lim_n \psi^n = 0.$$

Because ϕ *has only a single inverse image on* $(0, \psi)$,

$$\begin{array}{lclcl} \theta^{-1}(E) & = & \big(g(\phi), g(\psi)\big) & = & (\phi^1, \psi^1) \\ \theta^{-2}(E) & = & \big(g(\phi^1), g\psi^1)\big) & = & (\phi^2, \psi^2) \\ \vdots & & \vdots & & \vdots \\ \theta^{-i}(E) & = & \big(g(\theta^{i-1}), g(\psi^{i-1})\big) & = & (\phi^i, \psi^i) \end{array}$$

so $J_i^g = \{\tau(x) \in S | x \in \bigcup_{i=1}^{\infty} (\phi^i, \psi^i) = J \subset\subset Z_i\}$ *and* $p(J_i^g) = \sum_{i=1}^{\infty} p(\phi^i, \psi^i) <$ 1. *Obviously,* $\theta(F) \subset F$. *Moreover, by an argument similar to the preceding, let* $T_1^\delta = \{\tau(x) \in S | x \in \bigcup_{i=1}^{\infty} \theta^{-1}(F)$ *where* $\theta^{-1}(F) = \big(\theta^{-1}(\psi), \theta^{-1}(\gamma)\big)$. *Because* $\theta_{[\gamma_i, d_i]}$ *is monotonic increasing any trajectory in* S *must either enter* E *or* F. *Consequently,* $p(J_i^g) + p(T_i^\delta) = 1$.

(v) By the same argument as that used in case (iii), the probability of the growth event is positive. Hence, $p(J_i) > 0$. *Now consider the fluctuating set* $F = [\phi, \psi]$ *where* ϕ, ψ *are the preimages of* γ_{i+1}. *Because* $t_i = 2, \phi_i < \theta(\beta_i) < \psi_i$. *Hence, there exists a set* $E_j^1 \subset F$ *such that for all* $x \in D_j^1\, \theta(x) \in E_j$. *Hence, all trajectories that enter* $\cup_t \theta^{-t}(E_j^1)$ *escape. But by an argument similar to that used in case (iii), this is a measurable set, so* $p(J^{1c}) > 0$.

If $\theta'(x) < -1$ *for all* $x \in [\psi, \gamma_{i+1}]$, *then it is expansive and using Pianigiani [1979, 1981] almost all trajectories escape* F, *i.e.,* $p(J_i) = 1$. *An obvious extension of the argument can be used where* $t_z 3$.

(vi) Obviously, $p(\Gamma_{i+1}) = 0$. *Since* $t_i < 3$ *there exist sets* $E_1, \ldots, D_{t_i - 3}$ *such the* $\theta(x) \in Z_{i-j}$ *for all* $x \in E_j, j = 1, \ldots, t_{i-3}$. *But by our familiar techniques, we*

can show that for almost all $x \in \Gamma_i^* n > n_i(x)$ such that $\theta^n(x) \in E_j$ for some $j = 1, \ldots, t_{i-3}$ so almost all histories revert.

(vii) Combining the arguments of Cases (v) and (vi), the result is obtained.

This completes the discussion of Theorem 1.

NOTES

The present study was initially stimulated by discussions at Harvard in 1976 with J. Sablov concerning the Classic Mayan Collapse. Sablov later organized an advanced seminar of the Center for American Studies in Santa Fe in the Fall of 1978 involving archaeologists L. Cordell, C. Renfrew, and E. Zubrow, a mathematician K. Cooke, a philosopher J. Bell, and the first author, an economist. A book resulted with essays by the participants: Sablov [1980]. Further inspiration was provided by successive interdisciplinary conferences sponsored by Ilya Prigogine's Institute in Theoretical and Applied Thermodynamics in March 1982 and March 1984, and by a continuing interdisciplinary seminar led by A. Iberall, physics, and including A. Moore and D. White, anthropology, L. Goldberg, biology, R. Baum and D. Wilkenson, political science, P. Wohlmuth, law and the first author. All of the computations and diagrams for this study were prepared by Weihong Huang. Part 4 of the paper has benefited from discussions with Guilio Pianigiani. Although the first author's original notes on the subject were set down more than a decade ago, and these served as the basis for a lecture on the subject at the University of Paris in the Spring of 1985, it was during a month of research at Gunnar Eliasson's Industrial Institute for Economic and Social Research in October 1986 that he finally began to put these ideas in a form suitable for publication.

1. A splendid, boldly synthetic survey of this vast process that vividly portrays its dynamic character will be found in Barraclough [1984]. See also Sherratt [1980] for another helpful overview for the nonspecialist.

2. Deevey, for example, bases his survey on seven major epoches, while at another extreme, Easterlin [1983] uses three gross epoches, those of hunting and food collecting, settled agriculture and modern growth. The present theory can actually accommodate as many or as few distinct regimes as is meaningful or that are convenient for the purpose at hand. See Day [1978, 1987] for very general expositions.

3. Barraclough p. 52. See also Sagan [1985, pp. xvi– xxiii] on "complex societies" in this context.

4. If effort allocated to management were freely substitutable for work, then we could derive aggregate production as a function of group size alone by using an efficient combination of the two inputs. Thus we could define $f(x;N) := \max_{M,L}\{g(M)h(L)k(S)|M+L \leq x, S = N - x\}$. Indeed, any number of separate types of effort could be subsumed in this way. Suppose, however, that there is a residuum of effort that is not substitutable within a given regime. Such a socially nonfungible type of effort is what we associate with the variable M.

5. Evidently, $nf(x/n) \geq 0$ all $x \in V_n, n \leq \bar{n}$ and on $V_n \cap (0,\bar{x})$. Note that the effect of the externality is to compress the production function "downward and backward." Thus $V_n = (\bar{n}M, \min\{\bar{n}N, \bar{x}\})$ and $F(x) \leq \max_n\{nf(x)\}$. Note that $V_n \cap V_{n+1}$ may be empty. Thus, suppose $N \leq 2M$ then $V_1 \cap V_2 = \emptyset$. In general, if $nN \geq (n+1)M$ then $V_m \cap V_{m+1} = \emptyset, m = 1,\ldots,n$. From this it does *not* follow that $V_n \cap V_{n+1} = \emptyset$ all n. But suppose $nN \geq (n+1)M$ then $V_n \cap V_{n+1} \neq \emptyset$. If $N \geq 2M$ then it follows by induction that $V_n \cap V_{n+1} \neq \emptyset$ all n.

6. The set $V^i := \cup_{n\in\mathcal{N}} V_n^i \cap (0,\bar{x})$, where $V_n^i := (nM^i, nN^i)$, is the feasibility domain for each technology $i \in \tau$. and the set $V^* := \cup_{i\in\tau} V^i$ is the *aggregate feasibility domain* for G defined on $\mathcal{T}$. The set $\mathcal{R}^+ \setminus V^* := V^o$ is the *null domain* given the menu $\mathcal{T}$.

7. All of this can be encompassed within a standard economic decision-making framework as is outlined in Day, Kim and Macunovich [1986] which reviews the empirical background.

8. The externality factor could be written in terms of population density. Let d be the maximum possible average density and S the total space available for a given regime. Then $\bar{x} = Sd$.

9. Note that each phasestructure has three potential "sub-regimes" with corresponding "sub-phasestructures." These are

$$\theta^i(x_t) = \theta^{i_o}(x_t) := 0 \text{ when } F^i(x_t)/x_t \leq c^i \tag{7a}$$

$$\theta^i(x_t) = \theta^{i_1}(x_t) := \alpha^i F^i(x_t) - \alpha^i c_i x_t \text{ when } G^i(x_t)/x_t \leq c_i + \lambda^i/d^i \tag{7b}$$

$$\theta^i(x_t) = \theta^{i_2}(x_t) := \lambda^i x_t \text{ when } F^i(x_t)/x_t \geq c^i + \lambda^i/\alpha^i \tag{7c}$$

Of course, when $x_t \notin (M^i n, N^i n)$ for some n then subregime i_o holds. Suppose there exists an A such that $F^i(x)/x \geq \lambda^i/\alpha^i + c$, then there is a set made up of a finite union of intervals on which $F^i(x)/x \geq \lambda^i/\alpha^i + y^i$ where each interval corresponds to a particular value of n, the efficient number of groups. In this case there must be values of A where subphase i_1 holds.

The switching among subphases within a regime is governed by the average product. When it is below the threshold c people cannot or will not raise children. Above c_i but below $\lambda_i/\alpha_i + c_i$ the preference for children is manifest but may be less than or greater than the number required for a growing population. When the average product is above $c_i + \lambda_i/\alpha_i$ population growth is exponential, the rate of growth being determined by the adult mortality rate and the minimum of the maximum possible number of children and the maximum desired number of children, λ_i.

10. An attempt to estimate parameters using econometric methods would be interesting, perhaps even worthwhile, but not justified for our illustrative purposes and given the data

at hand. Crude estimates will be sufficient for illustrative purposes and gives us a good idea if further research along this line is warranted.

11. Using a function stretching argument analogous to that exploited in Day [1982], one can now easily derive conditions on the underlying group production functions in $\mathcal{T}$ for the existence of a "chaos" point $x \in \mathcal{V}^*$ such that $\theta^3(x) < x < \theta(x)$. The implication is that under these conditions, which essentially mean that some technologies are productive enough, there exists an uncountable scrambled set C such that $\omega(C) = C$ (where $\omega(C)$ is the limit set for all trajectories originating in C and such that $\tau(x)$ is chaotic in the sense of Li and York [1976] for all $x \in C$.

Moreover, if a point $z \in \mathcal{V}^*$ and an odd interger or such that $\theta^n(z) < z < \theta(z)$ (or such that $\theta^n(z) > z > \theta(z)$ then, similarly, according to Li, Misiurewicz, Pianigiani and York [1978], a scrambled set exists. Points that satisfy this $LMPY$ condition are easy to find in the sample trajectories shown in Figure 6.

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