IFN Working Paper No. 1207, 2018

Analysts' Disagreement and Investor Decisions

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Abstract

Earning forecasts disclosed by financial analysts are known to be overly optimistic. Since an investor relies on their expertise, the question arises whether he would take analyst recommendations at face value or instead structure consultation with differently upward-biased analysts in a way that would permit him to make more accurate investment decisions. We characterize disagreement in a strategic disclosure game where two analysts disclose to an investor who has commitment power. This setup delivers an explanation of why "de-biasing" occurs naturally when disagreement carries through the disclosure process itself. Our results suggest that consulting more than one analyst permits the investor to make more accurate decisions, even if both analysts overstate their recommendations. We generalize our findings to the case of noisy observation.

JEL classification: G11; G14; G17; D83.

Keywords: Strategic Information Transmission, Disagreement, Upward-biased Experts, Commitment Power, Noisy Observation.

*The author is indebted to Anjan Thakor for first and helpful suggestions. Mailing addresses: School of Economics, Free University of Bozen/Bolzano, Universitätsplatz 1, 39100 Bozen, Italy, and IFN, Research Institute of Industrial Economics, Grevgatan 34 - 2 fl, 10215 Stockholm, Sweden. E-mail addresses: wolfgang.gick@unibz.it and wolfgang.gick@ifn.se . This work was supported by the Free University of Bozen/Bolzano [grant numbers E-RA-12 and E-RA-12-2]

When the experts are agreed, the opposite opinion
cannot be held to be certain.
When they are not agreed, no opinion can be
regarded as certain by a non-expert.

Bertrand Russell

1 Introduction

There is little doubt about the value that financial analysts bring to the functioning of financial markets. Analysts make markets work as they deliver valuable trading recommendations to investors, and their disclosure helps to improve market efficiency. Sell-side analysts are inclined to overstate the earning potential of a stock, a known fact since Trueman (1994), who shows that analysts forecasts do not properly reflect investor's earnings expectations.

Malmendier and Shanthikumar (2007) use trade figures to document such distortions. The empirical puzzle that follows from their findings is that some investor groups could be systematically misled by analysts as a whole, thus making suboptimal investment decisions. The question that arises is whether investors are indeed misled by upward biased analysts and buy stock in expectation of much higher returns and lose money, to learn too late from past experience, or whether they actually *are* capable to offset too optimistic information conveyed by analysts in the underlying disclosure process.

There is some evidence that investors actually do *not* take analyst recommendations at face value. Agrawal and Chen (2008) suggest that even in the presence of overly optimistic stock recommendations, investors are able to put the strategically released information in context, to so play one analyst's forecast against the other:

> “The notion that investors are victims of biased stock research presumes that (1) analysts respond to the conflicts by inflating their stock recommendations and (2) investors take analysts’ recommendations at face value. Even if analysts are biased, it is possible that investors understand the conflicts of interest inherent in stock research and rationally discount analysts’ opinions. This alternative viewpoint, if accurate, would lead to very different conclusions about the consequences of analysts’ research." (Agrawal and Chen, p. 504)

A framework that would explain the underlying logic of their statement requires three ingredients. Firstly, it needs to take the concept of a “bias" seriously. Secondly, it needs to build on more than one sender. Clearly, a game with one investor and one analyst cannot explain disagreement between analysts. Thirdly, it needs to incorporate the impact of disagreement. In our view, the only sensical concept for such a framework lies i the use of disagreement that naturally emerges in a setup with two analysts. We present such a framework.

Disagreement among informed players is an argument that has been often overlooked. In the field of finance it has first been systematically studied in a recent two-player model by Thakor (2015). His contribution studies voluntary disclosure, with information being transmitted to an investor through the narrative section of a company’s annual report, via media, or via analysts. Specifically, Thakor’s theory builds on an agency setup where prior beliefs are heterogeneous. By applying Milgrom’s Representation Theorem (Milgrom, 1981) he is able to derive an endogenously determined level of disclosure. In a nutshell, a first agent (a company) has a binary disclosure space. After the disclosure of the firm, a second player (a bank) is invited to contribute to the project. If the first agent discloses nothing, the second agent cannot recur to priors. Under some form of commitment power, disagreement then positively affects equilibrium behavior in that the first agent may indeed be better off by not fully disclosing his information. The reason behind this is that disclosure now induces a trade-off between obtaining financing and the cost of financing, which itself varies with the disclosure level. In his model, disclosure comes with the transfer of control rights, and disagreement is linked to different levels of disclosure. Because of heterogeneous priors, better types of firms will disclose less, a property akin to the results found in the literature on counter-signaling (Feltovich et al., 2002; Chung and Esö, 2013).

Stocken (2000) shows in a model of repeated cheap talk that the sender can build up reputation, permitting the receiver to build her decision on truthful disclosure. Morgan and Stocken (2003) drop repeated interactions. Like our paper, they build on seminar work by Crawford and Sobel (1982) (CS hereafter), their model is similar to ours. However, they consider uncertainty about the bias level between analyst and investor. Overall, Morgan and Stocken (2003) find two general ways how an investor may structure communication with one analyst. Analysts can give advice to buy, sell or hold stocks, and expertise arises endogenously. When there is uncertainty about the degree of bias divergence, there exists a class of *ranking system equilibria* in which the sender will generally tend to issue too many favorable reports, while there are other equilibria with less biased information.[1] Our paper extends their work as we permit disagreement between the two senders. The intuition behind is that the more biased sender is less informed.

A clarification is in order. As Morgan and Stocken (2003), also our paper uses the concept of a "bias" as a fixed number by which the players diverge from their bliss point (or the point they consider optimal along a one-dimensional line). This clarifies the difference between our concept vis-a-vis the "systematic" bias concept as used in Hilary and Hsu (2013). Like in CS and the literature on strategic information disclosure, we understand bias as a fixed number by which the preferences of each players differ, independent of what the analyst observes. This permits us to disentangle the true observation of a piece of information (e.g. the accurately measured stock value) from what an analyst would like to make the investor believe this value should be to maximize his own in equilibrium. We so distinguish between the preferences and the resulting *equilibrium disclosure* in a game of strategic information transmission.

More generally, each analyst may well want to overstate the true value by some number (bias), but in contrast to Hilary and Hsu (2013) this does not imply that an investor will believe that this holds for any possible disclosure. As a result, the existence of biased analysts does not prevent an investor to "understand" the intended overstatement as expressed in Agrawal and Chen (2008) in the presence of the second analyst. To see this, assume that an analyst, capable of accurately forecasting the value of a stock performance - call it θ - overstates this value by a fixed amount called b. In equilibrium, this overstatement does *not* imply that the investor will *generally* believe that the true value be $\theta + b$ for all θ, even if this were best for the analyst. In other words: "de-biasing" has to

[1] See also Beyer et al. (2010) for a detailed overview of the literature

be considered *in equilibrium*, an this depends on the underlying disclosure game and the underlying equilbrium concept. Put differently: the divergence of interest between two analysts may be used by the investor to "de-bias" their disclosure to make more accurate decisions as the investor can commit to specific ways of incorporating the disclosure of one analyst versus the other in equilibrium.

As we differ from the existing multisender literature on strategic information transmission suche Krishna and Morgan (2001) (KM hereafter) as we introduce commitment power, our paper is also close in spirit to ongoing research based on the concept of Bayesian Persuasion. To our knowledge, Friedman et al. (2016) is the first paper that models a game between a firm who can design an information system, and an outsider. In their Bayesian Persuasion model, disclosure is considered in a similar way.[2]

The remaining relevant literature can be summarized as follows. Easterwood and Nutt (1999) as well as Li and You (2015) have ventured into research on the interplay between prior beliefs and resulting forecasts. Easterwood and Nutt (1999), with an eye on forecast precision, find that analysts primarily engage in too optimistic forecasts, delivering consistent support for analysts adding value by increasing demand. Yet, none of these authors take into account strategic disclosure as we do in the present paper (see also Hansen, 2015, for an overview) .

As we model disclosure in a game-theoretic setup, our paper also differs from purely Bayesian approaches such as found in West (1988), a paper that completely abstract from modeling disclosure and its inherent strategic considerations. West does not consider any equilibrium concept but assumes that the statement of any expert, be it through his distribution (completely or in quantile form) always updates the distribution, therefore ignoring incentives that may lie in the interest of one analyst to disclose differently when a competitor is present.[3]

The remainder of our paper is structured as follows. Section 2 briefly introduces the theory and presents the equilibrium concept, followed by a detailed example. Section 3 discusses the properties of commitment and disagreement, and Section 4 concludes.

[2]For a Bayesian Persuasion game between a regulator (supervisor) and a continuum of investors see Gick and Pausch (2012). We further refer to Bayesian Persuasion in sections 2.3 and 3.

[3]See also Lindley (1983) on multivariate logistic normal models for discrete probabilities in this context.

2 A Theory of Financial Advice

2.1 Game-theoretic background

In their seminal paper on strategic information transmission, CS model a sender-receiver game in which one informed expert R sends a signal to an uninformed decision maker, or receiver, R. The expert observes the state of nature, possibly under some form of noise. After this observation, he discloses a signal m to the decision maker who takes an action that affects the utility of both players. Both have single-peaked preferences based on a "uniform-quadratic" specification, which is standard in this class of games. Their relative bliss point is called their *bias*, which directly enters their utility function. Typically, R's bias is normalized to zero, while the expert has a positive bias, $b > 0$.

Neither player's utility depends on the signal m - a property inherent to cheap-talk models. In the CS game, there are always equilibria in which no information is transmitted. The main contribution is that information transmission is coarse in equilibrium, and it occurs in intervals. To which extent information is informative indeed depends on the bias. The existence of a bias inevitably leads the sender to withhold some information, and this is independent of the state (or the market situation) observed. CS thus tell *ex ante* what kind of equilibrium messages will result for all possible states of nature.

Except for the case of perfect preference alignment and no bias, there is no equilibrium in which the sender reports truthfully. The reason behind is that the receiver would then implement his best action, which differs with the sender's bias. Once a bias exists, it cannot be any longer an equilibrium strategy of the sender to tell the truth. CS show that for small biases, there exists a unique equilibrium with partial information revelation, which yields the highest expected utility for both players, with a more informative equilibrium emerging when preferences are more aligned: the closer the interests of the players are, the greater the possibilities for communication between sender and receiver.

Different messages induce different actions, and the number of equilibrium actions is determined by the bias. Once the bias gets large ($b > \frac{1}{4}$), there exists no feasible value of a state a to generate a meaningful equilibrium, thus no information is conveyed in equilibrium, and the sender "babbles" and sends messages that would not convey any information as the receiver remains as uninformed as before. For a sender bias between 0 and $\frac{1}{4}$, CS show that there exist partitional equilibria. More generally, R does better to pick an expert with a small bias, ideally with a bias close to zero, to so reach a high degree of information transmission.

Quality of information is an important criterion when dealing with multiple senders. With one sender, quality can be measured by his accuracy. An expert that observes the state of nature under lots of noise will be generally less valuable as a qualitatively reliable source than one who observes the state of nature accurately. Two-sender games show different properties, depending on their construction. The key issue of our two-sender model is that under like biases (or upward biases of both senders), *R* may use the information strategically revealed in a setup when only the first, less biased sender, is able to take into account the disclosure of the second sender.

2.2 Setting the stage

To give a first illustration of the transition from one to two senders, consider the borderline case with one sender and $b > \frac{1}{4}$. There is only a babbling equilibrium. We now ask, what would happen with two senders, one with $b < \frac{1}{4}$ and the other babbling? More generally, can we say something about how two combine two CS profiles under simultaneous disclosure that would increase the amount of information transmitted?

As a starting point, consider the following illustration with two senders. Each sender has a specific bias that, when playing the CS game alone with the receiver, would be able to break down the unit interval into two partitions. Assume the investor has access to two possible analysts, a less biased independent analyst with a bias of $b_1 = 1/12$, and a more biased institutional expert, with $b_2 = 1/8$. Playing the CS Best Equilibrium with the less biased sender will reveal him that the true state θ (a number between 0 and 1) is either below $1/4$ or above, that is between $1/4$ and 1.[4] When playing the same game with the more biased sender, it will permit him to learn that the state is either below $1/3$ or above. An illustration is given in *Fig*.1 below:

[4]A CS Best Equilibrium is the most informative one among all that are played in CS, that is, the equilibrium with the most partitions. It survives the so-called NITS refinement condition (Chen et al., 2008).

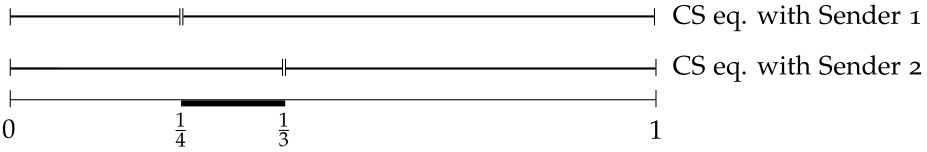

Fig. 1: How to combine two messages under simultaneous disclosure? ($b_1 = \frac{1}{12}, b_2 = \frac{1}{8}$)

What we analyze here is *how* such a disclosure game will be realistically played with two senders, given the biases being known by the receiver. The one-sender one-receiver game leads to a well-known result: whenever an uninformed receiver has the choice among senders with different biases and he can pick one sender, he will be best off taking the least-biased sender.

To proceed toward an analysis with two senders, we need to make additional assumptions on the disclosure process, to then determine how disclosure is structured and what equilibria emerge, given specific message rules, in short, we need to define the equilibrium concept. This has been done in previous articles, but in a different manner. In a seminal paper with two senders and a continuous state space, KM already mention that if two experts are unaware of each others' existence, the consultation of two would simply result in the overlap of two CS partitions. Given that, it would be clear, asking a second, differently biased sender, that a more informative equilibrium is generally being reached. Considering *Fig. 1* again, the assumption of the two senders being unaware of each other's existence, would in fact result in three possible actions: one between zero and $\frac{1}{4}$, one between $\frac{1}{4}$ and $\frac{1}{3}$, and a third between and $\frac{1}{3}$ and unity. Still, the assumption may be considered as way too strong and therefore difficult to maintain as it simply rules out any shared information between two experts.

The other polar case, which has been discussed at the center of KM's work, is the game with two senders and sequential disclosure. One sender is asked first, and the disclosure is observed by the second sender before he would disclose. While such a modeling option seems adequate for political experts and the optimal design of a cabinet of advisors that may typically have opposing biases. In that case, even full revelation may be the outcome.

KM's original setup for upward biases with two senders leads to results without disagreement. Once two experts are biased into the same direction and their disclosure is sequential *in that one expert discloses, and all players observe this disclosure, plus each player knows this being common knowledge*, there is no way to make consultation of both experts *ex-ante* Pareto superior compared to only consult the less biased sender. KM's model has been tailored to describe information transmission in political speech. One should add that this seemingly general result is highly specific. It unnecessarily precludes the second sender to improve on information transmitted as he is biased in the same direction. Sequential disclosure leaves him only a subset of the state space. Such a concept may be realistic to their strict form of disclosure; the world of finance is one that, in our view, clearly differs. To argue that an investor would publicly first consult one expert, this disclosure afterwards being exactly observed by a second expert who then would disclose last, again observed by all players, is an unrealistic setup in the field of financial disclosure.

To expand on the above observation of KM's two polar cases, we ask: is there an equilibrium construction that would permit to overcome the inherent limits to informativeness inherent to the construction in KM? In other words: is an investor able, through structuring communication with the two analysts, to improve upon information transmission?

To introduce this option, we utilize a simple ranking for structuring the disclosure process. Instead of giving both analysts the same role while keeping different biases, we follow some properties on disclosure with like-biased senders but now we make assumptions that further distinguish the senders according to their bias.[5] This is akin to Mechtenberg and Münster (2012) who characterize a game in which the third player becomes an outside mediator, a player without a bias who plays an exogenous role in the disclosure process. Lastly, Shimizu (2014), building on Austen-Smith (1993), has a setup with partially informed senders and shows that disagreement leads to a more informative result for R when both experts are biased into the same direction.[6]

[5] Some of these properties have been studied in Gick (2006) and Gick (2009).

[6] Lu (2017) has offered a complete treatment of coordination-free equilibria with two senders where all players are informed.

2.3 Structuring the disclosure process with analysts

Before characterizing our model, it is worthwhile to highlight other properties expressed in KM, to so distinguish our setup as a viable solution that differs from theirs. Note that KM *Lemma 2, (p. 760)* has shown that there are no fully revealing equilibria (FRE) with two like-biased senders. Already the earlier literature of expertise in political science (see Gilligan and Krehbiel, 1989; Austen-Smith, 1993) has shown existence of fully revealing equilibria (FRE) where the decision maker can extract full information, thus exactly learning the state of nature. In this equilibrium, both senders observe the true state and truthfully reveal it to R.[7] Specifically, KM have shown that when senders have like biases, the resulting equilibria induce at most a finite number of actions. This rules FRE out in their setup as FRE must be monotonic and involve an infinite number of equilibrium actions.[8]

FRE have been considered realistic in situations where the decision maker can structure communication in a way that permits him to directly play one expert's disclosure against the other.[9] An investor can hardly be assumed to be powerful enough to dictate punishments to two analysts in a way that would make an FRE the surviving equilibrium. Put differently: an investor will generally show a divergence of interest with "distant" and institutional analysts, while typically approaching an analyst with a lower upward bias. This permits the investor to structure the process with the latter, still disciplining him by committing to take the higher-biased analyst's disclosure into account.

What we analyze in our paper is a game that shows some distant similarity to KM in that there is asymmetry in the disclosure process as we introduce commitment power. To reach this result, we drop KM's assumption that disclosure is sequential. In a game with one less and one more-biased expert, we realistically assume that an investor will pick the

[7] Battaglini (2002) has delivered a full characterization of FRE. To illustrate, assume two like-biased senders and a receiver. Both senders observe the same state. The receiver must be able to construct punishment strategies (to implement a much lower state, closer to his own ideal point), to so "punish" both senders when deviating from truth-telling. Such equilibria exist, and they show attractive welfare properties. However, they are not stable under noise, as already Battaglini (2002) has shown.

[8] Note also that FRE rely on the assumption that the two experts observe the *exact* same state of nature. Battaglini (2002) has shown that fully revealing equilibria are not stable under noise. To illustrate, assume noise in the two expert's observations, leading to two distinct and different messages sent to the decision maker. According to his posterior beliefs, the latter will need to put a comparably high probability on the fact that one sender has deviated, but a very low one that both did. Knowing this, it becomes a dominant strategy for each sender to deviate from the fully revealing equilibrium upward in direction of the senders' ideal state – illustrating the brittleness of FRE. We show in the appendix that our result also holds under noisy observation.

[9] A known example is the one of a judge who can design disclosure in a court meeting to play the defendant's disclosure against the one of the plaintiff, to so ideally learn the true state. In the context of an investor playing information disclosed by multiple analysts against each other seems hardly applicable to financial expertise.

less biased expert as his private analyst and consult him together with a say larger, e.g. institutional analyst with a much higher bias, who routinely discloses recommendations for a specific asset. This is in line with the picture of CS according to which R would do best by picking the least biased sender among many. It is straightforward to argue that a decision maker may typically approach the less-biased expert and reveal him to know of another well-known source of information about financial investment options to be disclosed at a say fixed date by the more "distant" institutional analyst, e.g. through the narrative section of a firm's annual report as expressed in Thakor (2015).

Given the investor's option to structure the communication process with two differently (upward) biased senders, R can choose a response rule that need not be a best response to *all* senders' strategies all of the time. In essence, whenever the investor has the choice among differently biased analysts, he will so be able to create a consultation mechanism with specific message vectors that renders the disclosure of two analysts more valuable than consulting only the less biased expert. The disclosure process that we offer results of course superior to the sequential KM equilibrium as well.[10] Moreover, it is intuitive for consultation setups between investors and analysts.

2.4 Equilibrium

The timing of the game is as follows. Biases are known in the sense that R and S_1 know S_2's bias. We have $b_2 > b_1 > b_R = 0$. S_2 knows his own bias but is not aware of S_1's existence. In turn, S_1 knows all players. First, R discloses the message rule to the less biased analysts. The two analysts observe the state and disclose simultaneously. The player's utility functions are single-peaked in the inverted-loss function style of CS. For the rest of the paper, we use m_1^C to label the messages of Sender 1 for which R commits to consider m_2 simultaneously, whenever applicable.

[10]Our model can furthermore be extended to cover situations where each sender would hold beliefs about the decision maker still consulting other senders with some given probability that could be endogenously determined.

Definition. An equilibrium with R's commitment power for some messages m_1^C consists of a family of signaling rules for Sender 1, called $q_{S_1}(m_1|b_2,\theta)$ and Sender 2, denoted $q_{S_2}(m_2,\theta)$ and an action rule $y(m_1,m_2)$ for the receiver such that:

(1) for each $m \in [0,1]$, $\int_{M_{S_1}} q_{S_1}(m_1|b_2,\theta)dm_1 = 1$ and $\int_{M_{S_2}} q_{S_2}(m_2,\theta)dm_2 = 1$,
where the Borel sets M_{S_i} are the sets of feasible signals for S_1 given θ and b_2,
and for S_2 given θ.

If now m_1^* is in the support of $q_{S_1}(\cdot|b_2,\theta)$ and m_2^* in the support of $q_{S_2}(\cdot|\theta)$ and R has commitment power, then

m_1^C solves $\max_{m_1 \in M_{S_1}} U^{S_1}\left(y(m_1,b_2),\theta,b_{S_1},b_{S_2}\right)$ for any b_2, and

m_2^* solves $\max_{m_2 \in M_{S_2}} \left(U^{S_2} y(m_2),\theta,b_{S2}\right)$.

(2) for each signal pair $(m_1,m_2), y(m_1,m_2)$ solves $\max_y \int_0^1 U^R(y,\theta)p(\theta|m_1,m_2)d\theta$,

with $p(\theta|m_1,m_2) \equiv \frac{q_{S_1}(m_1|b_2,\theta)\cdot q_{S_2}(m_2,\theta)}{\int_0^1 q_{S_1}(m_1|\hat{\theta},b_2)q_{S_2}(m_2|\hat{\theta})f(\hat{\theta})d\hat{\theta}}$, with $\hat{\theta} \neq \theta$, respectively. □

Note that $q_{S_2}(m_2,\theta)$ is the probability that S_2 sends m_2 for the true state (as in CS, p. 1434), and $q_{S_1}(m_1|(b_2,\theta)$ is the probability that S_1 sends m_1 observing θ and expecting b_2 to be simultaneously sent by S_2, given that both senders observe θ.

2.5 An example

As a representative illustration, consider the numerical example with $b_1 = \frac{1}{18}, b_2 = \frac{1}{12}$. Sender 1 breaks the unit interval into three sub-intervals, Sender 2 into two. The two-sender CS best equilibrium is characterized as follows. Without commitment and only playing the Best CS equilibrium strategies with S_1 would lead to break points at $\theta = \frac{1}{9}$ and $\theta = \frac{4}{9}$. Playing this game with S_2 would lead to a break point at $\theta = \frac{1}{3}$. In short, since the break point of $\frac{1}{3}$ is effectively exogenous to Sender 1, there are two cutoffs and four equilibrium actions following by indifference conditions and receiver optimality conditions.

Proposition. *The characterized equilibrium is ex-ante Pareto superior to playing CS Best Equilibrium strategies with the less biased sender.*

Proof. We start by defining the message spaces for this equilibrium. Similar to the example in Lu (2017) we argue that S_1 has only three equivalent messages that would suggest three optimal messages. Sender 2 has two messages to indicate that the state is either between zero and $\frac{1}{3}$ or between $\frac{1}{3}$ and unity. No sender observes the other sender's disclosure and there are no message rules implemented other than both senders do best by playing CS best equilibrium strategies. Here, R plays a 3-partition best CS equilibrium profile with S_1, and, given the 2-partition best CS equilibrium profile with S_2 he commits to also consult S_2 whenever S_1 discloses message m_1^2.

- Sender 1's strategy:

$$\mu_1(\theta, b_1, b_2) = \begin{cases} m_1^1 \text{ if } \theta \in [0, \frac{5}{18}] \\ m_1^2 \text{ if } \theta \in [\frac{5}{18}, \frac{19}{36}] \\ m_1^3 \text{ if } \theta \in [\frac{19}{36}, 1] \end{cases} \tag{1}$$

- Sender 2's strategy:

$$\mu_2(\theta, b_2) = \begin{cases} m_2^1 \text{ if } \theta \in [0, \frac{1}{3}] \\ m_2^2 \text{ if } \theta \in [\frac{1}{3}, 1] \end{cases} \tag{2}$$

- R's posterior beliefs for given biases are

$$P(\cdot \mid m_1, m_2) = \begin{cases} \mathcal{U}[0, \frac{5}{18}] \text{ if } S_1 \text{ discloses } m_1^1 \\ \mathcal{U}[\frac{5}{18}, \frac{1}{3}] \text{ if } S_1 \text{ discloses } m_1^2 \text{ and } S_2 \text{ discloses } m_2^1 \\ \mathcal{U}[\frac{1}{3}, \frac{19}{36}] \text{ if } S_1 \text{ discloses } m_1^2 \text{ and } S_2 \text{ discloses } m_2^2 \\ \mathcal{U}[\frac{19}{36}, 1] \text{ if } S_1 \text{ discloses } m_1^3. \end{cases} \tag{3}$$

- R's strategy is:

$$y(m_1, m_2) = \begin{cases} y_1^* = \frac{5}{36} \text{ if } S_1 \text{ discloses } m_1^1 \\ y_2^* = \frac{11}{36} \text{ if } S_1 \text{ discloses } m_1^2 \text{ and } S_2 \text{ discloses } m_2^1 \\ y_3^* = \frac{31}{72} \text{ if } S_1 \text{ discloses } m_1^2 \text{ and } S_2 \text{ discloses } m_2^2 \\ y_4^* = \frac{55}{72} \text{ if } S_1 \text{ discloses } m_1^3. \end{cases} \tag{4}$$

Note that in this example, commitment leads to one additional break point compared to the equilibrium played by consulting only the less-biased sender. A graphic illustration is given in *Fig.* 2 below. It is easy to see that commitment power creates an additional break point and thus a refined information structures for an entire disagreement region, here between e_1^1 and e_1^2:

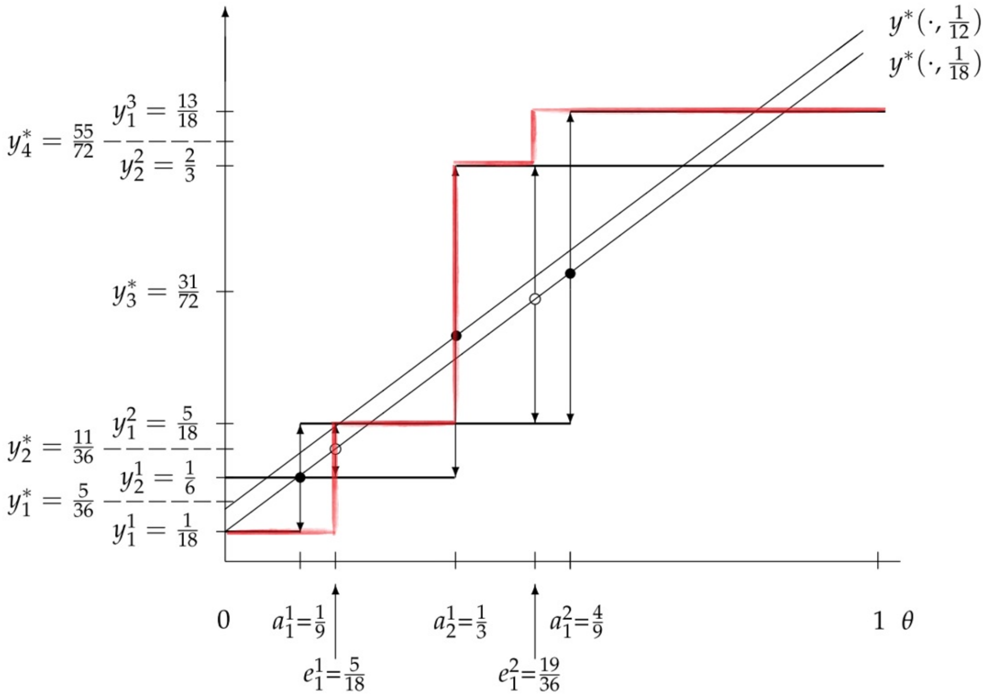

Fig. 2: Equilibrium with commitment power and $b_1 = \frac{1}{18}, b_2 = \frac{1}{12}$.

Lastly, we show that this equilibrium indeed performs better than the disclosure game with only one sender. Consider first the ex-ante utility of R in this two-sender equilibrium is

$$EU^R = -\left[\int_0^{\frac{5}{18}} \left(\frac{\frac{5}{18}}{2}\right)^2 + \int_{\frac{5}{18}}^{\frac{1}{3}} \left(\frac{\frac{1}{3}-\frac{5}{18}}{2}\right)^2 + \int_{\frac{1}{3}}^{\frac{19}{36}} \left(\frac{\frac{19}{36}-\frac{1}{3}}{2}\right)^2 + \int_{\frac{19}{36}}^{1} \left(\frac{1-\frac{19}{36}}{2}\right)^2\right] = \frac{29}{2592} \approx -0.0111188271.$$

When playing the CS game only with the less biased sender of $b_1 = \frac{1}{18}$ this would yield a lower expected utility to the R, namely

$$EU^R_{CS} = -\left[\int_0^{a_1^1} \left(\frac{a_1^1}{2}\right)^2 + \int_{a_1^1}^{a_1^2} \left(\frac{a_1^2 - a_1^1}{2}\right)^2 + \int_{a_1^2}^{1} \left(\frac{1 - a_1^2}{2}\right)^2 \right] = -\frac{17}{972} \approx -0.017489.$$

Comparing the results concludes the proof. ■

3 Commitment Power

Equilibria that exhibit commitment power have been studied in several contexts. Dessein (2002) has a paper where he considers different communication modes, depending on the bias. The idea of commitment also relates to the literature of Bayesian Persuasion as in Kamenica and Gentzkow (2011) and applied in Friedman et al. (2016). More generally, it is easy to show that commitment power exists when the receiver, given his options to design disclosure under some given biases b_1 and b_2, would receive a higher ex-ante utility. Call a specific Best CS equilibrium message m_1^C. When taking action y after observing the message pair (m_1^C, m_2), R's utility is

$$U_R^C = \max_{\hat{y}} \int_0^1 U_R(y, \theta) P(\theta | m_1^C, m_2) d\theta,$$

and thus higher compared to playing Best CS equilibrium strategies with S_1 for all messages for which he would not commit. In that latter case, he plays a strategy profile where he does not best respond to sender 2 but only to sender 1. Note that for messages m_1 for which the R does not consult Sender 2, he plays the CS game with S_1 and receives

$$U_R^{CS_1} \max_{y} \int_0^1 U_R(y, \theta) P(\theta | m_1) d\theta.$$

Some of the existing theories have explained the empirical puzzle of upward biased analysts by taking into account reputation effects. While such effects play a role for an analyst's career there is no reason to assume that investors cannot "de-bias" the disclosure of analysts. In other words, for most upward bias combinations there exist equilibria with two senders and commitment power that outperform the above mentioned CS equilibrium played with the less biased sender alone. Put differently: it may pay for investors to consult two analysts, even if the second is more biased than the first.

4 Conclusions

We have characterized a biased-ranked communication game where the less biased is given more information concerning the investor's decision rule, attributing a reduced influence to the more biased sender. In this way, we have accounted for a naturally emerging setup of expertise with two analysts. Such a setup may be considered intuitive when an institutional analyst regularly discloses upward biased earning forecasts and the investor profits from additionally consulting a less-biased analyst when relying on simultaneous disclosure. As typical for CS games, our game has multiple equilibria, among those also equilibria with commitment power. When committing to only consider a second analyst's disclosure for specific messages disclosed by the first, less biased analyst, the investor may make more informed decisions. The result is robust as the disclosure rule to play Best CS equilibrium with Sender 1 is known to the latter, and Sender 2's disclosure is effectively exogenous. In fact, talk is no longer "cheap" in the classic sense as the receiver has commitment power.

Our conclusion confirms and underpins the view illustrated in the introduction, namely that financial markets work, and this because of the informational contribution of financial analysts. When information is transmitted in a structured communication game, an investor can ultimately gain from consulting more than one upward biased sender. In other words, the idea that an investor is capable of "de-biasing" overly optimistic recommendation is natural, it directly already follows from the information disclosure process.

Our contribution also adds a new aspect to the literature on strategic information transmission with many senders. Although similar in spirit to the findings of Shimizu (2014), it adds additional value to games with upward biases. As illustrated in Section 2, the multisender literature so far has either shown that (1) with opposing biases, there is disagreement that could permit the receiver to learn the state perfectly (as in the FRE concept) or to further improve information quality, while (2) this is not possible for upward (or "like") biased experts as we observe in the situation typical for financial expertise, as shown in KM. While KM rests on several assumptions, including on information being transmitted sequentially, our contribution is less restrictive as it keeps disclosure simultaneous, including options of collecting information from analysts that arise naturally when an investor seeks information from multiple sources.

With our findings we open up additional avenues for research in financial disclosure as some further generalizations seem natural to expect. Given the growing literature on strategic information transmission that focuses on real-world settings where the consultation of several experts or players with upward biases is considered, our analysis quite naturally points toward some extensions. Consider the symmetric setup in which analysts are only aware of each other's existence by some probability. As we expect, this additional assumption might change our result only to a minor but still noteworthy extent, and confirm the direction of our analysis. Such work should broaden the applicability of multi-expert disclosure for the study of information transmission in finance. We leave these considerations for future research.

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Appendix: Noisy observation

In this appendix we relax the assumption made in the paper that both analysts would observe the true state truthfully but assume noisy observation with a correlated noise, occurring with probability ε. We therefore assume that, with probability $1-\varepsilon$, the analysts observe the true state perfectly, and with probability ε they observe a random state that is uniformly distributed over the state space $[0,1]$.

We know that the investor will best respond to any message indicating the boundaries for θ of a_1^{k-1}, a_1^k by updating his beliefs. Then, the action chosen will no longer follow beliefs according to which θ is uniformly distributed over the interval under consideration, but by "averaging" as follows, thus choosing action

$$y_\varepsilon^{a_i^{k-1},a_i^k} = (1-\varepsilon)\frac{a_1^{k-1}+a_1^k}{2} + \varepsilon\frac{1}{2}.$$

Reducing the following observation to this particular interval[11] one can express the first sender's expected utility by

$$E[U_1(\theta)] = -(1-\varepsilon)(\theta + b_1 - y_\varepsilon^{a_1^{k-1},a_1^k})^2 - \varepsilon\int_0^1 (\theta' + b_1 - y_\varepsilon^{a_1^{k-1},a_1^k})^2 \;\; d\theta'.$$

Because of the no-arbitrage condition in CS, sender types at the break point must be indifferent in inducing action $y_\varepsilon^{a_1^{k-1},a_1^k}$ and $y_\varepsilon^{a_1^k,a_1^{k+1}}$, which leads to

$$U_i(\theta, y_\varepsilon^{a_1^{k-1},a_1^k}) = U_i(\theta, y_\varepsilon^{a_1^k,a_1^{k+1}}).$$

In words, the last interval will grow by $\frac{4b}{1-\varepsilon}$ compared to the first. It is known from CS that intervals grow by $4b$ when moving one interval of the partition to the right. Under noise, partitional equilibria grow by $\frac{4b}{1-\varepsilon} > 4b$ for all $\varepsilon > 0$. While partitional equilibria are sustained under noise, they however become less informative compared to the situation without noise.

CS (p. 1441) furthermore have shown that, the number of equilibrium partitions are limited and given by b_i in the one-sender game, characterizing the largest number of steps in any equilibrium of one-sender CS model as

[11]This observation holds for each sender. With perfectly correlated noise the result is identical with the one-sender treatment, otherwise may, in the extreme, converge to the double of the loss compared to the one-sender setting.

$$N(b_i) = \left\lceil -\frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{2}{b}} \right\rceil,$$

where the number in brackets $\lceil z \rceil$ is the smallest integer that is greater than or equal to z. It is easy to show that under noise, the maximum number of steps in a partition reduces only slightly and proportional to ε:

$$N(b_i) = \left\lceil -\frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{(1-\varepsilon)2}{b}} \right\rceil.$$

Following CS (p. 1442) one can furthermore characterize for all $0 < \varepsilon < 1$ and all $0 < b_i < \frac{1}{4}$ the ex-ante utility of the receiver playing CS with noise:

$$EU_R = N(b_i) = -\frac{(1-\varepsilon)^2}{12N(b^2)} - \frac{b^2(N(b)^2 - 1)}{3\varepsilon^2} - \frac{\varepsilon(2-\varepsilon)}{12},$$

which is less than the receiver's expected utility without noise in the Pareto optimal equilibrium:

$$EU_R = N(b_i) = -\frac{1}{12N(b^2)} - \frac{(b^2)(N(b)^2 - 1)}{3}.$$

■